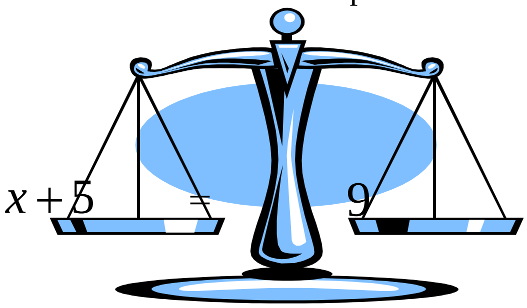
Topic 1: Equations

An algebraic equation can be represented by a balance like the one below. An equation contains an equal sign, indicating both sides are the same value. A balance will have equal masses on each side when the balance is level. A simple equation like $x+5=9$ can be represented on the balance as below.



$$x + 5 \quad = \quad 9$$

Solving an equation is finding the value of x that makes the equation true or balanced, that is, the left hand side will equal the right hand side.

This value is called the **solution** of the equation.

This equation is easy to solve, clearly the value $x=4$ is the solution of the equation, however, it is important to develop processes to cope with questions of higher difficulty.

$$x + 5 \quad = \quad 9$$

To find the value of x it is necessary to undo the operations on x. The variable x has had 5 added to it, so to undo this operation it is necessary to ask 'what is the opposite of adding 5?'

The opposite of adding 5 is subtracting 5, so if 5 is subtracted from the left hand side of the equation, the x will be on its own. However, if 5 is subtracted from the left side, the right had side of the balance will become heaver and tilt. For the balance to remain level (equal), 5 needs to be subtracted from the right side also.

Mathematically this is written as below and the solution is obtained.

$$x+5=9$$
$$x+5-5=9-5$$
$$x=4$$

The solution to the equation is $x=4$

Let's try another example: $2a=14$

$$2a=14$$

The variable a has been multiplied by 2, to undo this operation, the opposite of multiplying by 2 is dividing by 2, to maintain the balance, both sides must be divided by 2 and the solution obtained.

$$2a=14$$
$$\frac{\cancel{2}^{1}a}{\cancel{2}^{1}}=\frac{\cancel{14}^{7}}{\cancel{2}^{1}}$$
$$a=7$$

The solution to the equation is $a=7$.

Example: $p-7=3$

The variable p has had 7 subtracted from it, to undo this operation, the opposite of subtracting 7 is adding 7, to maintain the balance, both sides must have 7 added and the solution obtained.

$$p-7=3$$
$$p-7+7=3+7$$
$$p=10$$

The solution to the equation is $p=10$.

It is possible to check the answer by substituting $p=10$ into original equation.
Checking: Does $10-7=3$? Yes, so the solution is correct.

Example: $\frac{k}{6}=2$

The variable k has been divided by 6, to undo this operation, the opposite of dividing by 6 is multiplying by 6, to maintain the balance, both sides must be multiplied by 6 and the solution obtained.

$$\frac{k}{6}=2$$
$$\frac{k}{\cancel{6}^{1}}\times\cancel{6}^{1}=2\times 6$$
$$k=12$$

The solution to the equation is $k=12$.

Checking the answer by substituting $k=12$ into the original equation. Does $12\div 6=2$? Yes, so the solution is correct.

The equations solved so far have been one step equations. The next step is to look at **two step equations** such as $2a-5=7$. In these equations the 5 and the 2 have to be un-done, but it is important to do this in the correct order.

The story of this equation is that the variable a was firstly multiplied by 2 and then 5 was subtracted, this was implied from our knowledge of order of operations.

To find the value of a, the last operation must be undone first. The first step is to undo the subtraction of 5 by doing the opposite which is adding 5 to both sides to maintain the balance. The second step is to undo the multiplication of 2 by doing the opposite which is dividing both sides by 2 to maintain the balance.

The working for this is:

$$2a-5=7$$
$$2a-5+5=7+5 \quad \text{Add 5 to both sides}$$
$$2a=12 \quad \text{Tidy up}$$
$$\frac{\cancel{2}^{1}a}{\cancel{2}^{1}}=\frac{\cancel{12}^{6}}{\cancel{2}^{1}} \quad \text{Divide both sides by 2}$$
$$a=6 \quad \text{Tidy up}$$

Example: $\frac{x}{4}+1=6$ The story of this equation is that the variable x was firstly divided by 4 and then 1 was added. To find the value of x the first step is to undo the addition of 1 by doing the opposite which is subtracting 1 from both sides to maintain the balance. The next step is to undo the division by 4 by doing the opposite which is multiplication by 4 on both sides to maintain the balance.

$$\frac{x}{4}+1=6$$
$$\frac{x}{4}+1-1=6-1 \quad \text{subtract 1 from both sides}$$
$$\frac{x}{4}=5 \quad \text{tidy up}$$
$$\frac{x}{4}\times 4=5\times 4 \quad \text{multiply both sides by 4}$$
$$x=20 \quad \text{tidy up}$$

The next examples require three steps to get to a solution.

Example:

$$\frac{2x}{5}-7=3$$
$$\frac{2x}{5}-7+7=3+7 \quad \text{add 7 to both sides}$$
$$\frac{2x}{5}=10 \quad \text{tidy up}$$
$$\frac{2x}{\cancel{5}^1}\times \cancel{5}^1=10\times 5 \quad \text{multiply both sides by 5}$$
$$2x=50 \quad \text{tidy up}$$
$$\frac{\cancel{2}^1 x}{\cancel{2}^1}=\frac{50}{2} \quad \text{divide both sides by 2}$$
$$x=25$$

Example:

$$\frac{x-9}{4}+7={}^{-}5$$
$$\frac{x-9}{4}+7-7={}^{-}5-7 \quad \text{Subtract 7 from both sides}$$
$$\frac{x-9}{4}={}^{-}12 \quad \text{Tidy up}$$
$$\frac{x-9}{\cancel{4}^1}\times \cancel{4}^1={}^{-}12\times 4 \quad \text{Multiply both sides by 4}$$
$$x-9={}^{-}48 \quad \text{Tidy up}$$
$$x-9+9={}^{-}48+9 \quad \text{Add 9 to both sides}$$
$$x={}^{-}39$$

Some questions may contain other variables.

