



$-7 \times -7 \times -7 \times -7 \times -7$  is written in exponential form as  $(-7)^5$

It is important that brackets are used in this example to avoid confusion with a similar question  $-7^5$  (This will be covered below).

## The Language of Exponents

The **power**  $a^n$  can be written in expanded form as:  $a^n = a \times a \times a \times a \times a \dots \times a$  [for  $n$  factors]

The **power**  $a^n$  consists of a **base**  $a$  and an **exponent** (or index)  $n$ .



## Saying Powers

$3^5$  can be said as '3 to the power of 5', '3 to the 5<sup>th</sup> power', '3 to the 5<sup>th</sup>' or '3 raised to 5'.

$4^9$  can be said as '4 to the power of 9', '4 to the 9<sup>th</sup> power', '4 to the 9<sup>th</sup>' or '4 raised to 9'.

With common exponents such as 2 and 3, special language is used.

$8^2$  is most commonly said as 8 squared.

$5^3$  is most commonly said as 5 cubed.

$(2a)^3$  is said as '2a all cubed' to emphasize that both the 2 and the  $a$  are being cubed.

## Powers containing numbers can be evaluated.

$$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$$

**Watch out!**

Many students will perform the multiplication  $3 \times 5 = 15$  which is incorrect.

$$2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$$

$$(2.35)^4 = 2.35 \times 2.35 \times 2.35 \times 2.35 = 30.498 \text{ (to 3 decimal places)}$$

$$(3a)^3 = 3a \times 3a \times 3a = 27a^3$$

Be careful with negative numbers!

$$(-4)^6 = -4 \times -4 \times -4 \times -4 \times -4 \times -4 = +4096$$

When solving  $-4^6$ , use the order of operation rules where the powers are performed before the multiplication.

$$-4^6 = -1 \times 4^6 = -1 \times 4096 = -4096$$

$$(-2)^4 = -2 \times -2 \times -2 \times -2 = 16$$

$$-2^4 = -1 \times 2^4 = -1 \times 16 = -16$$

## **Powers containing variable bases can be expanded**

$$x^4 = x \times x \times x \times x$$

$$ab^3 = a \times b \times b \times b$$

$$(ab)^4 = ab \times ab \times ab \times ab = a \times a \times a \times a \times b \times b \times b \times b$$

One last thing to note: when a variable does not appear to have an exponent, the exponent is 1.

$$a = a^1$$

$$x = x^1$$

$$de = (de)^1 \text{ or } d^1e^1$$

## Common Powers and Roots

Squares	Square Roots	Cubes	Cube Roots	Other Powers	Other Roots
$1^2 = 1$	$\sqrt{1} = 1$	$1^3 = 1$	$\sqrt[3]{1} = 1$	$1^4 = 1$	$\sqrt[4]{1} = 1$
$2^2 = 4$	$\sqrt{4} = 2$	$2^3 = 8$	$\sqrt[3]{8} = 2$	$2^4 = 16$	$\sqrt[4]{16} = 2$
$3^2 = 9$	$\sqrt{9} = 3$	$3^3 = 27$	$\sqrt[3]{27} = 3$	$3^4 = 81$	$\sqrt[4]{81} = 3$
$4^2 = 16$	$\sqrt{16} = 4$	$4^3 = 64$	$\sqrt[3]{64} = 4$	$4^4 = 256$	$\sqrt[4]{256} = 4$
$5^2 = 25$	$\sqrt{25} = 5$	$5^3 = 125$	$\sqrt[3]{125} = 5$	$5^4 = 625$	$\sqrt[4]{625} = 5$
$6^2 = 36$	$\sqrt{36} = 6$	$6^3 = 216$	$\sqrt[3]{216} = 6$		
$7^2 = 49$	$\sqrt{49} = 7$			$1^5 = 1$	$\sqrt[5]{1} = 1$
$8^2 = 64$	$\sqrt{64} = 8$			$2^5 = 32$	$\sqrt[5]{32} = 2$
$9^2 = 81$	$\sqrt{81} = 9$			$3^5 = 243$	$\sqrt[5]{243} = 3$
$10^2 = 100$	$\sqrt{100} = 10$				

## Calculators

Some calculators contain a ^ to calculate powers.

For example:  $2 \cdot 35^4 = 30.49800625$

Other calculators use a w key

For example:  $2 \cdot 35w4 = 30.49800625$

Other calculators use a  $\square^{\square}$  key

For example:  $2 \cdot 35 \square^{\square} 4 = 30.49800625$

## Module contents

### Introduction

- Exponent Rules – Whole Number Exponents
- Exponent Rules – Integer and Fraction Exponents
- Equations with Exponents
- Scientific Notation
- Operations with Numbers in Scientific Notation

### Answers to activity questions

## Outcomes

- Name the parts of a number written in exponential form.
- Simplify exponents using rules.
- Solve equations containing exponents.
- Express numbers in scientific notation.
- Perform operations with numbers in scientific notations.

## Check your skills

This module covers the following concepts, if you can successfully answer these questions, you do not need to do this module. Check your answers from the answer section at the end of the module.

- (a) Simplify:  
 $x^6 \times 4x^3 \times x$

(b) Simplify:  
 $a^6 \div a^0$

(c) Simplify:  
 $(x^3)^2 \times 4x^4 \div (2x^2)^4$
- (a) Simplify:  
 $(x^{-2})^2 \times \left(x^{\frac{1}{2}}\right)^4 \times x^0$

(b) Simplify:  
 $\sqrt[3]{512}$

(c) Simplify:  
 $(27x^9y^6)^{\frac{2}{3}}$
- (a) Solve for x:  
 $x^4 = 81$

(b) Solve for x:  
 $\sqrt[3]{x+1} = 7$

(c) Solve for r:  
 $V = \pi r^3$
- (a) Write 634 000 000 in Scientific Notation

(b) Write  $9.12 \times 10^{-5}$  in Standard Notation

(c) Write 0.000 000 55 in Scientific Notation

(d) Write  $7.25 \times 10^8$  in Standard Notation
- (a) Calculate  $3.2 \times 10^8$  multiplied by  $5.3 \times 10^{-5}$

(b) Calculate  $2.8 \times 10^8$  divided by  $7.2 \times 10^{-12}$

(c) Add  $7.25 \times 10^8$  and  $2.61 \times 10^4$

## Topic 1: Exponent rules - whole number exponents

### The product rule

To help develop the rule, let's consider the question below:

$$\begin{aligned}
 &2^5 \times 2^4 \\
 &\quad \text{5 factors} \qquad \qquad \text{4 factors} \\
 &= (2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2) \\
 &\quad \text{altogether gives 9 factors} \\
 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\
 &= 2^9
 \end{aligned}$$

Generalising this:

$$a^m \times a^n = a^{m+n}$$

(When multiplying two powers with the same base, add the exponents.)

Examples:

$5^4 \times 5^6$ $= 5^{4+6}$ $= 5^{10}$ or 9765625	$(-3)^2 \times (-3)^5$ $= (-3)^{2+5}$ $= (-3)^7$	$p^5 \times p^3 \times p$ $= p^5 \times p^3 \times p^1$ $= p^{5+3+1}$ $= p^9$
$2^5 \times 3^4$ cannot be done the bases are not the same	$3x^4 \times 8x^5$ $= 3 \times 8 \times x^{4+5}$ $= 24x^9$	$c^4 \times c^b$ $= c^{4+b}$
$2^7 + 2^2$ cannot be done the rule is for products	$(2b)^4 \times (2b)^8$ $= (2b)^{4+8}$ $= (2b)^{12}$	$-2ab^3 \times 4a^5b^3$ $= -2 \times 4 \times a^1 \times a^5 \times b^3 \times b^3$ $= -8a^6b^6$

## The quotient rule

To help develop the rule, let's consider the question below:

$$\begin{aligned}
 &2^7 \div 2^4 \\
 &= \frac{2^7}{2^4} \quad \leftarrow \text{write in fraction form} \\
 &= \frac{2 \times 2 \times 2 \times \cancel{2^1} \times \cancel{2^1} \times \cancel{2^1} \times \cancel{2^1}}{\cancel{2^1} \times \cancel{2^1} \times \cancel{2^1} \times \cancel{2^1}} \quad \leftarrow \text{simplify factors} \\
 &= 2^3
 \end{aligned}$$

Generalising this:

$$a^m \div a^n = a^{m-n}$$

(When dividing two powers with the same base, subtract the exponents.)

Examples:

$  \begin{aligned}  &5^8 \div 5^6 \\  &= \frac{5^8}{5^6} \\  &= 5^{8-6} \\  &= 5^2 \text{ or } 25  \end{aligned}  $	$  \begin{aligned}  &(-3)^9 \div (-3)^5 \\  &= \frac{(-3)^9}{(-3)^5} \\  &= (-3)^{9-5} \\  &= (-3)^4 \text{ or } 81  \end{aligned}  $	$  \begin{aligned}  &p^{15} \div p^3 \\  &= \frac{p^{15}}{p^3} \\  &= p^{15-3} \\  &= p^{12}  \end{aligned}  $
$2^5 \div 3^4$ cannot be done the bases are not the same	$  \begin{aligned}  &(2b)^{10} \div (2b)^5 \\  &= (2b)^{10-5} \\  &= (2b)^5  \end{aligned}  $	$  \begin{aligned}  &c^4 \div c^b \\  &= c^{4-b}  \end{aligned}  $
$2^7 - 2^2$ cannot be done the rule is for quotients	$  \begin{aligned}  &4x^8 \div 8x^5 \\  &= \frac{1 \cancel{4} x^8}{2 \cancel{8} x^5} \\  &= \frac{x^{8-5}}{2} \\  &= \frac{x^3}{2}  \end{aligned}  $	$  \begin{aligned}  &12a^8b^3 \div 4a^5b^3 \\  &= \frac{3 \cancel{12} a^8b^3}{1 \cancel{4} a^5b^3} \\  &= \frac{3a^{8-5}b^{3-3}}{1} \\  &= 3a^3b^0 \\  &\text{As } b^0 = 1 \text{ (see next section)} \\  &= 3a^3  \end{aligned}  $

## Zero exponent rule

To help develop the rule, let's consider the question below where the solution is obtained by exponents and by evaluating the exponents:

$$\begin{array}{lcl}
 2^5 \div 2^5 & & 2^5 \div 2^5 \\
 = \frac{2^5}{2^5} & \text{or alternatively} & = \frac{2^5}{2^5} \\
 = 2^{5-5} & & = \frac{32}{32} \\
 = 2^0 & & = 1
 \end{array}
 \quad \text{means that } 2^0 = 1$$

Generalising this for many different examples gives:

$$a^0 = 1$$

(A power with a zero exponent is equal to 1.)

Examples:

$5^0 = 1$	$(-3)^0 = 1$	$p^5 \times p^0$ $= p^5 \times 1$ $= p^5$
$(5a)^0 = 1$	$5a^0$ $= 5 \times 1$ $= 5$	$(4a^2c^8)^0$ $= 1$
$16x^6y^3 \div 4x^6y$ $= \frac{16x^6y^3}{4x^6y^1}$ $= 4x^{6-6}y^{3-1}$ $= 4x^0y^2$ $= 4y^2$	$1\ 234\ 567^0 = 1$	$4ab^3 \div 4ab^3$ $= \frac{4^1a^1b^3}{4^1a^1b^3}$ $= a^{1-1}b^{3-3}$ $= a^0b^0$ $= 1$



## The power rule

To help develop the rule, let's consider the question below:

$$\begin{aligned}
 &(2^5)^3 \\
 &= \overset{5 \text{ factors}}{(2 \times 2 \times 2 \times 2 \times 2)} \times \overset{5 \text{ factors}}{(2 \times 2 \times 2 \times 2 \times 2)} \times \overset{5 \text{ factors}}{(2 \times 2 \times 2 \times 2 \times 2)} \\
 &= \overset{\text{altogether gives } 3 \times 5 = 15 \text{ factors}}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} \\
 &= 2^{15}
 \end{aligned}$$

Generalising this:

$$\boxed{(a^m)^n = a^{m \times n}}$$

(When a power is raised to a power, multiply the exponents.)