Example: Solve the following equation for x.

$$
\begin{aligned}
ax - b &= 6 \\
ax - b + b &= 6 + b && \text{adding } b \text{ to both sides} \\
ax &= 6 + b && \text{tidy up} \\
\frac{\cancel{a}^{1}x}{\cancel{a}^{1}} &= \frac{6+b}{a} && \text{dividing both sides by } a \\
x &= \frac{6+b}{a}
\end{aligned}
$$

Some questions may contain brackets. Expanding brackets is usually the easiest but can be done without expanding as shown below.

Example: Solve the following equation.

$$
\begin{aligned}
3(b-4) &= 18 \\
3b - 12 &= 18 && \text{Expand brackets} \\
3b - 12 + 12 &= 18 + 12 && \text{Add 12 to both sides} \\
3b &= 30 && \text{Tidy up} \\
\frac{\cancel{3}^{1}b}{\cancel{3}^{1}} &= \frac{\cancel{30}^{10}}{\cancel{3}^{1}} && \text{Divide both sides by 10} \\
b &= 10 && \text{Tidy up}
\end{aligned}
$$

or

$$
\begin{aligned}
3(b-4) &= 18 \\
\frac{\cancel{3}^{1}(b-4)}{\cancel{3}^{1}} &= \frac{\cancel{18}^{6}}{\cancel{3}^{1}} \\
b - 4 &= 6 \\
b - 4 + 4 &= 6 + 4 \\
b &= 10
\end{aligned}
$$

There some examples where care is required. The first of these is when there is a negative sign in front of the variable.

Example:

$$7 - y = 5$$

It is best to think of this as: 7 has been added to and y has been subtracted from the left hand side. Think: what is the opposite of adding 7? Subtracting 7 from both sides. The second method involves adding y to both sides.

$$
\begin{aligned}
7 - y &= 5 \\
7 - y - 7 &= 5 - 7 \\
{}^{-}y &= {}^{-}2 \\
\text{or } {}^{-}1y &= {}^{-}2 \\
\frac{{}^{-}\cancel{1}^{1}y}{{}^{-}\cancel{1}^{1}} &= \frac{{}^{-}\cancel{2}^{2}}{{}^{-}\cancel{1}^{1}} \\
y &= 2
\end{aligned}
$$

or

$$
\begin{aligned}
7 - y &= 5 \\
7 - y + y &= 5 + y \\
7 &= 5 + y \\
7 - 5 &= 5 + y - 5 \\
2 &= y
\end{aligned}
$$

Another tricky situation is when the variable is in the denominator like in the example below, once again there are two methods, the first involves taking the reciprocal of both sides, the second involves getting the variable out of the denominator by multiplying both sides by the variable.

Example:

$$\frac{3}{x} = 2$$

$$\frac{3}{x} = \frac{2}{1} \quad \text{write both as single fractions}$$

$$\frac{x}{3} = \frac{1}{2} \quad \text{take reciprocal of both sides}$$

$$\frac{x}{\cancel{3}^1} \times \cancel{3}^1 = \frac{1}{2} \times 3 \quad \text{multiply both sides by 3}$$

$$x = \frac{3}{2} \text{ or } 1\frac{1}{2}$$

or

$$\frac{3}{x} = 2$$

$$\frac{3}{\cancel{x}^1} \times \cancel{x}^1 = 2 \times x$$

$$3 = 2x$$

$$\frac{3}{2} = x$$

The method for solving equations shown above is logically the process that is required. However after doing many practice examples, many people notice that when a number is moved from one side of the equals sign to the other, it does the opposite operation. We'll call this the shortcut method.

$$4x = 24$$

$$x = \frac{24}{4} = 6$$

Multiplying by 4 on the LHS becomes divide by 4 on the RHS.

$$a - 7 = 2$$

$$a = 2 + 7 = 9$$

Subtracting 7 on the LHS becomes adding 7 on the RHS

A 3 step problem is performed in the same order.

$$\frac{2x-5}{3} = 7$$

$\div 3$ on the LHS becomes $\times 3$ on the RHS

$$2x - 5 = 7 \times 3$$

$$2x - 5 = 21$$

-5 on the LHS becomes $+5$ on the RHS

$$2x = 21 + 5$$

$$2x = 26$$

$\times 2$ on the LHS becomes $\div 2$ on the RHS

$$x = \frac{26}{2}$$

$$x = 13$$

Video 'Solving Equations'

Activity

1. Solve the equations below for the given variable

(a) $7x = 49$ (b) $x + 5 = 2$ (c) $\frac{w}{5} = {}^{-}3$

(d) $6 + k = 4$ (e) $3x - 7 = 11$ (f) $p - 1.5 = 4.2$

(g) ${}^{-}3h = 21$ (h) $\frac{g}{5} = \frac{2}{3}$ (i) $5 - x = 7$

(j) $6 = 2t - 7$ (k) $\frac{x}{3} + 2 = 5$ (l) $\frac{x-4}{2} = 7$

(m) $\frac{4}{x} = \frac{5}{7}$ (n) ${}^{-}x + 1 = 6$ (o) ${}^{-}4 + 3x = 15$

(p) $\frac{3r}{4} - 5 = 1$ (q) $\frac{5-2x}{3} = 2$ (r) $\frac{2x-5}{4} = {}^{-}7$

2. Solve the equations below for t.

(a) $7(t-1) = 9$ (b) $2t + a = 2$ (c) $\frac{w}{t} = {}^{-}3$

(d) $3(2t - a) = 4$ (e) $3t - x + y = {}^{-}7$ (f) $4 - 3t = 7$

Equations containing like terms

An equation containing like terms is similar to this example: $4x-7=x+5$.

The equation has like terms on opposite sides of the equals sign. To solve these equations all like terms need to be on one side of the equals sign and the constant terms on the other.

$$4x-7=x+5$$

The suggestion in this question is to move the like term to the LHS and the constant terms to the RHS. It is important to remember that initially terms are being moved not individual numbers.