Examples:

$  \begin{aligned}  &(3^4)^3 \\  &= 3^{4 \times 3} \\  &= 3^{12} \text{ or } 531441  \end{aligned}  $	$  \begin{aligned}  &\left[(-3)^2\right]^5 \\  &= (-3)^{2 \times 5} \\  &= (-3)^{10} \text{ or } 59049  \end{aligned}  $	$  \begin{aligned}  &(p^5)^3 \\  &= p^{5 \times 3} \\  &= p^{15}  \end{aligned}  $
$  \begin{aligned}  &(x^0)^4 & (x^0)^4 \\  &= x^{0 \times 4} & = 1^4 \\  &= x^0 & \text{ or } = 1 \\  &= 1  \end{aligned}  $	$  \begin{aligned}  &\left[(2x)^4\right]^4 \\  &= (2x)^{4 \times 4} \\  &= (2x)^{16}  \end{aligned}  $	$  \begin{aligned}  &(c^4)^b \\  &= c^{4 \times b} \\  &= c^{4b}  \end{aligned}  $
$  \begin{aligned}  &(x^6)^{\frac{1}{3}} \\  &= x^{6 \times \frac{1}{3}} \\  &= x^2  \end{aligned}  $	$  \begin{aligned}  &\left[(b^2)^4\right]^a \\  &= b^{2 \times 4 \times a} \\  &= b^{8a}  \end{aligned}  $	$  \begin{aligned}  &(c^{-4})^{-5} \\  &= c^{-4 \times -5} \\  &= c^{20}  \end{aligned}  $

## Two more useful power rules are

$$(ab)^m = a^m b^m \text{ or } a^m b^m = (ab)^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \text{ or } \frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

Examples:

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$$\begin{aligned}(3x)^3 \\&= 3^3 x^3 \\&= 27x^3\end{aligned}$$

$$\begin{aligned}4(xy)^2 \\&= 4x^2 y^2\end{aligned}$$

$$\begin{aligned}(4p^5)^3 \\&= 4^3 p^{5 \times 3} \\&= 64p^{15}\end{aligned}$$

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$$\begin{aligned}\left(\frac{5}{a}\right)^4 \\&= \frac{5^4}{a^4} \\&= \frac{625}{a^4}\end{aligned}$$

$$\begin{aligned}\left(\frac{a}{b}\right)^3 \\&= \frac{a^3}{b^3}\end{aligned}$$

$$\begin{aligned}\left(\frac{2x^2}{y^3}\right)^4 \\&= \frac{2^4 x^{2 \times 4}}{y^{3 \times 4}} \\&= \frac{16x^8}{y^{12}}\end{aligned}$$

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$$\begin{aligned}5(2x^6 y^2)^2 \\&= 5 \times 2^2 x^{6 \times 2} y^{2 \times 2} \\&= 5 \times 4x^{12} y^4 \\&= 20x^{12} y^4\end{aligned}$$

$$\begin{aligned}\left(\frac{3a^4}{b^3}\right)^2 \times \frac{4a^3}{b} \\&= \frac{3^2 a^{4 \times 2}}{b^{3 \times 2}} \times \frac{4a^3}{b} \\&= \frac{9a^8 \times 4a^3}{b^6 \times b^1} \\&= \frac{36a^{8+3}}{b^{6+1}} \\&= \frac{36a^{11}}{b^7}\end{aligned}$$

$$\begin{aligned}(3x^2 y^3)^2 \times 4(xy^4)^3 \\&= 3^2 x^{2 \times 2} y^{3 \times 2} \times 4x^3 y^{4 \times 3} \\&= 9x^4 y^6 \times 4x^3 y^{12} \\&= 36x^{4+3} y^{6+12} \\&= 36x^7 y^{18}\end{aligned}$$



[Video 'Exponent Rules – Whole Number Exponents'](#)

## Activity

1. Evaluate the following powers.

- |                      |                    |                  |
|----------------------|--------------------|------------------|
| (a) $3^5$            | (b) $5^3$          | (c) $7^3$        |
| (d) $2^9$            | (e) $14^3$         | (f) $5^5$        |
| (g) $3^0$            | (h) $(2^7)^0$      | (i) $(3^2)^3$    |
| (j) $2^4 \times 2^6$ | (k) $4^7 \div 4^5$ | (l) $-4^3$       |
| (m) $(-5)^3$         | (n) $0.85^6$       | (o) $4.5^{1.25}$ |
| (p) $(1.006)^{240}$  | (q) $(1.05)^{20}$  |                  |

2. Simplify the following (leave your answer in power form).

- |  |  |                                       |
|--|--|---------------------------------------|
| (a) $5^2 \times 5^5$                       | (b) $m^4 \times m^5$                   | (c) $a^4 \times a^3 \times a$         |
| (d) $10^4 \times 10 \times 10^0$           | (e) $\frac{r^4}{2} \times \frac{r}{4}$ | (f) $5c^2 \times 3c^4$                |
| (g) $a^2b^4 \times a^3bc$                  | (h) $4x^3y^4z^2 \times 5x^2y$          | (i) $5^2e f^3g^5 \times 5^3e^2f^2g^0$ |
| (j) $4^9 \div 4^5$                         | (k) $x^8 \div x^5$                     | (l) $h^5 \div h$                      |
| (m) $3^x \div 3^4$                         | (n) $\frac{16a^5}{4a^2}$               | (o) $\frac{5a^2b^4}{25a^2b}$          |
| (p) $a^{\frac{1}{2}} \div a^{\frac{1}{4}}$ | (q) $(2^4)^3$                          | (r) $(5^4)^{\frac{1}{2}}$             |
| (s) $(b^4)^5$                              | (t) $(3t)^4$                           | (u) $(2a^4b^3)^3$                     |
| (v) $\left(\frac{4}{y}\right)^3$           | (w) $\frac{(2y^4)^2}{(2x^2)^4}$        | (x) $(2^5)^b$                         |

3. Simplify the following.

- |  |   |   |
|--|---|---|
| (a) $(a^2)^4 \times a^3$                     | (b) $\frac{(3y^4)^2}{2y^5}$   | (c) $\frac{2x^5 \times x^4}{(x^2)^3}$                         |
| (d) $3b^0 + (3b)^0$                          | (e) $\frac{(3n)^2 \times 3n^2}{9n^4}$                                   | (f) $5(mn)^2 \times (mn^2)^3$                                 |
| (g) $m^2n^5p^7 \times m^3p^4 \times n^2p$    | (h) $-4(a^2b^5)^0$  | (i) $2a^2b^3 \times 3ab^4$                                    |
| (j) $\frac{2x^5y^4}{10x^2y^3}$               | (k) $\left(\frac{2x^3}{y^4}\right)^4 \times \left(\frac{y}{x}\right)^2$ | (l) $4a^0 - \left(\frac{a}{4}\right)^0$                       |
| (m) $\frac{25p^6}{36q^8} = \left(-\right)^2$ | (n) $27a^6b^9c^3 = (\quad)^3$   | (o) $\frac{(3xy^2)^3}{2x(2y)^2} \div \frac{6xy^0}{4(xy^2)^3}$ |

## Topic 2: Exponent rules – integers and fraction exponents

### Negative exponents

To help develop the rule, let's consider the question below:

$$\frac{4}{32} \quad - \quad \text{both the 4 and the 32 are powers of 2.}$$

This question can be solved two ways: by normal fraction methods and using exponents. Because the starting question is the same, the answers must also be the same.

$\frac{4^1}{32^8}$ $= \frac{1}{8}$ $= \frac{1}{2^3}$	<p>Question the same</p> $\longleftrightarrow$	$\frac{4}{32}$ $= \frac{2^2}{2^5}$ $= 2^{2-5}$ $= 2^{-3}$
<p>Answers must be the same</p> $\longleftrightarrow$		

This means that:  $\frac{1}{2^3} = 2^{-3}$

Generalising this:

$a^{-m} = \frac{1}{a^m} \quad \text{or} \quad \frac{1}{a^{-m}} = a^m$
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One good way to remember this rule is to say

**'Take the reciprocal and change the sign of the exponent'**

#### Note:

There is a convention in mathematics that answers to questions should be expressed with positive exponents only.

When solving a question, use the rules in the previous section first (if possible), if an answer still contains a negative exponent then use the rule above to express with positive exponents.

Examples:

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$$\begin{aligned}
 &3^{-4} \\
 &= \frac{3^{-4}}{1} \\
 &= \frac{1}{3^4} \text{ or } \frac{1}{81}
 \end{aligned}$$

$$\begin{aligned}
 &\frac{1}{5^{-3}} \\
 &= \frac{5^3}{1} \\
 &= 5^3 \text{ or } 125
 \end{aligned}$$

$$\begin{aligned}
 &\left(\frac{2}{7}\right)^{-2} \\
 &= \left(\frac{7}{2}\right)^2 \\
 &= \frac{49}{4}
 \end{aligned}$$


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$$\begin{aligned}
 &\frac{2^{-3}}{3^{-2}} \\
 &= \frac{3^2}{2^3} \\
 &= \frac{9}{8} \text{ or } 1\frac{1}{8}
 \end{aligned}$$

$$\begin{aligned}
 &4^{-1}e^2f^{-3} \\
 &= \frac{1}{4^1}e^2\frac{1}{f^3} \\
 &= \frac{e^2}{4f^3}
 \end{aligned}$$

$$\begin{aligned}
 &\left(\frac{a^3}{4}\right)^{-2} \\
 &= \left(\frac{4}{a^3}\right)^2 \\
 &= \frac{4^2}{a^6}
 \end{aligned}$$


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$$\begin{aligned}
 &2(p^{-3})^2q^4 \times (4p^2q^{-3})^{-4} \\
 &= 2p^{-3 \times 2}q^4 \times (2^2)^{-4}p^{2 \times -4}q^{-3 \times -4} \\
 &= 2^1p^{-6}q^4 \times 2^{-8}p^{-8}q^{12} \\
 &= 2^{1+^{-8}}p^{-6+^{-8}}q^{4+12} \\
 &= 2^{-7}p^{-14}q^{16} \\
 &= \frac{q^{16}}{2^7p^{14}} \text{ or } \frac{q^{16}}{128p^{14}}
 \end{aligned}$$

$$\begin{aligned}
 &\frac{2x^{-1}y^3}{3x^3y} \div \frac{xy^{-2}}{x^{-2}y} \\
 &= \frac{2x^{-1}y^3}{3x^3y^1} \times \frac{x^{-2}y^1}{x^1y^{-2}} \\
 &= \frac{2x^{-1+^{-2}}y^{3+1}}{3x^{3+1}y^{1+^{-2}}} \\
 &= \frac{2x^{-3}y^4}{3x^4y^{-1}} \\
 &= \frac{2}{3}x^{-3-4}y^{4-^{-1}} \\
 &= \frac{2}{3}x^{-7}y^5 \\
 &= \frac{2y^5}{3x^7}
 \end{aligned}$$

$$\begin{aligned}
 &\frac{(x^2y)^{-2}}{x^{-4}y} \\
 &= \frac{x^{2 \times -2}y^{1 \times -2}}{x^{-4}y} \\
 &= \frac{x^{-4}y^{-2}}{x^{-4}y^1} \\
 &= x^0y^{-2-1} \\
 &= y^{-3} \\
 &= \frac{1}{y^3}
 \end{aligned}$$


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## Fractional exponents

To help develop the rule, let's consider the information below:

We know using exponents that:  $x^{\frac{1}{3}} \times x^{\frac{1}{3}} \times x^{\frac{1}{3}} = x^{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = x^1 = x$

From the ideas about roots of numbers:  $\sqrt[3]{x} \times \sqrt[3]{x} \times \sqrt[3]{x} = x$

That means that  $x^{\frac{1}{3}} = \sqrt[3]{x}$

Generalising this:

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

(This is for unit fractions)

Before proceeding with some examples, let's have a look at the meaning of different roots.

Square root: the square root of a number is usually written as  $\sqrt{n}$  but more correctly should be written as  $\sqrt[2]{n}$ , the 2 indicating a relationship to squaring.

Square root of a number: for example  $\sqrt{36} = 6$ , because  $6 \times 6 = 6^2 = 36$ .

Cube root of a number: for example  $\sqrt[3]{125} = 5$ , because  $5 \times 5 \times 5 = 5^3 = 125$ .

Forth root of a number: for example  $\sqrt[4]{16} = 2$ , because  $2 \times 2 \times 2 \times 2 = 2^4 = 16$ .

Fifth root of a number: for example  $\sqrt[5]{243} = 3$ , because  $3 \times 3 \times 3 \times 3 \times 3 = 3^5 = 243$ .  
etc.

## Using Calculators

Calculators vary in the way roots are calculated.

Most calculators find square roots by:  $\sqrt{\square}$  2 5 = 5

Most calculators find cube roots by:  $(\sqrt[3]{\square})$  8 = 2

To find any root, most calculators use  $\sqrt[x]{\square}$

To find  $\sqrt[4]{625}$ , do 4 ( $\sqrt[x]{\square}$ ) 6 2 5 = 5

**Another option** is to use the 'raising to an exponent' key ( $x^{\square}$  or w) using the fraction or changing the fraction to a decimal:  $\sqrt[4]{625} = (625)^{\frac{1}{4}} = (625)^{0.25}$ , do 6 2 5  $x^{\square}$  0 . 2 5 = 5 or

6 2 5  $x^{\square}$  1  4 = 5

Examples:

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$$25^{\frac{1}{2}}$$

$$= \sqrt{25}$$

$$= 5$$

$$256^{\frac{1}{4}}$$

$$= \sqrt[4]{256}$$

$$= 4$$

$$256^{\frac{1}{4}}$$

$$= (2^8)^{\frac{1}{4}}$$

$$= 2^{8 \times \frac{1}{4}}$$

$$= 2^2 \text{ or } 4$$

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$$(20)^{\frac{1}{4}}$$

$$= 2.115 \text{ (to 3 d.p.)}$$

By calculator

$$(p^6)^{\frac{1}{3}}$$

$$= p^{6 \times \frac{1}{3}}$$

$$= p^2$$

$$(1.515\bar{3})^{\frac{1}{52}}$$

$$= \sqrt[52]{1.515\bar{3}}$$

$$= 1.008 \text{ to 3 d.p.}$$

By calculator

---


$$\left(\frac{9}{16}\right)^{\frac{1}{2}}$$

$$= \sqrt{\frac{9}{16}}$$

$$= \frac{\sqrt{9}}{\sqrt{16}}$$

$$= \frac{3}{4}$$

$$(9x^4)^{\frac{1}{2}}$$

$$= 9^{\frac{1}{2}} x^{4 \times \frac{1}{2}}$$

$$= \sqrt{9} x^2$$

$$= 3x^2$$

$$\sqrt{64x^{64}}$$

$$= \sqrt{64} \sqrt{x^{64}}$$

$$= 8x^{64 \times \frac{1}{2}}$$

$$= 8x^{32}$$

---


$$100^{-\frac{1}{2}}$$

$$= \frac{1}{100^{\frac{1}{2}}}$$

$$= \frac{1}{\sqrt{100}}$$

$$= \frac{1}{10}$$

$$\left(\frac{16x^4}{81y^8}\right)^{\frac{1}{4}}$$

$$= \left(\frac{2^4 x^4}{3^4 y^8}\right)^{\frac{1}{4}}$$

$$= \frac{2^{4 \times \frac{1}{4}} x^{4 \times \frac{1}{4}}}{3^{4 \times \frac{1}{4}} y^{8 \times \frac{1}{4}}}$$

$$= \frac{2^1 x^1}{3^1 y^2}$$

$$= \frac{2x}{3y^2}$$

$$\left(\frac{16x^4}{81y^8}\right)^{\frac{1}{4}}$$

$$= \frac{\sqrt[4]{16x^{4 \times \frac{1}{4}}}}{\sqrt[4]{81y^{8 \times \frac{1}{4}}}}$$

$$= \frac{2x^1}{3y^2}$$

$$= \frac{2x}{3y^2}$$


---

If the exponent is a non-unit fraction, the power of a power rule can assist.

$$a^{\frac{2}{3}} = (a^2)^{\frac{1}{3}} = \sqrt[3]{a^2}$$

Generalising this:

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \text{or} \quad \left(\sqrt[n]{a}\right)^m$$

Examples:

Evaluate  $8^{\frac{2}{3}}$

This question can be done in three ways.