Moving the 7 using the shortcut method, subtracting 7 on LHS becomes adding 7 on the RHS.

$$\begin{aligned} 4x-7&=x+5 \\ 4x&=x+5+7 \\ 4x&=x+12 \end{aligned}$$

Moving the x using the shortcut method, x has been added to the RHS so it will be subtracted on the LHS.

$$\begin{aligned} 4x&=x+12 \\ 4x-x&=12 \\ 3x&=12 \end{aligned}$$

Then this simple one step equation is solved.

$$\begin{aligned} 3x&=12 \\ x&=\frac{12}{3} \\ x&=4 \end{aligned}$$

Example: $4x+3=7-2x$

$$\begin{aligned} 4x+3&=7-2x \\ 4x&=7-2x-3 \\ 4x&=4-2x \\ 4x+2x&=4 \\ 6x&=4 \\ x&=\frac{6}{4} \text{ or } \frac{3}{2} \text{ or } 1\frac{1}{2} \end{aligned}$$

+3 on RHS becomes -3 on LHS
Add like terms ($7-3=5$)

$-2x$ on RHS becomes $+2x$ on LHS
Add like terms ($4x+2x=6x$)

$\times 6$ on LHS becomes $\div 6$ on RHS

Example: $5-2a=a+11$

$$
\begin{aligned}
5-2a &= a+11\\
-2a &= a+11-5\\
-2a &= a+6\\
-2a-a &= 6\\
-3a &= 6\\
a &= \frac{6}{-3} = {}^{-}2
\end{aligned}
$$

Or

$$
\begin{aligned}
5-2a &= a+11\\
5-11-2a &= a\\
-6-2a &= a\\
-6 &= a+2a\\
-6 &= 3a\\
\frac{-6}{3} &= a\\
{}^{-}2 &= a
\end{aligned}
$$

<u>There are two ways of performing this question.</u>

5 was added to the LHS, and becomes -5 on the RHS. 11-5=6

a was added to the RHS and becomes $-a$ on the LHS. $-2a-a=-3a$

$\times{}^{-}3$ on the LHS becomes $\div{}^{-}3$ on the RHS (remember it is the operation that changes not the sign of the number)

11 was added to the RHS and becomes -11 on the LHS. 5-11=-6

$-2a$ on the LHS become $+2a$ on the RHS, $a+2a=3a$

$\times 3$ on the LHS becomes $\div 3$ on the RHS, $-6\div 3=-2$

Example: $6y-7=4y$

$$
\begin{aligned}
6y-7 &= 4y\\
6y &= 4y+7\\
6y-4y &= 7\\
2y &= 7\\
y &= \frac{7}{2} = 3\frac{1}{2}
\end{aligned}
$$

7 was subtracted from the LHS and becomes +7 on the RHS.

$4y$ was added to the RHS and becomes $-4y$ on the LHS.

$\times 2$ on the LHS becomes $\div 2$ on the RHS

Example: $2(3x-1)=4(2-x)$

$$
\begin{aligned}
2(3x-1) &= 4(2-x)\\
6x-2 &= 8-4x\\
6x &= 8-4x+2\\
6x &= 10-4x\\
6x+4x &= 10\\
10x &= 10\\
x &= \frac{10}{10} = 1
\end{aligned}
$$

Expand brackets

-2 on the LHS becomes +2 on the RHS, 8+2=10

$-4x$ on the LHS becomes $+4x$ on the RHS, $6x+4x=10x$

$\times 10$ on the LHS becomes $\div 10$ on the RHS, $10\div 10=1$

[Video ‘Solving Equations II’]

Activity (continued)

3. Solve the equations below for the variable used.

(a) $7t - 1 = 5t + 5$ (b) $3x + 11 = 3 - x$ (c) $15 - p = 2p$

(d) $2(a - 3) = 4a - 6$ (e) $3(2s - 7) = 2(3s - 1)$ (f) $5(y - 2) = 3(4 - y)$

(g) $0.6(3y - 1) = 0.4y - 1$ (h) $4 - 7a = 3(1 - 3a)$ (i) $2 - (3x + 9) = 4(5 - 3x)$

Transposition of formulae

A formula is an established rule that relate variables together for well defined situations. For example: $V = IR$ is a formula showing the mathematical relationship between Voltage (V), Current (I) and Resistance (R) for electrical circuits. The subject of this formula is Voltage (V) because this variable is expressed in terms of other variables. The word transposition means to rearrange the formula to obtain a different subject. Transposing formulae uses the same skills required to solve equations.

The word formulae is the plural of formula.

Examples:

Transpose the formula $V = IR$ to make R the subject.

$V = IR$	opposite of $\times I$ is $\div I$ on both sides	or	$V = IR$	
$\frac{V}{I} = \frac{\cancel{I}^1 \times R}{\cancel{I}^1}$			$\frac{V}{I} = R$	$\times I$ on RHS becomes $\div I$ on LHS
$\frac{V}{I} = R$				

Transpose the simple interest formula $I = PRT$ to make T the subject.

$I = PRT$	opposite to $\times PR$ is $\div PR$ on both sides	or	$I = PRT$	$\times PR$ on RHS
$\frac{I}{PR} = \frac{\cancel{PR}^1 T}{\cancel{PR}^1}$			$\frac{I}{PR} = T$	becomes $\div PR$ on RHS
$\frac{I}{PR} = T$				

Transpose the Perimeter of a Rectangle formula $P = 2l + 2b$ to make b the subject.

$$P = 2l + 2b \qquad \text{Move the term } 2l \text{ first}$$
$$P - 2l = 2l - 2l + 2b \qquad \text{Opposite of adding } 2l$$
$$P - 2l = 2b \qquad \text{is subtracting } 2l \text{ from both sides}$$
$$\frac{P - 2l}{2} = \frac{\cancel{2}^1 b}{\cancel{2}^1} \qquad \text{Opposite of } \times 2 \text{ is } \div 2 \text{ on both sides}$$

or

$$P = 2l + 2b$$
$$P - 2l = 2b \qquad +2l \text{ on RHS becomes } -2l \text{ on LHS}$$
$$\frac{P - 2l}{2} = b \qquad \times 2 \text{ on RHS becomes } \div 2 \text{ on LHS}$$

Transpose the final velocity of a body equation $v = u + at$ to make t the subject.

$$v = u + at \qquad u \text{ first to go}$$
$$v - u = u - u + at \qquad \text{opposite to } +u \text{ is } -u \text{ from both sides}$$
$$v - u = at \qquad \text{opposite to } \times a \text{ is } \div a \text{ from both sides}$$
$$\frac{v - u}{a} = \frac{\cancel{a}^1 t}{\cancel{a}^1}$$
$$\frac{v - u}{a} = t$$

or

$$v = u + at$$
$$v - u = at \qquad +u \text{ on RHS becomes } -u \text{ on LHS}$$
$$\frac{v - u}{a} = t \qquad \times a \text{ on RHS becomes } \div a \text{ on LHS}$$

With fractions, multiply through by the denominator to remove the fraction, in this question, multiply both sides by 2.

Transpose the kinetic energy of a body formula $E = \frac{1}{2}mv^2$ to make m the subject.

$$E = \frac{1}{2}mv^2 \quad \text{multiply both sides by 2 to remove the fraction}$$

$$2E = \cancel{2}^1 \times \frac{1}{\cancel{2}^1}mv^2 \quad \text{the opposite to } \times v^2 \text{ is } \div v^2 \text{ on both sides}$$

$$2E = mv^2$$

$$\frac{2E}{v^2} = m$$

or

$$E = \frac{1}{2}mv^2 \quad \text{multiply both sides by 2 to remove the fraction}$$

$$2E = mv^2 \quad \times v^2 \text{ on the RHS becomes } \div v^2 \text{ on the LHS}$$

$$\frac{2E}{v^2} = m$$

Transpose the area of a circle formula $A = \pi r^2$ to make r the subject.

$$A = \pi r^2 \quad \text{the opposite to } \times \pi \text{ is } \div \pi \text{ on both sides}$$

$$\frac{A}{\pi} = \frac{\cancel{\pi}^1 r^2}{\cancel{\pi}^1} \quad \text{the opposite to squaring is square root of both sides}$$

$$\sqrt{\frac{A}{\pi}} = \sqrt{r^2}$$

$$\sqrt{\frac{A}{\pi}} = r$$

or

$$A = \pi r^2 \quad \times \pi \text{ on RHS becomes } \div \pi \text{ on LHS}$$

$$\frac{A}{\pi} = r^2 \quad \text{squaring on the RHS becomes square root on RHS}$$

$$\sqrt{\frac{A}{\pi}} = r$$

Transpose this equation $Z = \sqrt{R^2 + \omega^2 L^2}$ to make L the subject.

$$Z = \sqrt{R^2 + \omega^2 L^2}$$

$$Z^2 = \left(\sqrt{R^2 + \omega^2 L^2}\right)^2 \quad \text{the opposite of square root is squaring on both sides}$$

$$Z^2 = R^2 + \omega^2 L^2 \quad \text{the opposite of adding } R^2 \text{ is subtracting } R^2 \text{ on both sides}$$

$$Z^2 - R^2 = R^2 - R^2 + \omega^2 L^2$$

$$Z^2 - R^2 = \omega^2 L^2 \quad \text{the opposite of multiplying by } \omega^2 \text{ is dividing by } \omega^2 \text{ on both sides}$$

$$\frac{Z^2 - R^2}{\omega^2} = \frac{\cancel{\omega^2}^{1} L^2}{\cancel{\omega^2}^{1}}$$

$$\frac{Z^2 - R^2}{\omega^2} = L^2 \quad \text{the opposite of squaring is taking the square root on both sides}$$

$$\sqrt{\frac{Z^2 - R^2}{\omega^2}} = \sqrt{L^2}$$

$$\sqrt{\frac{Z^2 - R^2}{\omega^2}} = L$$

or

$$Z = \sqrt{R^2 + \omega^2 L^2} \quad \text{the square root on the RHS becomes squaring on the LHS}$$

$$Z^2 = R^2 + \omega^2 L^2 \quad \text{adding } R^2 \text{on the RHS becomes subtracting } R^2 \text{ on the LHS}$$

$$Z^2 - R^2 = \omega^2 L^2 \quad \text{multiplying by } \omega^2 \text{on the RHS becomes dividing by} \omega^2 \text{ on the LHS}$$

$$\frac{Z^2 - R^2}{\omega^2} = L^2 \quad \text{the square on the RHS become a square root on the LHS}$$

$$\sqrt{\frac{Z^2 - R^2}{\omega^2}} = L$$

The formula for finding the total capacitance of two capacitors in series is given by the equation $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$, transpose to make C_2 the subject. This is quite a difficult example. At some stage it is important to add the fractions with a common denominator.

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{the opposite to adding } \frac{1}{C_1} \text{ is subtracting } \frac{1}{C_1} \text{ from both sides}$$

$$\frac{1}{C} - \frac{1}{C_1} = \frac{1}{C_1} - \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C} - \frac{1}{C_1} = \frac{1}{C_2} \quad \text{Make a single fraction on both sides to then take reciprocal of both sides}$$

$$\frac{C_1 - C}{CC_1} = \frac{1}{C_2} \quad \text{Now take reciprocal of both sides}$$

$$\frac{CC_1}{C_1 - C} = \frac{C_2}{1}$$

$$C_2 = \frac{CC_1}{C_1 - C}$$

Or

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}$$ the term $\frac{1}{C_1}$ is added on the RHS becomes $-\frac{1}{C_1}$ on the LHS

$$\frac{1}{C}-\frac{1}{C_1}=\frac{1}{C_2}$$ make both sides into a single fraction

$$\frac{C_1-C}{CC_1}=\frac{1}{C_2}$$ take the reciprocal of both sides

$$\frac{CC_1}{C_1-C}=\frac{C_2}{1}$$

$$C_2=\frac{CC_1}{C_1-C}$$

Video 'Transposing formulae'

Activity (continued)

4. Transpose the following equations for the subject given in the brackets.

(a) $P = \dfrac{W}{t}$ (W)