In the first method, 8 is written in power form as  $2^3$  and the power of a power rule is used.

$$\begin{aligned} 8^{\frac{2}{3}} &= \left(2^3\right)^{\frac{2}{3}} \\ &= 2^{3^1 \times \frac{2}{3^1}} \\ &= 2^2 \\ &= 4 \end{aligned}$$

The second method uses a calculator with a fraction exponent.

$$\begin{aligned} 8^{\frac{2}{3}} &= \left(8^2\right)^{\frac{1}{3}} \\ &= \sqrt[3]{8^2} \\ &= ({}^3\sqrt{\phantom{x}}) \quad 8 \quad x^2 \quad = 4 \end{aligned}$$

The third method involves changing the fraction exponent to a decimal.

$$\begin{aligned} 8^{\frac{2}{3}} &= 8^{0.666666666666} \\ &= 4 \end{aligned}$$

Calculator sequence:  $8 \quad x^{\frac{\square}{\square}} \quad 2 \quad = 4$

Calculator sequence:  $8 \quad x^{\frac{\square}{\square}} \quad 0.6666666666 \quad = 4$

It is important to fill the display of your calculator with the recurring digit. Be careful with calculating with recurring decimals.

Evaluate  $243^{\frac{3}{5}}$

$$\begin{aligned} 243^{\frac{3}{5}} &= \left(3^5\right)^{\frac{3}{5}} \\ &= 3^{5^1 \times \frac{3}{5^1}} \\ &= 3^3 \\ &= 27 \end{aligned}$$

$$\begin{aligned} 243^{\frac{3}{5}} &= \left(243^3\right)^{\frac{1}{5}} \\ &= \sqrt[5]{243^3} \\ &= ({}^5\sqrt{\phantom{x}}) \quad 2 \quad 4 \quad 3 \quad x^3 \quad = 27 \end{aligned}$$

$$\begin{aligned} 243^{\frac{3}{5}} &= 243^{0.6} \\ &= 27 \end{aligned}$$

Calculator sequence:  $2 \quad 4 \quad 3 \quad x^{\frac{\square}{\square}} \quad 0.6 \quad = 27$

or

Calculator sequence:  $2 \quad 4 \quad 3 \quad w \quad 0 \quad 0.6 \quad = 27$

Evaluate  $50^{\frac{2}{5}}$

As 50 cannot be written in power form, only a calculator solution is possible.

$$50^{\frac{2}{5}}$$

$$\begin{aligned} 50^{\frac{2}{5}} &= \sqrt[5]{50^2} \end{aligned}$$

Calculator sequence:  $({}^5\sqrt{\phantom{x}}) \quad 5 \quad 0 \quad x^2 \quad = 4.781762499$

$$\begin{aligned} 50^{\frac{2}{5}} &= 50^{0.4} \end{aligned}$$

Calculator sequence:  $5 \quad 0 \quad x^{\frac{\square}{\square}} \quad 0.4 \quad = 4.781762499$

Calculator sequence:  $5 \quad 0 \quad x^{\frac{\square}{\square}} \quad 2 \quad =$



Evaluate  $128^{-\frac{4}{7}}$ 

$$\begin{aligned}
 128^{-\frac{4}{7}} &= \frac{1}{128^{\frac{4}{7}}} \\
 &= \frac{1}{(2^7)^{\frac{4}{7}}} \\
 &= \frac{1}{2^{7 \times \frac{4}{7}}} \\
 &= \frac{1}{2^4} \\
 &= \frac{1}{16} \text{ or } 0.0625
 \end{aligned}$$

$$\begin{aligned}
 128^{-\frac{4}{7}} &= \sqrt[7]{128^{-4}} \\
 &= 0.0625
 \end{aligned}$$

Calculator input:  $7 \text{ (}\sqrt[7]{\square}\text{)} 1 2 8 x^{\square} (-) 4 = 0.0625$

$$\begin{aligned}
 128^{-\frac{4}{7}} &= 128^{-0.571428571} \\
 &= 0.0625
 \end{aligned}$$

Calculator input:  $1 2 8 x^{\square} (-) 0 . 5 7 1 4 2 8 5 7 1 = 0.0625$

or

$$\begin{aligned}
 &= 0.0625
 \end{aligned}$$

Calculator input:  $1 2 8 x^{\square} = (-) 4 = 0.0625$

Simplify  $(625a^4b^8)^{\frac{3}{4}}$ 

$$\begin{aligned}
 (625a^4b^8)^{\frac{3}{4}} &= 5^{1 \times \frac{3}{4}} a^{4 \times \frac{3}{4}} b^{8 \times \frac{3}{4}} \\
 &= 5^{\frac{3}{4}} a^3 b^6 \\
 &= 125a^3b^6
 \end{aligned}$$

$$\begin{aligned}
 (625a^4b^8)^{\frac{3}{4}} &= \sqrt[4]{625^3 a^{12} b^{24}} \\
 &= 125a^3b^6
 \end{aligned}$$

$$\begin{aligned}
 (625a^4b^8)^{\frac{3}{4}} &= 625^{0.75} a^{1 \times \frac{3}{4}} b^{8 \times \frac{3}{4}} \\
 &= 125a^3b^6
 \end{aligned}$$

$$\begin{aligned}
 &= 125
 \end{aligned}$$

Calculator input:  $4 \text{ (}\sqrt[4]{\square}\text{)} 6 2 5 x^{\square} 3 = 125$

$$\begin{aligned}
 &= 12
 \end{aligned}$$

Calculator input:  $6 2 5 x^{\square} 0 . 7 5 = 12$

Simplify  $\sqrt[3]{64a^3b^5}$ 

$$\begin{aligned}
 \sqrt[3]{64a^3b^5} &= (64a^3b^5)^{\frac{1}{3}} \\
 &= 4^{\frac{1}{3} \times \frac{1}{1}} a^{\frac{1}{3} \times \frac{1}{1}} b^{5 \times \frac{1}{3}} \\
 &= 4ab^{\frac{5}{3}}
 \end{aligned}$$

$$\begin{aligned}
 \sqrt[3]{64a^3b^5} &= \sqrt[3]{64(a^3b^5)} \\
 &= 4a^{\frac{1}{3} \times \frac{1}{1}} b^{5 \times \frac{1}{3}} \\
 &= 4ab^{\frac{5}{3}}
 \end{aligned}$$

$$\begin{aligned}
 \sqrt[3]{64a^3b^5} &= (64a^3b^5)^{\frac{1}{3}} \\
 &= 64^{0.333} a^{\frac{1}{3} \times \frac{1}{1}} b^{5 \times \frac{1}{3}} \\
 &= 4ab^{\frac{5}{3}}
 \end{aligned}$$

$$\begin{aligned}
 &= 4
 \end{aligned}$$

Calculator input:  $3 \text{ (}\sqrt[3]{\square}\text{)} 6 4 = 4$

$$\begin{aligned}
 &= 4
 \end{aligned}$$

Calculator input:  $6 4 x^{\square} 0 . 3 3 3 = 4$

---

Simplify  $\left(\frac{625a^6}{b^8}\right)^{-\frac{3}{4}}$

$$\begin{aligned} & \left(\frac{b^8}{625a^6}\right)^{\frac{3}{4}} \\ &= \left(\frac{b^8}{5^4a^6}\right)^{\frac{3}{4}} \\ &= \frac{b^{8^2 \times \frac{3}{4}}}{5^{4^1 \times \frac{3}{4}} a^{6^3 \times \frac{3}{4^2}}} \\ &= \frac{b^6}{5^3 a^{\frac{9}{2}}} \text{ or } \frac{b^6}{125a^{\frac{9}{2}}} \end{aligned}$$

$$\begin{aligned} & \left(\frac{b^8}{625a^6}\right)^{\frac{3}{4}} \\ &= \left(\frac{b^8}{625^{\frac{3}{4}}a^6}\right)^{\frac{3}{4}} \\ &= \frac{b^{8^2 \times \frac{3}{4^1}}}{\sqrt[4]{625^3} a^{6^3 \times \frac{3}{4^2}}} \\ &= \frac{b^6}{125a^{\frac{9}{2}}} \end{aligned}$$

$$\begin{aligned} & \left(\frac{b^8}{625a^6}\right)^{\frac{3}{4}} \\ &= \left(\frac{b^8}{625^{\frac{3}{4}}a^6}\right)^{\frac{3}{4}} \\ &= \frac{b^{8^2 \times \frac{3}{4^1}}}{625^{0.75} a^{6^3 \times \frac{3}{4^2}}} \\ &= \frac{b^6}{125a^{\frac{9}{2}}} \end{aligned}$$


 6 2 5 x<sup>3</sup> ÷ 3  
 4 = 125

6 2 5 x<sup>3</sup> 0 ÷ 7 5 = 125

4 (√) 6 2 5 x<sup>3</sup> 3  
 = 125

---



[Video 'Exponent Rules – Integer & Fraction Exponents'](#)

## Activity

### 1. Evaluate

- |                        |                                     |                          |
|------------------------|-------------------------------------|--------------------------|
| (a) $36^{-1}$          | (b) $5^{-2}$                        | (c) $10^{-4}$            |
| (d) $\frac{1}{4^{-3}}$ | (e) $\left(\frac{1}{4}\right)^{-4}$ | (f) $\frac{5^{-2}}{2^3}$ |
| (g) $7 \times 2^{-4}$  | (h) $(3 \times 2^{-2})^{-2}$        |                          |

### 2. Simplify

- |                                      |  |   |
|--------------------------------------|--|---|
| (a) $a^{-4}$                         | (b) $c^{-7}$                                   | (c) $\frac{1}{x^{-2}}$  |
| (d) $2x^{-3}$                        | (e) $(3b)^{-2}$                                | (f) $ab^{-1}$   |
| (g) $a^3 \times b^{-4}$              | (h) $4p^{-1}$                                  | (i) $(4p)^{-1}$   |
| (j) $(3^{-1}ab^{-2})^3$              | (k) $4p^2q^{-3} \times 3p^{-1}q^2$             | (l) $\left(\frac{4}{b^2}\right)^{-3}$                                     |
| (m) $\frac{(xy^3)^{-4}}{4x^4y^{-2}}$ | (n) $\frac{2(x^2y^{-1})^{-3}}{(3x^{-1}y^3)^2}$ | (o) $\frac{2x^2}{3y^{-3}} \times \left(\frac{4xy^2}{9x^{-4}}\right)^{-1}$ |

### 3. Evaluate without using a calculator

- |                                  |   |  |
|----------------------------------|---|--|
| (a) $49^{\frac{1}{2}}$           | (b) $8^{\frac{2}{3}}$                           | (c) $32^{\frac{3}{5}}$                         |
| (d) $625^{\frac{1}{4}}$          | (e) $1024^{\frac{7}{10}}$                       | (f) $256^{-\frac{1}{4}}$                       |
| (g) $16^{-\frac{1}{2}}$          | (h) $1024^{-\frac{7}{10}}$                      | (i) $\frac{3}{27^{-\frac{2}{3}}}$              |
| (j) $7 \times 128^{\frac{3}{7}}$ | (k) $\frac{16^{\frac{3}{4}}}{27^{\frac{1}{3}}}$ | (l) $36^{\frac{1}{2}} \times 32^{\frac{3}{5}}$ |

### 4. Use a calculator to answer the following

- |                                  |                         |                               |
|----------------------------------|-------------------------|-------------------------------|
| (a) $10^{\frac{3}{4}}$           | (b) $450^{\frac{1}{7}}$ | (c) $1.864297^{\frac{1}{10}}$ |
| (d) $2.1209527^{-\frac{1}{240}}$ | (e) $4.95^{2.5}$        | (f) $1.01^{-0.75}$            |

### 5. Simplify

- |  |  |   |
|--|--|---|
| (a) $\left(\frac{8}{a^6}\right)^{\frac{1}{3}}$ | (b) $\frac{4(a^6b^3)^{\frac{1}{3}}}{(16a^8b^4)^{\frac{1}{4}}}$ | (c) $\left(\frac{a^2b^6}{16}\right)^{-\frac{1}{2}}$ |
| (d) $\sqrt[3]{64a^2b^4}$                       | (e) $\sqrt[4]{32(a^3b^2)^2}$                                   | (f) $\frac{\sqrt{x^2y^6}}{\sqrt[3]{x^6y^3}}$        |

## Topic 3: Equations with exponents

Before starting to solve equations containing exponents, it is important to think about opposite operations just like in the Equations Topic.