(b) $P = \dfrac{W}{t}$ (t)

(c) $A = l \times b \quad (l)$

(d) $C = Vq + F \quad (q)$

(e) $V = \dfrac{1}{3}\pi r^2 h \quad (h)$

(f) $V = \dfrac{1}{3}\pi r^2 h \quad (r)$

(g) $R = \dfrac{l}{\mu a} \quad (l)$

(h) $R = \dfrac{l}{\mu a} \quad (a)$

(i) $A = \dfrac{1}{2}bh \quad (b)$

(j) $C = 2\pi r \quad (r)$

(k) $c^2 = a^2 + b^2 \quad (a)$

(l) $PV = nRT \quad (P)$

(m) $PV = nRT \quad (n)$

(n) $Q_1 = P(Q_2 - Q_1) \quad (P)$

(o) $Q_1 = P(Q_2 - Q_1) \quad (Q_2)$

(p) $Q_1 = P(Q_2 - Q_1) \quad (Q_1)$
Hint: expand brackets, collect like terms, take a common factor, solve.

(q) $R_2 = R_1(1 + \alpha t) \quad (t)$

(r) $E = mc^2 \quad (c)$

(s) $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v} \quad (v)$

(t) $T = mg + m\omega^2 r \quad (r)$

(u) $P = \dfrac{V_1(V_2 - V_1)}{gJ} \quad (V_2)$

(v) $Z = \sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2} \quad (\omega)$

Topic 2: Equations with exponents

Before starting to solve equations containing exponents, it is important to think about opposite operations just like in the Equations topic.

You will recall the opposites below:

Operation	Opposite Operation
Addition +	Subtraction -
Subtraction -	Addition +
Multiplication x	Division ÷
Division ÷	Multiplication x
Squaring n^2	Square root $\sqrt{n}$
Square root $\sqrt{n}$	Squaring n^2

Revision example:

$$
\begin{aligned}
2x^2+3&=11\\
2x^2+3-3&=11-3 \quad \rightarrow \text{take 3 from both sides}\\
2x^2&=8\\
\frac{2x^2}{2}&=\frac{8}{2} \quad \rightarrow \text{divide both sides by 2}\\
x^2&=4\\
\sqrt{x^2}&=\pm\sqrt{4} \quad \rightarrow \text{take square root of both sides}\\
x&=\pm 2
\end{aligned}
$$

There are two solutions here because both 2 and -2 satisfy the equation. This always occurs with even roots.

$$
\begin{aligned}
2x^2+3&=11\\
2x^2&=11-3 \quad \rightarrow +3 \text{ becomes -3 on the RHS}\\
2x^2&=8\\
x^2&=\frac{8}{2} \quad \rightarrow \times 2 \text{ becomes } \div 2 \text{ on the RHS}\\
x^2&=4\\
x&=\pm\sqrt{4} \quad \rightarrow {}^2 \text{ becomes } \sqrt{\ } \text{ on the RHS}\\
x&=\pm 2
\end{aligned}
$$

This list can be expanded to cubes, cube roots, to the 4, forth root, etc.

Operation	Opposite Operation
Cubing ( )^3	Cube root ($\sqrt[3]{\ }$) or $^{\frac{1}{3}}$
To the 4th power	Forth root ($\sqrt[4]{\ }$) or $^{\frac{1}{4}}$

To the 5th power	Fifth root ($\sqrt[5]{\ }$) or $\frac{1}{5}$
To the 6th power	Sixth root ($\sqrt[6]{\ }$) or $\frac{1}{6}$
To the 7th power	Seventh root ($\sqrt[7]{\ }$) or $\frac{1}{7}$
etc.	etc.
To the power $\frac{2}{3}$	To the power $\frac{3}{2}$
To the power $\frac{3}{5}$	To the power $\frac{5}{3}$

Examples: Solve for x

Both sides method	**Short -cut Method**
$x^4 = 256$ $\sqrt[4]{x^4} = \sqrt[4]{256}$ $\rightarrow$ take the fourth root of both sides $x = \pm 4$ There are two solutions here because both 4 and -4 satisfy the equation. This always occurs with even roots	$x^4 = 256$ $x = \sqrt[4]{256}$ $\rightarrow$ ^{4} becomes $\sqrt[4]{\ }$ on the RHS $x = \pm 4$
$\sqrt[6]{x} = 3$ $\left(\sqrt[6]{x}\right)^6 = 3^6$ raise both sides to the power of 6 $x = 729$	$\sqrt[6]{x} = 3$ $x = 3^6$ $\rightarrow$ $\sqrt[6]{\ }$ becomes ^{6} on the RHS $x = 729$
$3\sqrt[5]{x} - 1 = 5$ $3\sqrt[5]{x} - 1 + 1 = 5 + 1 \rightarrow$ add 1 to both sides $3\sqrt[5]{x} = 6$ $\frac{3\sqrt[5]{x}}{3} = \frac{6}{3} \rightarrow$ divide both sides by 3 $\sqrt[5]{x} = 2$ $\left(\sqrt[5]{x}\right)^5 = 2^5 \rightarrow$ raise both side to the power of 5 $x = 32$	$3\sqrt[5]{x} - 1 = 5$ $3\sqrt[5]{x} = 5 + 1 \rightarrow$ - 1 becomes + 1 on the RHS $3\sqrt[5]{x} = 6$ $\sqrt[5]{x} = \frac{6}{3} \rightarrow \times$ by 3 becomes $\div$ by 3 on the RHS $\sqrt[5]{x} = 2$ $x = 2^5 \rightarrow$ $\sqrt[5]{\ }$ becomes $(\ )^5$ on the RHS $x = 32$
$4x^{-2} = 40$ $\frac{\cancel{4}^1 x^{-2}}{\cancel{4}^1} = \frac{\cancel{40}^{10}}{\cancel{4}^1} \rightarrow$ divide both sides by 4 $x^{-2} = 10 \rightarrow$ raise both sides to the $-\frac{1}{2}$ $\left(x^{-2}\right)^{-\frac{1}{2}} = 10^{-\frac{1}{2}}$ $x = \frac{1}{10^{\frac{1}{2}}}$ $x = \frac{1}{\sqrt{10}}$ $x = \pm 0.3162$ (to 4 d.p.)	$4x^{-2} = 40$ $x^{-2} = \frac{\cancel{40}^{10}}{\cancel{4}^1} \rightarrow \times 4$ becomes $\div 4$ on the RHS $x^{-2} = 10$ $x = 10^{-\frac{1}{2}} \rightarrow$ $^{-2}$ becomes $^{-\frac{1}{2}}$ on the RHS $x = \frac{1}{\sqrt{10}}$ $x = \pm 0.3162$ (to 4 d.p.)

$(2x-1)^2=49$ $\sqrt{(2x-1)^2}=\sqrt{49}\rightarrow$ take square root of both sides $2x-1=\pm 7$ $2x-1+1=7+1$ or $-7+1\rightarrow$ add 1 to both sides $2x=8$ or -6 $\frac{2x}{2}=\frac{8}{2}$ or $\frac{-6}{2}\rightarrow$ divide both sides by 2 $x=4$ or -3	$(2x-1)^2=49$ $2x-1=\sqrt{49}\rightarrow$ ^{2} becomes $\sqrt{\ }$ on the RHS $2x-1=\pm 7$ $2x=7+1$ or $-7+1\rightarrow$ -1 becomes $+1$ on the RHS $2x=8$ or -6 $x=\frac{8}{2}$ or $\frac{-6}{2}\rightarrow$ $\times 2$ becomes $\div 2$ on the RHS $x=4$ or -3
In the question below, the opposite of raising a number to a power is raising it to the reciprocal power. Remember a number multiplied by its reciprocal gives 1.	In the question below, the fraction exponent becomes the reciprocal on the other side of the equation.
$x^{\frac{2}{3}}=50$ $\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}}=50^{\frac{3}{2}}$ $x=\sqrt{50^3}$ $x=353.55$	$x^{\frac{2}{3}}=50$ $x=50^{\frac{3}{2}}$ $x=\sqrt{50^3}$ $x=353.55$

It is also important to be able to rearrange formulae that contain powers.

Rearrange the formulae below to make the pronumeral in brackets the subject.	
$A=\pi r^2 \quad [r]$ $\frac{A}{\pi}=\frac{\cancel{\pi}^1 r^2}{\cancel{\pi}^1}\rightarrow$ divide both sides by π $\frac{A}{\pi}=r^2\rightarrow$ take the square root of both sides $\sqrt{\frac{A}{\pi}}=\sqrt{r^2}$ $r=\sqrt{\frac{A}{\pi}}$	$A=\pi r^2 \quad [r]$ $\frac{A}{\pi}=r^2\rightarrow\times\pi$ becomes $\div\pi$ on LHS $\sqrt{\frac{A}{\pi}}=r\rightarrow{}^2$ becomes $\sqrt{\ }$ on the LHS $r=\sqrt{\frac{A}{\pi}}$
$V=\frac{4}{3}\pi r^3 [r]$ The first step is to remove the fraction $3V=\frac{4}{\cancel{3}^1}\pi r^3\times\cancel{3}^1\rightarrow$ Multiple both sides by 3 $3V=4\pi r^3$ $\frac{3V}{4\pi}=\frac{\cancel{4\pi}r^3}{\cancel{4\pi}}\rightarrow$ Divide both sides by 4π $\frac{3V}{4\pi}=r^3$ $\sqrt[3]{\frac{3V}{4\pi}}=\sqrt[3]{r^3}\rightarrow$ take cube root of both sides $r=\sqrt[3]{\frac{3V}{4\pi}}$	$V=\frac{4}{3}\pi r^3 [r]$ The first step is to remove the fraction $3V=4\pi r^3\rightarrow\div 3$ becomes $\times 3$ on LHS $\frac{3V}{4\pi}=r^3\rightarrow$ $\times 4\pi$ becomes $\div 4\pi$ on LHS $\sqrt[3]{\frac{3V}{4\pi}}=r\rightarrow$ ^{3} becomes $\sqrt[3]{\ }$ on the LHS $r=\sqrt[3]{\frac{3V}{4\pi}}$

$$f=\frac{1}{2\pi\sqrt{LC}} \quad [C]$$

$$f\times\sqrt{LC}=\frac{1}{2\pi\cancel{\sqrt{LC}}}\times\cancel{\sqrt{LC}} \rightarrow \text{multiply both sides by } \sqrt{LC} \text{ to get C into the numerator}$$

$$\frac{\cancel{f}\times\sqrt{LC}}{\cancel{f}}=\frac{1}{2\pi f} \rightarrow \text{ divide both sides by } f$$

$$\left(\sqrt{LC}\right)^2=\left(\frac{1}{2\pi f}\right)^2 \rightarrow \text{ square both sides}$$

$$LC=\frac{1}{4\pi^2 f^2}$$

$$\frac{\cancel{L}C}{\cancel{L}}=\frac{1}{4\pi^2 f^2 L} \rightarrow \text{ divide both sides by } L$$

$$C=\frac{1}{4\pi^2 f^2 L}$$

Or

$$f=\frac{1}{2\pi\sqrt{LC}} \quad [C]$$

$$f\times\sqrt{LC}=\frac{1}{2\pi} \rightarrow \div\sqrt{LC} \text{ becomes } \times\sqrt{LC} \text{ on the LHS}$$

$$\sqrt{LC}=\frac{1}{2\pi f} \rightarrow \times f \text{ becomes } \div f \text{ on the RHS}$$

$$LC=\left(\frac{1}{2\pi f}\right)^2 \rightarrow \sqrt{\ } \text{ becomes } {}^2 \text{ on the RHS}$$

$$LC=\frac{1}{4\pi^2 f^2}$$

$$C=\frac{1}{4\pi^2 f^2 L} \rightarrow \times L \text{ becomes } \div L \text{ on the RHS}$$

Video ‘Equations containing exponents’

Activity

1. Solve for x:

(a) $x^3 = 27$ (b) $x^5 + 1 = 244$ (c) $3\sqrt[4]{x} = 6$

(d) $\sqrt[3]{x+1} = 5$ (e) $\sqrt[12]{x} = 2$ (f) $4\sqrt{x} + 31 = 63$

(g) $5x^2 + 4 = 9$ (h) $7(x^4 + 1) = 119$ (i) $x^{-4} = 20$ (use calc.)