You will recall the opposites below:

Operation	Opposite Operation
<b>Addition +</b>	Subtraction -
<b>Subtraction -</b>	Addition +
<b>Multiplication x</b>	Division ÷
<b>Division ÷</b>	Multiplication x
<b>Squaring <math>n^2</math></b>	Square root $\sqrt{n}$
<b>Square root <math>\sqrt{n}</math></b>	Squaring $n^2$

Revision example:

$2x^2 + 3 = 11$ $2x^2 + 3 - 3 = 11 - 3 \rightarrow$ take 3 from both sides $2x^2 = 8$ $\frac{2x^2}{2} = \frac{8}{2} \rightarrow$ divide both sides by 2 $x^2 = 4$ $\sqrt{x^2} = \pm\sqrt{4} \rightarrow$ take square root of both sides $x = \pm 2$	$2x^2 + 3 = 11$ $2x^2 = 11 - 3 \rightarrow +3$ becomes $-3$ on the RHS $2x^2 = 8$ $x^2 = \frac{8}{2} \rightarrow \times 2$ becomes $\div 2$ on the RHS $x^2 = 4$ $x = \pm\sqrt{4} \rightarrow^2$ becomes $\sqrt{\quad}$ on the RHS $x = \pm 2$
---	--

There are two solutions here because both 2 and -2 satisfy the equation. This always occurs with even roots.

This list can be expanded to cubes, cube roots, to the 4, forth root, etc.

Operation	Opposite Operation
<b>Cubing ( )<sup>3</sup></b>	Cube root ( $\sqrt[3]{\phantom{x}}$ ) or $\frac{1}{3}$
<b>To the 4<sup>th</sup> power</b>	Forth root ( $\sqrt[4]{\phantom{x}}$ ) or $\frac{1}{4}$
<b>To the 5<sup>th</sup> power</b>	Fifth root ( $\sqrt[5]{\phantom{x}}$ ) or $\frac{1}{5}$
<b>To the 6<sup>th</sup> power</b>	Sixth root ( $\sqrt[6]{\phantom{x}}$ ) or $\frac{1}{6}$
<b>To the 7<sup>th</sup> power</b>	Seventh root ( $\sqrt[7]{\phantom{x}}$ ) or $\frac{1}{7}$
<b>etc.</b>	<b>etc.</b>
<b>To the power <math>\frac{2}{3}</math></b>	To the power $\frac{3}{2}$
<b>To the power <math>\frac{3}{5}</math></b>	To the power $\frac{5}{3}$

Examples: Solve for  $x$

Both sides method	Short -cut Method
$x^4 = 256$ $\sqrt[4]{x^4} = \sqrt[4]{256} \rightarrow$ take the fourth root of both sides $x = \pm 4$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>There are two solutions here because both 4 and -4 satisfy the equation. This always occurs with even roots</p> </div>	$x^4 = 256$ $x = \sqrt[4]{256} \rightarrow$ $\sqrt[4]{\phantom{x}}$ becomes $\sqrt[4]{\phantom{x}}$ on the RHS $x = \pm 4$
$\sqrt[6]{x} = 3$ $(\sqrt[6]{x})^6 = 3^6$ raise both sides to the power of 6 $x = 729$	$\sqrt[6]{x} = 3$ $x = 3^6 \rightarrow \sqrt[6]{\phantom{x}}$ becomes $\phantom{x}^6$ on the RHS $x = 729$
$3\sqrt[5]{x} - 1 = 5$ $3\sqrt[5]{x} - 1 + 1 = 5 + 1 \rightarrow$ add 1 to both sides $3\sqrt[5]{x} = 6$ $\frac{3\sqrt[5]{x}}{3} = \frac{6}{3} \rightarrow$ divide both sides by 3 $\sqrt[5]{x} = 2$ $(\sqrt[5]{x})^5 = 2^5 \rightarrow$ raise both side to the power of 5 $x = 32$	$3\sqrt[5]{x} - 1 = 5$ $3\sqrt[5]{x} = 5 + 1 \rightarrow$ - 1 becomes + 1 on the RHS $3\sqrt[5]{x} = 6$ $\sqrt[5]{x} = \frac{6}{3} \rightarrow$ $\times$ by 3 becomes $\div$ by 3 on the RHS $\sqrt[5]{x} = 2$ $x = 2^5 \rightarrow \sqrt[5]{\phantom{x}}$ becomes $(\phantom{x})^5$ on the RHS $x = 32$

$4x^{-2} = 40$ $\frac{4^1 x^{-2}}{4^1} = \frac{40^{10}}{4^1} \rightarrow$ divide both sides by 4 $x^{-2} = 10 \rightarrow$ raise both sides to the $-\frac{1}{2}$ $(x^{-2})^{-\frac{1}{2}} = 10^{-\frac{1}{2}}$ $x = \frac{1}{10^{\frac{1}{2}}}$ $x = \frac{1}{\sqrt{10}}$ $x = \pm 0.3162$ ( to 4 d.p.)	$4x^{-2} = 40$ $x^{-2} = \frac{40^{10}}{4^1} \rightarrow$ $\times 4$ becomes $\div 4$ on the RHS $x^{-2} = 10$ $x = 10^{-\frac{1}{2}} \rightarrow$ $^{-2}$ becomes $\frac{1}{2}$ on the RHS $x = \frac{1}{\sqrt{10}}$ $x = \pm 0.3162$ ( to 4 d.p.)
$(2x-1)^2 = 49$ $\sqrt{(2x-1)^2} = \sqrt{49} \rightarrow$ take square root of both sides $2x-1 = \pm 7$ $2x-1+1 = 7+1$ or $-7+1 \rightarrow$ add 1 to both sides $2x = 8$ or $-6$ $\frac{2x}{2} = \frac{8}{2}$ or $\frac{-6}{2} \rightarrow$ divide both sides by 2 $x = 4$ or $-3$	$(2x-1)^2 = 49$ $2x-1 = \sqrt{49} \rightarrow$ $^2$ becomes $\sqrt{\quad}$ on the RHS $2x-1 = \pm 7$ $2x = 7+1$ or $-7+1 \rightarrow$ $-1$ becomes $+1$ on the RHS $2x = 8$ or $-6$ $x = \frac{8}{2}$ or $\frac{-6}{2} \rightarrow$ $\times 2$ becomes $\div 2$ on the RHS $x = 4$ or $-3$
In the question below, the opposite of raising a number to a power is raising it to the reciprocal power. Remember a number multiplied by its reciprocal gives 1.	In the question below, the fraction exponent becomes the reciprocal on the other side of the equation.
$x^{\frac{2}{3}} = 50$ $\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = 50^{\frac{3}{2}}$ $x = \sqrt{50^3}$ $x = 353.55$	$x^{\frac{2}{3}} = 50$ $x = 50^{\frac{3}{2}}$ $x = \sqrt{50^3}$ $x = 353.55$

It is also important to be able to rearrange formulae that contain powers.

Rearrange the formulae below to make the pronumeral in brackets the subject.	
$A = \pi r^2 \quad [r]$ $\frac{A}{\pi} = \frac{\pi^1 r^2}{\pi^1} \rightarrow$ divide both sides by $\pi$ $\frac{A}{\pi} = r^2 \rightarrow$ take the square root of both sides $\sqrt{\frac{A}{\pi}} = \sqrt{r^2}$ $r = \sqrt{\frac{A}{\pi}}$	$A = \pi r^2 \quad [r]$ $\frac{A}{\pi} = r^2 \rightarrow$ $\times \pi$ becomes $\div \pi$ on LHS $\sqrt{\frac{A}{\pi}} = r \rightarrow$ $^2$ becomes $\sqrt{\quad}$ on the LHS $r = \sqrt{\frac{A}{\pi}}$

$V = \frac{4}{3}\pi r^3 [r]$ The first step is to remove the fraction $3V = \frac{4}{\cancel{3}^1}\pi r^3 \times \cancel{3}^1 \rightarrow$ Multiple both sides by 3 $3V = 4\pi r^3$ $\frac{3V}{4\pi} = \frac{\cancel{4\pi} r^3}{\cancel{4\pi}} \rightarrow$ Divide both sides by $4\pi$ $\frac{3V}{4\pi} = r^3$ $\sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{r^3} \rightarrow$ take cube root of both sides $r = \sqrt[3]{\frac{3V}{4\pi}}$	$V = \frac{4}{3}\pi r^3 [r]$ The first step is to remove the fraction $3V = 4\pi r^3 \rightarrow \div 3$ becomes $\times 3$ on LHS $\frac{3V}{4\pi} = r^3 \rightarrow \times 4\pi$ becomes $\div 4\pi$ on LHS $\sqrt[3]{\frac{3V}{4\pi}} = r \rightarrow \sqrt[3]{\phantom{x}}$ becomes $\sqrt[3]{\phantom{x}}$ on the LHS $r = \sqrt[3]{\frac{3V}{4\pi}}$
--	--

$$f = \frac{1}{2\pi\sqrt{LC}} [C]$$

$$f \times \sqrt{LC} = \frac{1}{2\pi\cancel{\sqrt{LC}}} \times \cancel{\sqrt{LC}} \rightarrow \text{multiply both sides by } \sqrt{LC} \text{ to get C into the numerator}$$

$$\frac{f \times \sqrt{LC}}{f} = \frac{1}{2\pi f} \rightarrow \text{divide both sides by } f$$

$$(\sqrt{LC})^2 = \left(\frac{1}{2\pi f}\right)^2 \rightarrow \text{square both sides}$$

$$LC = \frac{1}{4\pi^2 f^2}$$

$$\frac{LC}{L} = \frac{1}{4\pi^2 f^2 L} \rightarrow \text{divide both sides by } L$$

$$C = \frac{1}{4\pi^2 f^2 L}$$

Or

$$f = \frac{1}{2\pi\sqrt{LC}} [C]$$

$$f \times \sqrt{LC} = \frac{1}{2\pi} \rightarrow \div \sqrt{LC} \text{ becomes } \times \sqrt{LC} \text{ on the LHS}$$

$$\sqrt{LC} = \frac{1}{2\pi f} \rightarrow \times f \text{ becomes } \div f \text{ on the RHS}$$

$$LC = \left(\frac{1}{2\pi f}\right)^2 \rightarrow \sqrt{\phantom{x}} \text{ becomes } ^2 \text{ on the RHS}$$

$$LC = \frac{1}{4\pi^2 f^2}$$

$$C = \frac{1}{4\pi^2 f^2 L} \rightarrow \times L \text{ becomes } \div L \text{ on the RHS}$$



[Video 'Equations containing exponents'](#)

## Activity

1. Solve for  $x$ :

- |                             |                        |                                 |
|-----------------------------|------------------------|---------------------------------|
| (a) $x^3 = 27$              | (b) $x^5 + 1 = 244$    | (c) $3\sqrt[4]{x} = 6$          |
| (d) $\sqrt[3]{x+1} = 5$     | (e) $\sqrt[12]{x} = 2$ | (f) $4\sqrt{x} + 31 = 63$       |
| (g) $5x^2 + 4 = 9$          | (h) $7(x^4 + 1) = 119$ | (i) $x^{-4} = 20$ (use calc.)   |
| (j) $\frac{x^{-1}}{4} = 11$ | (k) $4x^{-5} = 5$      | (l) $3x^{\frac{1}{2}} + 7 = 19$ |
| (m) $3x^4 + 256 = 1024$     |                        |                                 |

2. Rearrange these formulae for the pronumeral in brackets

- |   |                                 |                                      |
|---|---------------------------------|--------------------------------------|
| (a) $A = 4\pi r^2$ $[r]$                  | (b) $V = \pi r^2 h$ $[r]$       | (c) $c^2 = a^2 + b^2$ $[b]$          |
| (d) $F = \frac{1}{2}mv^2$ $[v]$           | (e) $Q = SLd^2$ $[d]$           | (f) $v^2 = u^2 + 2as$ $[u]$          |
| (g) $Z = \sqrt{R^2 + \omega^2 L^2}$ $[L]$ | (h) $R = \frac{2GM}{c^2}$ $[c]$ | (i) $P = \frac{\pi^2 EI}{L^2}$ $[L]$ |

The next questions are challenging, take care!

- |   |   |   |
|---|---|---|
| (j) $T = 2\pi\sqrt{\frac{r^3}{GM}}$ $[r]$ | (k) $F = m\left(g - \frac{v^2}{R}\right)$ $[v]$ | (l) $R = \frac{\pi r^4 (p_2 - p_1)}{8nL}$ $[r]$ |
| (m) $A = P(1+i)^n$ $[i]$                  |   |   |



## Topic 4: Scientific notation

Scientific Notation is a way of representing numbers that are very large or very small usually to avoid writing down numbers that contain many zeros.

When numbers are written in scientific notation they usually written in this form:

This number  
must be between  
1 and 9.999999...

$n \times 10^e$

The exponent is  
an integer.

The number 4 000 000 can be written as  $4 \times 1\,000\,000$  and then  $4 \times 10^6$ .


The number 0.000 8 can be written as  $\frac{8}{10\,000}$  and then  $\frac{8}{10^4} = 8 \times 10^{-4}$ .

The number 532 000 can be written as  $5.32 \times 100\,000 = 5.32 \times 10^5$ .


Mostly though, students look at the number of places that the decimal point has moved.

### Writing numbers in Scientific Notation

Write 395 000 in scientific notation.

Step 1	From your number, obtain the number between 1 and 9.9999999 $395\,000 = 3.95 \times 10^?$
Step 2	Calculate how many places the decimal point moved. <div style="text-align: center;">  <math>395\,000 = 3.95 \times 10^?</math> </div> The decimal point moved 5 places, so the ? becomes 5. $395\,000 = 3.95 \times 10^5$
Step 3	The exponent can be positive, negative or 0. In this step, the sign of the exponent is checked. If the original number is smaller than 1, then the exponent is negative. Otherwise it is positive or possibly zero. Answer is $395\,000 = 3.95 \times 10^5$


Write 0.000452 in scientific notation.

Step 1	From your number, obtain the number between 1 and 9.9999999 $0.000452 = 4.52 \times 10^?$
Step 2	Calculate how many places the decimal point moved.  $0.000452 = 4.52 \times 10^?$ The decimal point moved 4 places, so the ? becomes 4. $0.000452 = 4.52 \times 10^4$
Step 3	The original number is smaller than 1, so the exponent is negative. Answer is $0.000452 = 4.52 \times 10^{-4}$

Write 5.6 in scientific notation.

Step 1	From your number, obtain the number between 1 and 9.9999999 $5.6 = 5.6 \times 10^?$
Step 2	Calculate how many places the decimal point moved. $5.6 = 5.6 \times 10^?$ The decimal point has not moved, so the ? becomes 0. $5.6 = 5.6 \times 10^0$
Step 3	The exponent is zero (from the previous step). Answer is $5.6 = 5.6 \times 10^0$

Write 13600000 in scientific notation.

Step 1	From your number, obtain the number between 1 and 9.9999999 $13600000 = 1.36 \times 10^?$
Step 2	Calculate how many places the decimal point moved.  $13600000 = 1.36 \times 10^?$ The decimal point moved 7 places, so the ? becomes 7. $13600000 = 1.36 \times 10^7$
Step 3	The original number is greater than 1 so the exponent is positive or zero. Answer is $13600000 = 1.36 \times 10^7$

## Changing numbers in scientific notation to standard form

Write  $4.2 \times 10^5$  in standard form.

Step 1	<p>An exponent of 5 means that the number is greater than 10. Write the number with trailing zeros.</p> $4.2 \times 10^5 = 4200000000$
Step 2	<p>Move the decimal point 5 places in a direction that produces a larger number (to the right).</p> $4.2 \times 10^5 = 420000.0000$ <p>The number in standard form is:</p> $4.2 \times 10^5 = 420000$

Write  $8.2 \times 10^{-3}$  in standard form.

Step 1	<p>An exponent of -3 means that the number is smaller than 1. Write the number with preceding zeros.</p> $8.2 \times 10^{-3} = 00008.2$
Step 2	<p>Move the decimal point 3 places in a direction that produces a smaller number (to the left).</p> $8.2 \times 10^{-3} = 00.0082$ <p>The number in standard form is:</p> $8.2 \times 10^{-3} = 0.0082$

Write  $6.354 \times 10^1$  in standard form.

Step 1	<p>An exponent of 1 means that the number is greater than 1. Write the number with trailing zeros.</p> $6.354 \times 10^1 = 6.35400$
Step 2	<p>Move the decimal point 1 place in a direction that produces a smaller number (to the left).</p> $6.354 \times 10^1 = 63.5400$ <p>The number in standard form is:</p> $6.354 \times 10^1 = 63.54$

Write  $1.92 \times 10^{-9}$  in standard form.

Step 1	An exponent of -9 means that the number is smaller than 1. Write the number with preceding zeros. $1.92 \times 10^{-9} = 000000001.92$
Step 2	Move the decimal point 9 places in a direction that produces a smaller number (to the left). $1.92 \times 10^{-9} = 000000001.92$ The number in standard form is: $1.92 \times 10^{-9} = 0.00000000192$



[Video 'Scientific Notation – changing notation'](#)

## Activity

1. Write these numbers in scientific notation

- |               |                |                       |
|---------------|----------------|-----------------------|
| (a) 4 000 000 | (b) 500 000    | (c) 9 300 000 000 000 |
| (d) 345 900   | (e) 29 500 000 | (f) 354.85            |
| (g) 5.56      | (h) 0.000 04   | (i) 0.000 000 000 955 |
| (j) 745       | (k) 9          | (l) 0.25              |

2. Write these numbers in standard notation

- |                          |                          |                                 |
|--------------------------|--------------------------|---------------------------------|
| (a) $6 \times 10^5$      | (b) $7.5 \times 10^{10}$ | (c) $4.2 \times 10^1$           |
| (d) $1.325 \times 10^6$  | (e) $3.15 \times 10^0$   | (f) $6.1 \times 10^{-5}$        |
| (g) $9.1 \times 10^{-2}$ | (h) $9.5 \times 10^0$    | (i) $5.945 \times 10^{-6}$      |
| (j) $6.1 \times 10^5$    | (k) $3.125 \times 10^1$  | (l) $1.1933\bar{3} \times 10^3$ |

## Topic 5: Operations with numbers in scientific notation

### Multiplying

When multiplying numbers in scientific notation, the number part is multiplied and the exponent part is multiplied using exponent rules.

Example

$$\begin{aligned} & (3.1 \times 10^6) \times (1.4 \times 10^{-2}) \\ &= (3.1 \times 1.4) \times (10^6 \times 10^{-2}) \rightarrow \text{rearrange} \\ &= 4.34 \times 10^{6+(-2)} \\ &= 4.34 \times 10^4 \end{aligned}$$

It is important to look at the solution to see if the answer is still in scientific notation.

In the example below, an adjustment is required to put the answer into scientific notation.

$$\begin{aligned} & (5.2 \times 10^{-5}) \times (3.5 \times 10^{-9}) \\ &= (5.2 \times 3.5) \times (10^{-5} \times 10^{-9}) \rightarrow \text{rearrange} \\ &= 18.2 \times 10^{-5+(-9)} \\ &= 18.2 \times 10^{-14} \rightarrow \text{The number 18.2 is not in the range 1 to 9.9999...,} \\ &= 1.82 \times 10^1 \times 10^{-14} \rightarrow 18.2 = 1.82 \times 10^1 \\ &= 1.82 \times 10^{1+(-14)} \\ &= 1.82 \times 10^{-13} \end{aligned}$$



The speed of light is approximately  $3 \times 10^8$  m/sec. How far will it travel in 5 hours?

$$\begin{aligned} & 5 \text{ hours} \\ &= 5 \times 60 \text{ minutes} \\ &= 5 \times 60 \times 60 \text{ seconds} \\ &= 18000 \text{ seconds} \\ &= 1.8 \times 10^4 \text{ seconds} \end{aligned}$$

$$\begin{aligned} \text{Distance covered} &= \text{speed} \times \text{time} \\ &= (3 \times 10^8) \times (1.8 \times 10^4) \\ &= 5.4 \times 10^{12} \text{ metres} \end{aligned}$$

## Dividing

Dividing numbers in scientific notation is similar to multiplication. The number part is divided as usual and the exponent part is divided using exponent rules.

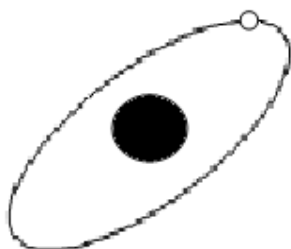
Example:

$$\begin{aligned} & \frac{6.25 \times 10^7}{5.1 \times 10^{-2}} \\ &= (6.25 \div 5.1) \times (10^7 \div 10^{-2}) \\ &= 1.2255 \times 10^{7-(-2)} \\ &= 1.2255 \times 10^9 \end{aligned}$$

As in multiplying, it is important to look at the solution to see if the answer is still in scientific notation.

In the example below, an adjustment is required to put the answer into scientific notation.

$$\begin{aligned} & \frac{3.12 \times 10^{-7}}{7.36 \times 10^4} \\ &= (3.12 \div 7.36) \times (10^{-7} \div 10^4) \\ &= 0.4239 \times 10^{-7-4} \\ &= 0.4239 \times 10^{-11} \rightarrow \text{The number } 0.4239 \text{ is not in the range } 1 \text{ to } 9.9999\dots, \\ &= 4.239 \times 10^{-1} \times 10^{-11} \rightarrow 0.4239 = 4.239 \times 10^{-1} \\ &= 4.239 \times 10^{-12} \end{aligned}$$



The mass of a hydrogen atom is  $1.6 \times 10^{-24}$  grams. How many atoms are there in 1 mg ( $10^{-3}$  gram) of hydrogen?

$$\begin{aligned} \text{Number of atoms} &= \frac{1 \times 10^{-3}}{1.6 \times 10^{-24}} \\ &= (1 \div 1.6) \times (10^{-3} \div 10^{-24}) \\ &= 0.625 \times 10^{-3-(-24)} \\ &= 0.625 \times 10^{21} \\ &= 6.25 \times 10^{-1} \times 10^{21} \\ &= 6.25 \times 10^{20} \end{aligned}$$

## Adding and Subtracting

When adding or subtracting any numbers, it is important that numbers of the same place value are added or subtracted. When adding or subtracting whole numbers or decimals these means lining up the numbers in place value rows.

With numbers written in scientific notation, lining up numbers in place values is achieved by having exponent parts the same.

Add  $(3 \times 10^5)$  and  $(4 \times 10^7)$

$$\begin{aligned} & (3 \times 10^5) + (4 \times 10^7) \\ &= 3 \times 10^{-2} \times 10^7 + 4 \times 10^7 \\ &= 0.03 \times 10^7 + 4 \times 10^7 \rightarrow \text{Both numbers are in the same place value} \\ &= 4.03 \times 10^7 \end{aligned}$$

One exponent must be changed to be the same as the other exponent. Choosing the higher exponent makes the finding then solution slightly easier.

Take  $4.3 \times 10^4$  from  $1.95 \times 10^3$

$$\begin{aligned} & 1.95 \times 10^3 - 4.3 \times 10^4 \\ &= 1.95 \times 10^{-1} \times 10^4 - 4.3 \times 10^4 \\ &= 0.195 \times 10^3 - 4.3 \times 10^4 \\ &= (0.195 - 4.3) \times 10^4 \\ &= -4.105 \times 10^4 \end{aligned}$$

$$\begin{array}{r} \phantom{0}^2 \cancel{10}^{\%} 10 \\ 4.\cancel{3}\cancel{0}\cancel{0} \\ -0.1\ 9\ 5 \\ \hline 4.1\ 0\ 5 \end{array}$$

The largest number is negative  
→ answer is negative  
→  $-4.105$

Add  $3.652 \times 10^{-4}$  and  $4.2 \times 10^{-7}$

$$\begin{aligned} & 3.652 \times 10^{-4} + 4.2 \times 10^{-7} \\ &= 3.652 \times 10^{-4} + 4.2 \times 10^{-3} \times 10^{-4} \\ &= 3.652 \times 10^{-4} + 0.0042 \times 10^{-4} \\ &= (3.652 + 0.0042) \times 10^{-4} \\ &= 3.6562 \times 10^{-4} \end{aligned}$$

Remember  $^{-4}$  is larger than  $^{-7}$ .

## Calculator use

To use scientific notation on your calculator, some calculators have a key marked  $\boxed{\text{Exp}}$ , others  $\boxed{\text{K}}$ .

$2 \times 10^{11}$  is entered by the sequence:  $2 \boxed{\times 10^x} 11$  or  $2 \boxed{\text{Exp}} 11$

$4.25 \times 10^{-6}$  is entered by the sequence  $\boxed{4} \boxed{\cdot} \boxed{2} \boxed{5} \boxed{\times 10^x} \boxed{z} \boxed{6}$  or  $\boxed{4} \boxed{\cdot} \boxed{2} \boxed{5} \boxed{\text{Exp}} \boxed{(-)} \boxed{6}$

$-8.75 \times 10^{15}$  is entered by the sequence  $\boxed{(-)} \boxed{8} \boxed{\cdot} \boxed{7} \boxed{5} \boxed{\times 10^x} \boxed{1} \boxed{5}$

$-1.6 \times 10^{-19}$  is entered by the sequence  $\boxed{(-)} \boxed{1} \boxed{\cdot} \boxed{6} \boxed{\times 10^x} \boxed{(-)} \boxed{1} \boxed{9}$

If you enter a number expressed in scientific notation using  $\boxed{\text{Exp}}$  or the  $\boxed{\times 10^x}$  keys, the calculator treats this as a single number (which it is) 😊.

However, if you enter your number using the  $(10^\blacksquare)$  key, for example  $\boxed{2} \boxed{\times} \boxed{(10^\blacksquare)} \boxed{1} \boxed{1}$ , the calculator treats this as two numbers being multiplied and this can lead to incorrect answers when performing calculations ☹.

Using your calculator to change between scientific notation and standard notation can be quite difficult. If you require this you will need to obtain the instructions for your calculator and find out how to achieve this.

Performing an operation such as  $2 \times 10^{11} \times 4.25 \times 10^{-6}$  is entered as:

$\boxed{2} \boxed{\times 10^x} \boxed{1} \boxed{1} \boxed{\times} \boxed{4} \boxed{\cdot} \boxed{2} \boxed{5} \boxed{\times 10^x} \boxed{(-)} \boxed{6} \boxed{=}$  850000

The format of your answer will vary from calculator to calculator. Because the answer to this question is able to be displayed on a calculator in standard notation, the answer displayed could be 850 000, so you have to change this to  $8.5 \times 10^5$  if you specifically need the answer in scientific notation. Other calculators may automatically give the answer in scientific notation. If numbers are outside the range of what can be displayed in standard form on the calculator, your calculator should display the answer in scientific notation. Even when using a calculator, it is still important to be able to change from standard form to scientific notation and vice-versa.