(j) $\dfrac{x^{-1}}{4} = 11$ (k) $4x^{-5} = 5$ (l) $3x^{\frac{1}{2}} + 7 = 19$

(m) $3x^4 + 256 = 1024$

2. Rearrange these formulae for the pronumeral in brackets

(a) $A = 4\pi r^2$ $[r]$ (b) $V = \pi r^2 h$ $[r]$ (c) $c^2 = a^2 + b^2$ $[b]$

(d) $F = \dfrac{1}{2}mv^2$ $[v]$ (e) $Q = SLd^2$ $[d]$ (f) $v^2 = u^2 + 2as$ $[u]$

(g) $Z = \sqrt{R^2 + \omega^2 L^2}$ $[L]$ (h) $R = \dfrac{2GM}{c^2}$ $[c]$ (i) $P = \dfrac{\pi^2 EI}{L^2}$ $[L]$

The next questions are challenging, take care!

(j) $T = 2\pi\sqrt{\dfrac{r^3}{GM}}$ $[r]$ (k) $F = m\left(g - \dfrac{v^2}{R}\right)$ $[v]$ (l) $R = \dfrac{\pi r^4 (p_2 - p_1)}{8nL}$ $[r]$

(m) $A = P(1+i)^n$ $[i]$

Equations

Rearranging linear equations was covered thoroughly in the Equations topic.

In this section Quadratic Equations are covered as they are linked to the work already performed in this module.

Topic 3: Quadratic equations

A quadratic equation is one that can be written in the form:

$$ax^2 + bx + c = 0$$
$$\text{where } a \neq 0$$

In a previous section, you learnt how to factorise a trinomial. The first step to solve the equation is to attempt to factorise the trinomial. If this fails, the General Quadratic Formula can be used. The General Quadratic Formula was obtained by a process called completing the square.

A simple example is: Solve for x: $x^2 - 6x + 5 = 0$

This is written in the format required, that is, $ax^2 + bx + c = 0$

Now factorise: $(x-1)(x-5) = 0$ Factorising trinomials was covered in the Topic 'Factorisation'.

When two factors multiply to give 0, then either factor could be 0.
Either

$$x - 1 = 0 \quad \text{or} \quad x - 5 = 0$$
$$x = 1 \quad \text{or} \quad x = 5$$

Solutions can be checked just like any other equation.
When $x = 1$: $x^2 - 6x + 5 = 1^2 - 6\times1 + 5 = 1 - 6 + 5 = 0$ ✓
When $x = 5$: $x^2 - 6x + 5 = 5^2 - 6\times5 + 5 = 25 - 30 + 5 = 0$ ✓

Example: Solve for x: $4x^2 + 14x - 8 = 0$

When inspecting this equation, you will notice that the coefficients (4, 14, -8) have a common factor of 2.

$$4x^2+14x-8=0$$
$$2(2x^2+7x-4)=0$$
$$2(2x-1)(x+4)=0$$
Either $2x-1=0$ or $x+4=0$

$2x=1$ $\quad x=-4$

$$x=\frac{1}{2}$$

Example: Solve for *x*: $x^2+x=2$

In the correct format, this gives $x^2+x-2=0$
$$x^2+x-2=0$$
$$(x-1)(x+2)=0$$
Either $x-1=0$ or $x+2=0$

$x=1$ $\quad x=-2$

Example: Solve for *x*: $x^2=x+42$

$$x^2=x+42$$
$$x^2-x-42=0$$
$$(x-7)(x+6)=0$$
$x-7=0$ or $x+6=0$

$x=7$ $\quad x=-6$

Example: Solve for *x*: $x^2+7x=0$
$$x^2+7x=0$$
$$x(x+7)=0$$
Either $x=0$ or $x+7=0$

$x=0$ $\quad x=-7$

Example: Solve for *x*: $0.2x^2+0.4x-0.6=0$

To remove the decimals, the equation can be multiplied through by 5.

$$
\begin{aligned}
0.2x^2+0.4x-0.6&=0\\
x^2+2x-3&=0\\
(x+3)(x-1)&=0\\
x+3=0 \quad &\text{or} \quad x-1=0\\
x=-3 \quad &\qquad\quad x=1
\end{aligned}
$$

Example: Solve for *x*: $x^2=9$

Although this equation can be solved without bringing all terms to the left hand side, following the method is still preferred.

$$
\begin{aligned}
x^2&=9\\
x^2-9&=0\\
(x-3)(x+3)&=0\\
x-3=0 \quad &\text{or} \quad x+3=0\\
x=3 \quad &\qquad\quad x=-3
\end{aligned}
$$

Example: Solve for *x*: $(x-2)^2=9$

$$
\begin{aligned}
(x-2)^2&=9\\
x^2-4x+4&=9\\
x^2-4x-5&=0\\
(x-5)(x+1)&=0\\
x-5=0 \quad &\text{or} \quad x+1=0\\
x=5 \quad &\qquad\quad x=-1
\end{aligned}
$$

There may be instances when either answer is not applicable to the context given in the question.

For example: ‘A rectangle has an Area of 20m^2. The width of the rectangle is 8m less that its length. Calculate the length and width of this rectangle.’

We know: $A = l \times w = 20m^2$ and $w = l - 8$

Now

$$l \times w = 20$$
$$l \times (l - 8) = 20$$
$$l^2 - 8l - 20 = 0$$
$$(l - 10)(l + 2) = 20$$
$$l = 10 \text{ or } -2$$

If the length is 10m then the width will be $l - 8 = 2m$. The solution -2 does not fit the context of the question.

Completing the square

At this stage, completing the square will give a method of solving quadratic equations. It is also used to obtain the General Quadratic Equation. After this is obtained, most people prefer to substitute into this. When graphing quadratics, completing the square gives the location of the turning point of the parabola.