[Video 'Operations using Scientific Notation'](#)



## Activity

1. Perform the operations indicated. (Leave your answers in scientific notation)

(a)  $6.02 \times 10^{23} \times 1.6 \times 10^{-19}$

(b)  $(3.432 \times 10^3) \div (3 \times 10^{-12})$

(c)  $3.4 \times 10^6 + 2.125 \times 10^8$

(d)  $2.5 \times 10^{-6} - 8.12 \times 10^{-5}$

2. Perform the operations indicated by changing to scientific notation.

(a)  $2\,000\,000\,000 \times 0.000\,000\,03$

(b)  $0.000\,000\,45 \div 0.000\,000\,000\,31$

(c)  $340\,000\,000\,000 \times 0.000\,000\,125$

(d)  $0.65 \times 12\,000\,000 \div 0.000\,000\,085$

## Answers to activity questions

### Check your skills

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1.	$x^6 \times 4x^3 \times x$	$a^6 \div a^0$	$(x^3)^2 \times 4x^4 \div (2x^2)^4$
(a)	$= 4x^{6+3+1}$ $= 4x^{10}$	(b) $= a^6 \div 1$ $= a^6$	(c) $= \frac{x^6 \times 4x^4}{2^4 x^8}$ $= \frac{4x^{10}}{16x^8}$ $= \frac{x^{10-8}}{4}$ $= \frac{x^2}{4}$

---

2.	$(x^{-2})^2 \times \left(x^{\frac{1}{2}}\right)^4 \times x^0$	$\sqrt[3]{512}$	$(27x^9y^6)^{\frac{2}{3}}$
(a)	$= x^{-2 \times 2} \times x^{\frac{1}{2} \times 4} \times 1$ $= x^{-4} \times x^2$ $= x^{-2} \text{ or } \frac{1}{x^2}$	(b) $= (2^9)^{\frac{1}{3}}$ $= 2^{9 \times \frac{1}{3}}$ $= 2^3$ $= 8$	(c) $= (3^3 x^9 y^6)^{\frac{2}{3}}$ $= 3^{3 \times \frac{2}{3}} x^{9 \times \frac{2}{3}} y^{6 \times \frac{2}{3}}$ $= 9x^6 y^4$

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3.	$x^4 = 81$	$\sqrt[4]{x} + 1 = 7$	$V = \pi r^3$
(a)	$x = \pm \sqrt[4]{81}$ $x = \pm 3$	$\sqrt[4]{x} = 7 - 1$ $\sqrt[4]{x} = 6$	(c) $\frac{V}{\pi} = r^3$
		(b) $\sqrt[4]{x} = \frac{6}{3}$ $\sqrt[4]{x} = 2$ $x = 2^4$ $x = 16$	$\sqrt[3]{\frac{V}{\pi}} = r$

---

- 4.
- (a) 634 000 000 in Scientific Notation is  $6.34 \times 10^8$
  - (b)  $9.12 \times 10^{-5}$  in Standard Notation is 0.000 091 2
  - (c) 0.000 000 55 in Scientific Notation is  $5.5 \times 10^{-7}$
  - (d)  $7.25 \times 10^8$  in Standard Notation is 725 000 000
-

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<p>5.</p> <p>(a) <math>3.2 \times 10^8 \times 5.3 \times 10^{-5}</math></p> <p><math>= 3.2 \times 5.3 \times 10^{8+(-5)}</math></p> <p><math>= 16.96 \times 10^3</math></p> <p><math>= 1.696 \times 10^4</math></p>	<p>(b) <math>\frac{(2.8 \times 10^8)}{(7.2 \times 10^{-12})}</math></p> <p><math>= 0.388\overline{8} \times 10^{8-(-12)}</math></p> <p><math>= 0.388\overline{8} \times 10^{20}</math></p> <p><math>= 3.88\overline{8} \times 10^{19}</math></p>	<p>(c) <math>7.25 \times 10^8 + 2.61 \times 10^4</math></p> <p><math>= 7.25 \times 10^8 + 2.61 \times 10^{-4} \times 10^8</math></p> <p><math>= 7.25 \times 10^8 + 0.000261 \times 10^8</math></p> <p><math>= 7.250261 \times 10^8</math></p>
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## Exponent rules – Whole number exponents

1. Evaluate the following powers.

<p>(a) <math>3^5</math></p> <p><math>= 3 \times 3 \times 3 \times 3 \times 3</math></p> <p><math>= 243</math></p>	<p>(b) <math>5^3</math></p> <p><math>= 5 \times 5 \times 5</math></p> <p><math>= 125</math></p>	<p>(c) <math>7^3</math></p> <p><math>= 7 \times 7 \times 7</math></p> <p><math>= 343</math></p>
<p>(d) <math>2^9</math></p> <p><math>= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2</math></p> <p><math>= 512</math></p>	<p>(e) <math>14^3</math></p> <p><math>= 14 \times 14 \times 14</math></p> <p><math>= 2744</math></p>	<p>(f) <math>5^5</math></p> <p><math>= 5 \times 5 \times 5 \times 5 \times 5</math></p> <p><math>= 3125</math></p>
<p>(g) <math>3^0</math></p> <p><math>= 1</math></p>	<p>(h) <math>(2^7)^0</math></p> <p><math>= 1</math></p>	<p>(i) <math>(3^2)^3</math></p> <p><math>= 3^{2 \times 3}</math></p> <p><math>= 3^6 = 729</math></p>
<p>(j) <math>2^4 \times 2^6</math></p> <p><math>= 2^{4+6}</math></p> <p><math>= 2^{10}</math></p> <p><math>= 1024</math></p>	<p>(k) <math>4^7 \div 4^5</math></p> <p><math>= \frac{4^7}{4^5}</math></p> <p><math>= 4^{7-5}</math></p> <p><math>= 4^2 = 16</math></p>	<p>(l) <math>-4^3</math></p> <p><math>= -(4 \times 4 \times 4)</math></p> <p><math>= -64</math></p>
<p>(m) <math>(-5)^3</math></p> <p><math>= -5 \times -5 \times -5</math></p> <p><math>= -125</math></p>	<p>(n) <math>0.85^6</math></p> <p>0.85W6é0.377149515</p>	<p>(o) <math>4.5^{1.25}</math></p> <p>4.5W1.25é</p> <p>6.554138918</p>
<p>(p) <math>(1.006)^{240}</math></p> <p>1.006W240é</p> <p>4.202574033</p>	<p>(q) <math>(1.05)^{20}</math></p> <p>1.05W20é</p> <p>2.653297705</p>	

2. Simplify the following (leave your answer in power form).

<p>(a) <math>5^2 \times 5^5</math></p> <p><math>= 5^{2+5}</math></p> <p><math>= 5^7</math></p>	<p>(b) <math>m^4 \times m^5</math></p> <p><math>= m^{4+5}</math></p> <p><math>= m^9</math></p>	<p>(c) <math>a^4 \times a^3 \times a</math></p> <p><math>= a^{4+3+1}</math></p> <p><math>= a^8</math></p>
<p>(d) <math>10^4 \times 10 \times 10^0</math></p> <p><math>= 10^{4+1+0}</math></p> <p><math>= 10^5</math></p>	<p>(e) <math>\frac{r^4}{2} \times \frac{r}{4}</math></p> <p><math>= \frac{r^{4+1}}{8}</math></p> <p><math>= \frac{r^5}{8}</math></p>	<p>(f) <math>5c^2 \times 3c^4</math></p> <p><math>= 15c^{2+4}</math></p> <p><math>= 15c^6</math></p>

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$$\begin{aligned} \text{(g)} \quad & a^2 b^4 \times a^3 b c \\ & = a^{2+3} b^{4+1} c \\ & = a^5 b^5 c \end{aligned}$$

$$\begin{aligned} \text{(h)} \quad & 4x^3 y^4 z^2 \times 5x^2 y \\ & = 20x^{3+2} y^{4+1} z^2 \\ & = 20x^5 y^5 z^2 \end{aligned}$$

$$\begin{aligned} \text{(i)} \quad & 5^2 e f^3 g^5 \times 5^3 e^2 f^2 g^0 \\ & = 5^{2+3} e^{1+2} f^{3+2} g^{5+0} \\ & = 5^5 e^3 f^5 g^5 \end{aligned}$$

$$\begin{aligned} \text{(j)} \quad & 4^9 \div 4^5 \\ & = 4^{9-5} \\ & = 4^4 \end{aligned}$$

$$\begin{aligned} \text{(k)} \quad & x^8 \div x^5 \\ & = x^{8-5} \\ & = x^3 \end{aligned}$$

$$\begin{aligned} \text{(l)} \quad & h^5 \div h \\ & = h^{5-1} \\ & = h^4 \end{aligned}$$

$$\begin{aligned} \text{(m)} \quad & 3^x \div 3^4 \\ & = 3^{x-4} \end{aligned}$$

$$\begin{aligned} \text{(n)} \quad & \frac{16a^5}{4a^2} \\ & = 4a^{5-2} \\ & = 4a^3 \end{aligned}$$

$$\begin{aligned} \text{(o)} \quad & \frac{5a^2 b^4}{25a^2 b} \\ & = \frac{a^{2-2} b^{4-1}}{5} \\ & = \frac{a^0 b^3}{5} \\ & = \frac{b^3}{5} \end{aligned}$$

$$\begin{aligned} \text{(p)} \quad & a^{\frac{1}{2}} \div a^{\frac{1}{4}} \\ & = a^{\frac{1}{2} - \frac{1}{4}} \\ & = a^{\frac{2}{4} - \frac{1}{4}} \\ & = a^{\frac{1}{4}} \end{aligned}$$

$$\begin{aligned} \text{(q)} \quad & (2^4)^3 \\ & = 2^{4 \times 3} \\ & = 2^{12} \end{aligned}$$

$$\begin{aligned} \text{(r)} \quad & (5^4)^{\frac{1}{2}} \\ & = 5^{4 \times \frac{1}{2}} \\ & = 5^2 \end{aligned}$$

$$\begin{aligned} \text{(s)} \quad & (b^4)^5 \\ & = b^{4 \times 5} \\ & = b^{20} \end{aligned}$$

$$\begin{aligned} \text{(t)} \quad & (3t)^4 \\ & = 3^4 t^4 \end{aligned}$$

$$\begin{aligned} \text{(u)} \quad & (2a^4 b^3)^3 \\ & = 2^3 a^{4 \times 3} b^{3 \times 3} \\ & = 2^3 a^{12} b^9 \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad & \left( \frac{4}{y} \right)^3 \\ & = \frac{4^3}{y^3} \end{aligned}$$

$$\begin{aligned} \text{(w)} \quad & \frac{(2y^4)^2}{(2x^2)^4} \\ & = \frac{2^2 y^{4 \times 2}}{2^4 x^{2 \times 4}} \\ & = \frac{y^8}{2^2 x^8} \end{aligned}$$

$$\begin{aligned} \text{(x)} \quad & (2^5)^b \\ & = 2^{5 \times b} \\ & = 2^{5b} \end{aligned}$$

3. Simplify the following.

$$\begin{aligned}
 \text{(a)} \quad & (a^2)^4 \times a^3 \\
 & = a^{2 \times 4} \times a^3 \\
 & = a^{8+3} \\
 & = a^{11}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & \frac{(3y^4)^2}{2y^5} \\
 & = \frac{3^2 y^{4 \times 2}}{2y^5} \\
 & = \frac{9y^8}{2y^5} \\
 & = \frac{9y^{8-5}}{2} \\
 & = \frac{9y^3}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad & \frac{2x^5 \times x^4}{(x^2)^3} \\
 & = \frac{2x^{5+4}}{x^{2 \times 3}} \\
 & = \frac{2x^9}{x^6} \\
 & = 2x^{9-6} \\
 & = 2x^3
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad & 3b^0 + (3b)^0 \\
 & = 3 \times 1 + 1 \\
 & = 4
 \end{aligned}$$

$$\begin{aligned}
 \text{(e)} \quad & \frac{(3n)^2 \times 3n^2}{9n^4} \\
 & = \frac{3^2 n^2 \times 3n^2}{9n^4} \\
 & = \frac{9n^4}{9n^4} \\
 & = \frac{9^1 \times 3 \times n^{2+2}}{9^1 n^4} \\
 & = \frac{3 \cancel{n^4}^1}{\cancel{n^4}^1} \\
 & = 3
 \end{aligned}$$

$$\begin{aligned}
 \text{(f)} \quad & 5(mn)^2 \times (mn^2)^3 \\
 & = 5m^2 n^2 \times m^3 n^{2 \times 3} \\
 & = 5m^{2+3} n^{2+6} \\
 & = 5m^5 n^8
 \end{aligned}$$

$$\begin{aligned}
 \text{(g)} \quad & m^2 n^5 p^7 \times m^3 p^4 \times n^2 p \\
 & = m^{2+3} n^{5+2} p^{7+4+1} \\
 & = m^5 n^7 p^{12}
 \end{aligned}$$

$$\begin{aligned}
 \text{(h)} \quad & -4(a^2 b^5)^0 \\
 & = -4 \times 1 \\
 & = -4
 \end{aligned}$$

$$\begin{aligned}
 \text{(i)} \quad & 2a^2 b^3 \times 3ab^4 \\
 & = 6a^{2+1} b^{3+4} \\
 & = 6a^3 b^7
 \end{aligned}$$

$$\begin{aligned}
 \text{(j)} \quad & \frac{2^1 x^5 y^4}{10^5 x^2 y^3} \\
 & = \frac{x^{5-2} y^{4-3}}{5} \\
 & = \frac{x^3 y^1}{5} \text{ or } \frac{x^3 y}{5}
 \end{aligned}$$

$$\begin{aligned}
 \text{(k)} \quad & \left( \frac{2x^3}{y^4} \right)^4 \times \left( \frac{y}{x} \right)^2 \\
 & = \frac{2^4 x^{3 \times 4}}{y^{4 \times 4}} \times \frac{y^2}{x^2} \\
 & = \frac{16x^{12} y^2}{x^2 y^{16}} \\
 & = 16x^{12-2} y^{2-16} \\
 & = 16x^{10} y^{-14} \text{ or } \frac{16x^{10}}{y^{14}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(l)} \quad & 4a^0 - \left( \frac{a}{4} \right)^0 \\
 & = 4 \times 1 - 1 \\
 & = 3
 \end{aligned}$$

$$\begin{aligned}
 \text{(m)} \quad & \frac{25p^6}{36q^8} \\
 &= \frac{(5p^3)^2}{(6q^4)^2} \\
 &= \left( \frac{5p^3}{6q^4} \right)^2
 \end{aligned}$$

$$\begin{aligned}
 \text{(n)} \quad & 27a^6b^9c^3 \\
 &= 3^3 a^6 b^9 c^3 \\
 &= (3a^2b^3c)^3
 \end{aligned}$$

$$\begin{aligned}
 \text{(o)} \quad & \frac{(3xy^2)^3}{2x(2y)^2} \div \frac{6xy^0}{4(xy^2)^3} \\
 &= \frac{(3xy^2)^3}{2x(2y)^2} \times \frac{4(xy^2)^3}{6xy^0} \\
 &= \frac{3^3 x^3 y^{2 \times 3}}{2x \times 2^2 y^2} \times \frac{4x^3 y^{2 \times 3}}{6x} \\
 &= \frac{\cancel{2}^9 \times \cancel{4}^1 \times x^{3+3} y^{6+6}}{\cancel{8}^2 \times \cancel{6}^2 \times x^2 y^2} \\
 &= \frac{9x^6 y^{12}}{4x^2 y^2} \\
 &= \frac{9x^{6-2} y^{12-2}}{4} \\
 &= \frac{9x^4 y^{10}}{4}
 \end{aligned}$$

## Exponent rules – Integer and fraction exponents

1. Evaluate

$$\begin{aligned}
 \text{(a)} \quad & 36^{-1} \\
 &= \frac{1}{36^1} \\
 &= \frac{1}{36}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & 5^{-2} \\
 &= \frac{1}{5^2} \\
 &= \frac{1}{25}
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad & 10^{-4} \\
 &= \frac{1}{10^4} \\
 &= \frac{1}{10000}
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad & \frac{1}{4^{-3}} \\
 &= 4^3 \\
 &= 64
 \end{aligned}$$

$$\begin{aligned}
 \text{(e)} \quad & \left( \frac{1}{4} \right)^{-4} \\
 &= (4)^4 \\
 &= 256
 \end{aligned}$$

$$\begin{aligned}
 \text{(f)} \quad & \frac{5^{-2}}{2^3} \\
 &= \frac{1}{2^3 \times 5^2} \\
 &= \frac{1}{8 \times 25} \\
 &= \frac{1}{200}
 \end{aligned}$$

$$\begin{aligned}
 \text{(g)} \quad & 7 \times 2^{-4} \\
 &= 7 \times \frac{1}{2^4} \\
 &= \frac{7}{16}
 \end{aligned}$$

$$\begin{aligned}
 \text{(h)} \quad & (3 \times 2^{-2})^{-2} \\
 &= \frac{1}{(3 \times 2^{-2})^2} \\
 &= \frac{1}{3^2 \times 2^{-4}} \\
 &= \frac{2^4}{3^2} \\
 &= \frac{16}{9} \text{ or } 1\frac{7}{9}
 \end{aligned}$$

2. Simplify

$$(a) \quad a^{-4} \\ = \frac{1}{a^4}$$

$$(b) \quad c^{-7} \\ = \frac{1}{c^7}$$

$$(c) \quad \frac{1}{x^{-2}} \\ = x^2$$

$$(d) \quad 2x^{-3} \\ = \frac{2}{x^3}$$

$$(e) \quad (3b)^{-2} \\ = \frac{1}{(3b)^2} \\ = \frac{1}{3^2 b^2} \\ = \frac{1}{9b^2}$$

$$(f) \quad ab^{-1} \\ = \frac{a}{b}$$

$$(g) \quad a^3 \times b^{-4} \\ = \frac{a^3}{b^4}$$

$$(h) \quad 4p^{-1} \\ = \frac{4}{p}$$

$$(i) \quad (4p)^{-1} \\ = \frac{1}{4p}$$

$$(j) \quad (3^{-1}ab^{-2})^3 \\ = \left(\frac{a}{3b^2}\right)^3 \\ = \frac{a^3}{3^3 b^{2 \times 3}} \\ = \frac{a^3}{27b^6}$$

$$(k) \quad 4p^2q^{-3} \times 3p^{-1}q^2 \\ = \frac{4p^2}{q^3} \times \frac{3q^2}{p} \\ = \frac{12p^2q^2}{pq^3} \\ = 12p^{2-1}q^{2-3} \\ = 12p^1q^{-1} \\ = \frac{12p}{q}$$

$$(l) \quad \left(\frac{4}{b^2}\right)^{-3} \\ = \left(\frac{b^2}{4}\right)^3 \\ = \frac{b^{2 \times 3}}{4^3} \\ = \frac{b^6}{64}$$

$$(m) \quad \frac{(xy^3)^{-4}}{4x^4y^{-2}} \\ = \frac{x^{-4}y^{3 \times -4}}{4x^4y^{-2}} \\ = \frac{x^{-4-4}y^{-12-^{-2}}}{4} \\ = \frac{x^{-8}y^{-10}}{4} \\ = \frac{1}{4x^8y^{10}}$$

$$(n) \quad \frac{2(x^2y^{-1})^{-3}}{(3x^{-1}y^3)^2} \\ = \frac{2x^{2 \times -3}y^{-1 \times -3}}{3^2 x^{-1 \times 2}y^{3 \times 2}} \\ = \frac{2x^{-6}y^3}{9x^{-2}y^6} \\ = \frac{2x^{-6-^{-2}}y^{3-6}}{9} \\ = \frac{2x^{-4}y^{-3}}{9} \\ = \frac{2}{9x^4y^3}$$

$$(o) \quad \frac{2x^2}{3y^{-3}} \times \left(\frac{4xy^2}{9x^{-4}}\right)^{-1} \\ = \frac{2x^2}{3y^{-3}} \times \left(\frac{9x^{-4}}{4xy^2}\right)^1 \\ = \frac{2^1 \times 2^1 \times x^2 \times^{-4}}{3^1 \times 4^1 \times xy^{-3+2}} \\ = \frac{3x^{-2}}{2xy^{-1}} \\ = \frac{3x^{-2-1}}{2y^{-1}} \\ = \frac{3x^{-3}}{2y^{-1}} \\ = \frac{3y}{2x^3}$$

3. Evaluate without using a calculator

$$(a) \quad 49^{\frac{1}{2}} \\ = (7^{2^1})^{\frac{1}{2^1}} \\ = 7^1 \text{ or } 7$$

$$(b) \quad 8^{\frac{2}{3}} \\ = (2^{3^1})^{\frac{2}{3^1}} \\ = 2^2 \\ = 4$$

$$(c) \quad 32^{\frac{3}{5}} \\ = (2^{5^1})^{\frac{3}{5^1}} \\ = 2^3 \\ = 8$$

$$\begin{aligned}
 \text{(d)} \quad & 625^{\frac{1}{4}} \\
 &= \left(5^4\right)^{\frac{1}{4}} \\
 &= 5^1 = 5
 \end{aligned}$$

$$\begin{aligned}
 \text{(e)} \quad & 1024^{\frac{7}{10}} \\
 &= \left(2^{10}\right)^{\frac{7}{10}} \\
 &= 2^7 \\
 &= 128
 \end{aligned}$$

$$\begin{aligned}
 \text{(f)} \quad & 256^{-\frac{1}{4}} \\
 &= \frac{1}{256^{\frac{1}{4}}} \\
 &= \frac{1}{\left(2^8\right)^{\frac{1}{4}}} \\
 &= \frac{1}{2^2} \\
 &= \frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 \text{(g)} \quad & 16^{-\frac{1}{2}} \\
 &= \frac{1}{\left(2^4\right)^{\frac{1}{2}}} \\
 &= \frac{1}{2^2} \\
 &= \frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 \text{(h)} \quad & 1024^{\frac{7}{10}} \\
 &= \frac{1}{\left(2^{10}\right)^{\frac{7}{10}}} \\
 &= \frac{1}{2^7} \\
 &= \frac{1}{128}
 \end{aligned}$$

$$\begin{aligned}
 \text{(i)} \quad & \frac{3}{27^{\frac{2}{3}}} \\
 &= 3 \times 27^{\frac{2}{3}} \\
 &= 3 \times \left(3^3\right)^{\frac{2}{3}} \\
 &= 3^1 \times 3^2 \\
 &= 3^3 \text{ or } 27
 \end{aligned}$$

$$\begin{aligned}
 \text{(j)} \quad & 7 \times 128^{\frac{3}{7}} \\
 &= 7 \times \left(2^7\right)^{\frac{3}{7}} \\
 &= 7 \times 2^3 \\
 &= 56
 \end{aligned}$$

$$\begin{aligned}
 \text{(k)} \quad & \frac{16^{\frac{3}{4}}}{27^{\frac{1}{3}}} \\
 &= \frac{\left(2^4\right)^{\frac{3}{4}}}{\left(3^3\right)^{\frac{1}{3}}} \\
 &= \frac{2^3}{3} \\
 &= \frac{8}{3} \text{ or } 2\frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{(l)} \quad & 36^{\frac{1}{2}} \times 32^{\frac{3}{5}} \\
 &= \left(6^2\right)^{\frac{1}{2}} \times \left(2^5\right)^{\frac{3}{5}} \\
 &= 6 \times 2^3 \\
 &= 48
 \end{aligned}$$

4. Use a calculator to answer the following

$$\begin{aligned}
 \text{(a)} \quad & 10^{\frac{3}{4}} \\
 &= 5.623413252
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & 450^{\frac{1}{7}} \\
 &= 2.393483066
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad & 1.864297^{\frac{1}{10}} \\
 &= 1.06426924
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad & 2.1209527^{-\frac{1}{240}} \\
 &= 0.996872129
 \end{aligned}$$

$$\begin{aligned}
 \text{(e)} \quad & 4.95^{2.5} \\
 &= 54.51462103
 \end{aligned}$$

$$\begin{aligned}
 \text{(f)} \quad & 1.01^{-0.75} \\
 &= 0.992565029
 \end{aligned}$$

5. Simplify



$$\begin{aligned}
 \text{(a)} \quad & \left(\frac{8}{a^6}\right)^{\frac{1}{3}} \\
 &= \frac{\left(2^3\right)^{\frac{1}{3}}}{a^{2 \times \frac{1}{3}}} \\
 &= \frac{2^1}{a^2} \\
 &= \frac{2}{a^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & \frac{4(a^6b^3)^{\frac{1}{3}}}{(16a^8b^4)^{\frac{1}{4}}} \\
 &= \frac{4a^{6 \times \frac{1}{3}}b^{3 \times \frac{1}{3}}}{\left(2^4\right)^{\frac{1}{4}}a^{8 \times \frac{1}{4}}b^{4 \times \frac{1}{4}}} \\
 &= \frac{4a^2b^1}{2a^2b^1} \\
 &= 2
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad & \left(\frac{a^2b^6}{16}\right)^{-\frac{1}{2}} \\
 &= \left(\frac{4^2}{(ab^3)^2}\right)^{\frac{1}{2}} \\
 &= \frac{4^{2 \times \frac{1}{2}}}{(ab^3)^{2 \times \frac{1}{2}}} \\
 &= \frac{4}{ab^3}
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad & \sqrt[3]{64a^2b^4} \\
 &= \left(2^6a^2b^4\right)^{\frac{1}{3}} \\
 &= 2^{6 \times \frac{1}{3}}a^{2 \times \frac{1}{3}}b^{4 \times \frac{1}{3}} \\
 &= 2^2a^{\frac{2}{3}}b^{\frac{4}{3}} \\
 &= 4a^{\frac{2}{3}}b^{\frac{4}{3}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(e)} \quad & \sqrt[4]{32(a^3b^2)^2} \\
 &= \left[32(a^3b^2)^2\right]^{\frac{1}{4}} \\
 &= 2^{5 \times \frac{1}{4}}(a^3b^2)^{2 \times \frac{1}{4}} \\
 &= 2^{\frac{5}{4}}(a^3b^2)^{\frac{1}{2}} \\
 &= 2^{\frac{5}{4}}a^{\frac{3}{2}}b^1 \text{ or } 2.3784a^{1.5}b
 \end{aligned}$$

$$\begin{aligned}
 \text{(f)} \quad & \frac{\sqrt{x^2y^6}}{\sqrt[3]{x^6y^3}} \\
 &= \frac{(x^2y^6)^{\frac{1}{2}}}{(x^6y^3)^{\frac{1}{3}}} \\
 &= \frac{xy^3}{x^2y} \\
 &= \frac{y^2}{x}
 \end{aligned}$$

## Equation with exponents

1. Solve for  $x$ :

$$\begin{aligned}
 \text{(a)} \quad & x^3 = 27 \\
 & x = \sqrt[3]{27} \\
 & x = 3
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & x^5 + 1 = 244 \\
 & x^5 + 1 - 1 = 244 - 1 \\
 & x^5 = 243 \\
 & \sqrt[5]{(x^5)} = \sqrt[5]{243} \\
 & x = 3
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad & 3\sqrt[4]{x} = 6 \\
 & \frac{3\sqrt[4]{x}}{3} = \frac{6}{3} \\
 & \sqrt[4]{x} = 2 \\
 & \left(\sqrt[4]{x}\right)^4 = 2^4 \\
 & x = 16
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad & \sqrt[3]{x+1} = 5 \\
 & \left(\sqrt[3]{x+1}\right)^3 = 5^3 \\
 & x+1 = 125 \\
 & x+1-1 = 125-1 \\
 & x = 124
 \end{aligned}$$

$$\begin{aligned}
 \text{(e)} \quad & \sqrt[12]{x} = 2 \\
 & \left(\sqrt[12]{x}\right)^{12} = 2^{12} \\
 & x = 4096
 \end{aligned}$$

$$\begin{aligned}
 \text{(f)} \quad & 4\sqrt{x} + 31 = 63 \\
 & 4\sqrt{x} + 31 - 31 = 63 - 31 \\
 & 4\sqrt{x} = 32 \\
 & \frac{4\sqrt{x}}{4} = \frac{32}{4} \\
 & \sqrt{x} = 8 \\
 & \left(\sqrt{x}\right)^2 = 8^2 \\
 & x = 64
 \end{aligned}$$

$$\begin{aligned}
 \text{(g)} \quad 5x^2 + 4 &= 9 \\
 5x^2 + 4 - 4 &= 9 - 4 \\
 5x^2 &= 5 \\
 \frac{5x^2}{5} &= \frac{5}{5} \\
 x^2 &= 1 \\
 x &= \pm 1
 \end{aligned}$$

$$\begin{aligned}
 \text{(h)} \quad 7(x^4 + 1) &= 119 \\
 \frac{7(x^4 + 1)}{7} &= \frac{119}{7} \\
 x^4 + 1 - 1 &= 17 - 1 \\
 x^4 &= 16 \\
 x &= \sqrt[4]{16} \\
 x &= \pm 2
 \end{aligned}$$

$$\begin{aligned}
 \text{(i)} \quad x^{-4} &= 20 \\
 \frac{1}{x^4} &= \frac{20}{1} \\
 x^4 &= \frac{1}{20} \\
 \sqrt[4]{x^4} &= \sqrt[4]{\frac{1}{20}} \\
 x &\approx 0.47287
 \end{aligned}$$

$$\begin{aligned}
 \text{(j)} \quad \frac{x^{-1}}{4} &= 11 \\
 \frac{x^{-1}}{4} \times 4 &= 11 \times 4 \\
 x^{-1} &= 44 \\
 x &= \frac{1}{44}
 \end{aligned}$$

$$\begin{aligned}
 \text{(k)} \quad 4x^{-5} &= 5 \\
 \frac{4x^{-5}}{4} &= \frac{5}{4} \\
 x^{-5} &= 1.25 \\
 x^5 &= \frac{1}{1.25} \\
 x^5 &= 0.8 \\
 \sqrt[5]{x^5} &= \sqrt[5]{0.8} \\
 x &\approx 0.9565
 \end{aligned}$$

$$\begin{aligned}
 \text{(l)} \quad 3x^{\frac{1}{2}} + 7 &= 19 \\
 3x^{\frac{1}{2}} + 7 - 7 &= 19 - 7 \\
 3x^{\frac{1}{2}} &= 12 \\
 \frac{3x^{\frac{1}{2}}}{3} &= \frac{12}{3} \\
 x^{\frac{1}{2}} &= 4 \\
 \left(x^{\frac{1}{2}}\right)^2 &= 4^2 \\
 x &= 16
 \end{aligned}$$

$$\begin{aligned}
 \text{(m)} \quad 3x^4 + 256 &= 1024 \\
 3x^4 + 256 - 256 &= 1024 - 256 \\
 \frac{3x^4}{3} &= \frac{768}{3} \\
 x^4 &= 256 \\
 \sqrt[4]{x^4} &= \sqrt[4]{256} \\
 x &= 4
 \end{aligned}$$

2. Rearrange these formulae for the pronumeral in brackets

$$\begin{aligned}
 \text{(a)} \quad A &= 4\pi r^2 \quad [r] \\
 \frac{A}{4\pi} &= \frac{4\pi r^2}{4\pi} \\
 \frac{A}{4\pi} &= r^2 \\
 \sqrt{\frac{A}{4\pi}} &= \sqrt{r^2} \\
 \sqrt{\frac{A}{4\pi}} &= r
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad V &= \pi r^2 h \quad [r] \\
 \frac{V}{\pi h} &= \frac{\pi r^2 h}{\pi h} \\
 \frac{V}{\pi h} &= r^2 \\
 \sqrt{\frac{V}{\pi h}} &= \sqrt{r^2} \\
 \sqrt{\frac{V}{\pi h}} &= r
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad c^2 &= a^2 + b^2 \quad [b] \\
 c^2 - a^2 &= a^2 + b^2 - a^2 \\
 c^2 - a^2 &= b^2 \\
 \sqrt{c^2 - a^2} &= \sqrt{b^2} \\
 \sqrt{c^2 - a^2} &= b
 \end{aligned}$$

$$\begin{array}{lll}
 \text{(d)} & F = \frac{1}{2}mv^2 \quad [v] & \text{(e)} \quad Q = SLd^2 \quad [d] \quad \text{(f)} \quad v^2 = u^2 + 2as \quad [u] \\
 & 2 \times F = 2 \times \frac{1}{2}mv^2 & \frac{Q}{SL} = \frac{SLd^2}{SL} & v^2 - 2as = u^2 + 2as - 2as \\
 & 2F = mv^2 & \frac{Q}{SL} = d^2 & v^2 - 2as = u^2 \\
 & \frac{2F}{m} = \frac{mv^2}{m} & \sqrt{\frac{Q}{SL}} = \sqrt{d^2} & \sqrt{v^2 - 2as} = \sqrt{u^2} \\
 & \frac{2F}{m} = v^2 & \sqrt{\frac{Q}{SL}} = d & \sqrt{v^2 - 2as} = u \\
 & \sqrt{\frac{2F}{m}} = \sqrt{v^2} & & \\
 & \sqrt{\frac{2F}{m}} = v & & 
 \end{array}$$

$$\begin{array}{lll}
 \text{(g)} & Z = \sqrt{R^2 + \omega^2 L^2} \quad [L] & \text{(h)} \quad R = \frac{2GM}{c^2} \quad [c] \quad \text{(i)} \quad P = \frac{\pi^2 EI}{L^2} \quad [L] \\
 & Z^2 = \left( \sqrt{R^2 + \omega^2 L^2} \right)^2 & c^2 R = \frac{2GM}{c^2} \times c^2 & L^2 \times P = \frac{\pi^2 EI}{L^2} \times L^2 \\
 & Z^2 = R^2 + \omega^2 L^2 & c^2 R = 2GM & L^2 P = \pi^2 EI \\
 & Z^2 - R^2 = R^2 + \omega^2 L^2 - R^2 & \frac{c^2 R}{R} = \frac{2GM}{R} & \frac{L^2 P}{P} = \frac{\pi^2 EI}{P} \\
 & Z^2 - R^2 = \omega^2 L^2 & c^2 = \frac{2GM}{R} & L^2 = \frac{\pi^2 EI}{P} \\
 & \frac{Z^2 - R^2}{\omega^2} = \frac{\omega^2 L^2}{\omega^2} & \sqrt{c^2} = \sqrt{\frac{2GM}{R}} & \sqrt{L^2} = \sqrt{\frac{\pi^2 EI}{P}} \\
 & \frac{Z^2 - R^2}{\omega^2} = L^2 & c = \sqrt{\frac{2GM}{R}} & L = \sqrt{\frac{\pi^2 EI}{P}} \\
 & \sqrt{\frac{Z^2 - R^2}{\omega^2}} = \sqrt{L^2} & & \\
 & \sqrt{\frac{Z^2 - R^2}{\omega^2}} = L & & 
 \end{array}$$

The next questions are challenging, take care!

$$\begin{array}{lll}
 \text{(j)} & T = 2\pi \sqrt{\frac{r^3}{GM}} \quad [r] & \text{(k)} \quad F = m \left( g - \frac{v^2}{R} \right) \quad [v] \quad \text{(l)} \quad R = \frac{\pi r^4 (p_2 - p_1)}{8nL} \quad [r] \\
 & \frac{T}{2\pi} = \frac{2\pi \sqrt{\frac{r^3}{GM}}}{2\pi} & \frac{F}{m} = \frac{m \left( g - \frac{v^2}{R} \right)}{m} & 8nL \times R = \frac{\pi r^4 (p_2 - p_1)}{8nL} \times 8nL \\
 & \frac{T}{2\pi} = \sqrt{\frac{r^3}{GM}} & \frac{F}{m} = g - \frac{v^2}{R} & 8nLR = \pi r^4 (p_2 - p_1) \\
 & \left( \frac{T}{2\pi} \right)^2 = \left( \sqrt{\frac{r^3}{GM}} \right)^2 & \frac{F}{m} - g = g - \frac{v^2}{R} - g & \frac{8nLR}{\pi (p_2 - p_1)} = r^4 \\
 & \left( \frac{T}{2\pi} \right)^2 = \frac{r^3}{GM} & \frac{F}{m} - g = -\frac{v^2}{R} & \sqrt[4]{\frac{8nLR}{\pi (p_2 - p_1)}} = \sqrt[4]{r^4} \\
 & GM \times \left( \frac{T}{2\pi} \right)^2 = \frac{r^3}{GM} \times GM & g - \frac{F}{m} = \frac{v^2}{R} & \sqrt[4]{\frac{8nLR}{\pi (p_2 - p_1)}} = r \\
 & GM \left( \frac{T}{2\pi} \right)^2 = r^3 & R \left( g - \frac{F}{m} \right) = \frac{v^2}{R} \times R & \\
 & \sqrt[3]{GM \left( \frac{T}{2\pi} \right)^2} = \sqrt[3]{r^3} & R \left( g - \frac{F}{m} \right) = v^2 & \\
 & \sqrt[3]{GM \left( \frac{T}{2\pi} \right)^2} = r & \sqrt{R \left( g - \frac{F}{m} \right)} = \sqrt{v^2} & \\
 & & \sqrt{R \left( g - \frac{F}{m} \right)} = v & 
 \end{array}$$

$$\begin{aligned}
 \text{(m)} \quad A &= P(1+i)^n \quad [i] \\
 \frac{A}{P} &= \frac{P(1+i)^n}{P} \\
 \frac{A}{P} &= (1+i)^n \\
 \sqrt[n]{\frac{A}{P}} &= \sqrt[n]{(1+i)^n} \\
 \sqrt[n]{\frac{A}{P}} &= 1+i \\
 \sqrt[n]{\frac{A}{P}} - 1 &= 1+i-1 \\
 \sqrt[n]{\frac{A}{P}} - 1 &= i
 \end{aligned}$$

## Scientific notation

1. Write these numbers in scientific notation

- |                                      |  |   |
|--------------------------------------|--|---|
| (a) 4 000 000<br>$= 4 \times 10^6$   | (b) 500 000<br>$= 5 \times 10^5$       | (c) 9 300 000 000 000<br>$= 9.3 \times 10^{12}$   |
| (d) 345 900<br>$= 3.459 \times 10^5$ | (e) 29 500 000<br>$= 2.95 \times 10^7$ | (f) 354.85<br>$= 3.5485 \times 10^2$              |
| (g) 5.56<br>$= 5.56 \times 10^0$     | (h) 0.000 04<br>$= 4 \times 10^{-5}$   | (i) 0.000 000 000 955<br>$= 9.55 \times 10^{-10}$ |
| (j) 745<br>$= 7.45 \times 10^2$      | (k) 9<br>$= 9 \times 10^0$             | (l) 0.25<br>$= 2.5 \times 10^{-1}$                |

2. Write these numbers in standard notation

- |                                       |   |  |
|---------------------------------------|---|--|
| (a) $6 \times 10^5$<br>$= 600\,000$   | (b) $7.5 \times 10^{10}$<br>$= 75\,000\,000\,000$ | (c) $4.2 \times 10^1$<br>$= 42$                        |
| (d) $1.325 \times 10^6$<br>1325 000   | (e) $3.15 \times 10^0$<br>$= 3.15$                | (f) $6.1 \times 10^{-5}$<br>$= 0.000\,061$             |
| (g) $9.1 \times 10^{-2}$<br>$= 0.091$ | (h) $9.5 \times 10^0$<br>$= 9.5$                  | (i) $5.945 \times 10^{-6}$<br>$= 0.000\,005\,945$      |
| (j) $6.1 \times 10^5$<br>$= 610\,000$ | (k) $3.125 \times 10^1$<br>$= 31.25$              | (l) $1.1933\bar{3} \times 10^3$<br>$= 1\,193.3\bar{3}$ |

## Operations with numbers in scientific notation

1. Perform the operations indicated. (Leave your answers in scientific notation)

- |   |  |
|---|--|
| (a) $6.02 \times 10^{23} \times 1.6 \times 10^{-19}$<br>$= 6.02 \times 1.6 \times 10^{23+(-19)}$<br>$= 9.632 \times 10^4$ | (b) $(3.432 \times 10^3) \div (3 \times 10^{-12})$<br>$= (3.432 \div 3) \times 10^{3-(-12)}$<br>$= 1.144 \times 10^{15}$ |
|---|--|

$$\begin{aligned}
 \text{(c)} \quad & 3.4 \times 10^6 + 2.125 \times 10^8 \\
 & = 3.4 \times 10^{-2} \times 10^8 + 2.125 \times 10^8 \\
 & = 0.034 \times 10^8 + 2.125 \times 10^8 \\
 & = 2.159 \times 10^8
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad & 2.5 \times 10^{-6} - 8.12 \times 10^{-5} \\
 & = 2.5 \times 10^{-1} \times 10^{-5} - 8.12 \times 10^{-5} \\
 & = 0.25 \times 10^{-5} - 8.12 \times 10^{-5} \\
 & = -7.87 \times 10^{-5}
 \end{aligned}$$

2. Perform the operations indicated by changing to scientific notation.

$$\begin{aligned}
 \text{(a)} \quad & 2000000000 \times 0.00000003 \\
 & = 2 \times 10^9 \times 3 \times 10^{-8} \\
 & = 6 \times 10^1 \\
 & \text{or } 60
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & 0.00000045 \div 0.0000000031 \\
 & = (4.5 \times 10^{-7}) \div (3.1 \times 10^{-10}) \\
 & = (4.5 \div 3.1) \times 10^{-7 - (-10)} \\
 & = 1.452 \times 10^3 \\
 & \text{or } 1452
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad & 340000000000 \times 0.000000125 \\
 & = 3.4 \times 10^{11} \times 1.25 \times 10^{-7} \\
 & = 4.25 \times 10^4 \\
 & \text{or } 42500
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad & 0.65 \times 12000000 \div 0.000000085 \\
 & = 6.5 \times 10^{-1} \times 1.2 \times 10^7 \div (8.5 \times 10^{-8}) \\
 & = 7.8 \times 10^6 \div (8.5 \times 10^{-8}) \\
 & = 0.9176 \times 10^{6 - (-8)} \\
 & = 0.9176 \times 10^{14} \\
 & = 9.176 \times 10^{13}
 \end{aligned}$$