When completing the square, the idea is to write the equation with a perfect square. Remember, a perfect square is written as $(x \pm a)^2$ or $(ax \pm b)^2$.

As	$x^2 + 2ax + a^2$ $= (x + a)^2$	**For example:**	$x^2 + 6x + 9$ $= (x + 3)^2$

It is possible to observe a relationship

As	$x^2 + 2ax + a^2$ ÷2 square $= (x + a)^2$	**For example:**	$x^2 + 6x + 9$ ÷2 square $= (x + 3)^2$

Example:

Write the expression $x^2+6x+10$ in completing the square form.

$$x^2+6x+10$$

$$=x^2+6x+? \quad +10-?$$ • add a constant to make the perfect square

$$=\left(x^2+6x+9\right)+10-9$$ • subtract the constant for a zero net effect

$\div 2$ square

$$= \quad (x+3)^2+1$$

Write the expression $x^2-10x-20$ in completing the square form.

$$x^2-10x-20$$ • add a constant to make the perfect square

$$=\left(x^2-10x+25\right)-20-25$$ • subtract the constant for a zero net effect

$$=(x-5)^2-45$$

Write the expression x^2+7x-5 in completing the square form.

$$x^2+7x-5$$ • add a constant to make the perfect square

$$=\left(x^2+7x+\frac{49}{4}\right)-5-\frac{49}{4}$$ • subtract the constant for a zero net effect

$$=\left(x+\frac{7}{2}\right)^2-\frac{69}{4}$$

Write the expression $2x^2-8x+9$ in completing the square form.

$$2x^2-8x+9$$

$$=2\left[x^2-4x+\frac{9}{2}\right]$$ • add a constant to make the perfect square

$$=2\left[\left(x^2-4x+4\right)+\frac{9}{2}-4\right]$$ • subtract the constant for a zero net effect

$\div 2$ square

$$= \quad 2\left[(x-2)^2+\frac{1}{2}\right]$$

$$=2(x-2)^2+1$$

Write the expression $2x^2-7x-10$ in completing the square form.

$$2x^2 - 7x - 10$$

$$= 2\left[x^2 - \frac{7}{2}x - 5\right]$$ • add a constant to make the perfect square

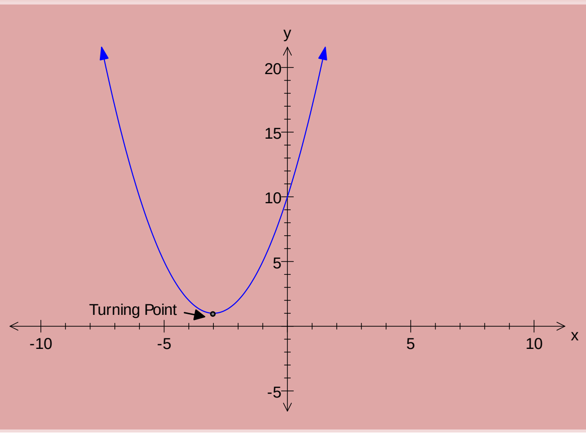
$$= 2\left[\left(x^2 - \frac{7}{2}x + \frac{49}{16}\right) - 5 - \frac{49}{16}\right]$$ • subtract the constant for a zero net effect

$$= \quad 2\left[\left(x - \frac{7}{4}\right)^2 - \frac{129}{16}\right]$$

$$= 2\left(x - \frac{7}{4}\right)^2 - \frac{129}{8}$$

Why is this useful?

The process of writing a trinomial in completing the square form is important in function study. For example: the graph of the function $y = x^2 + 6x + 10$ is a distinctive shape called a parabola. The graph is drawn below:



The turning point of the graph can be found by writing the function in completing the square form.

$$y = x^2 + 6x + 10$$

$$y = \left(x^2 + 6x + 9\right) + 10 - 9$$

$$y = \left(x + 3\right)^2 + 1$$

The turning point of the graph is (-3,1). (More on this in Function Study)

Solving quadratic equations by completing the square

So far, it has been possible to solve quadratic equations by factorising. The equations below cannot be factorised, so a new technique is required. The completing the square method, similar to the section above, is used.

Solve the equation $x^2 - 8x - 13 = 0$ by completing the square.

$x^2 - 8x - 13 = 0$ • Put the x^2 and x terms on the LHS, constant on the RHS

$x^2 - 8x = 13$ • Add constant to make perfect square

$x^2 - 8x + 16 = 13 + 16$ • Balance the equation

÷2 square

$$(x-4)^2 = 29$$

$$x - 4 = \pm\sqrt{29}$$

$$x = 4 \pm \sqrt{29}$$

$$x \approx 9.39 \text{ or } -1.39$$

Solve the equation $3x^2 - 9x + 1 = 0$ by completing the square.

$3x^2 - 9x + 1 = 0$ • Put the x^2 and x terms on the LHS, constant on the RHS

$3x^2 - 9x = -1$ • Add constant to make perfect square

$3\left(x^2 - 3x + \frac{9}{4}\right) = -1 + \frac{27^*}{4}$ • Balance the equation (with care!)

$$3\left(x - \frac{3}{2}\right)^2 = \frac{23}{4}$$

*The constant in this question was obtained from $3 \times \frac{9}{4}$

$$\left(x - \frac{3}{2}\right)^2 = \frac{23}{12}$$

$$x - \frac{3}{2} = \pm\sqrt{\frac{23}{12}}$$

$$x = \frac{3}{2} \pm \sqrt{\frac{23}{12}}$$

$$x \approx 2.88 \text{ or } 0.12$$

Solve the equation $2x^2 + x + 12 = 0$ by completing the square.

$$2x^2 + x + 12 = 0$$
$$2x^2 + x = -12$$
$$2\left(x^2 + \frac{1}{2}x\right) = -12$$
$$2\left(x^2 + \frac{1}{2}x + \frac{1}{16}\right) = -12 + \frac{1}{8}$$
$$2\left(x + \frac{1}{4}\right)^2 = -\frac{95}{8}$$
$$\left(x + \frac{1}{4}\right)^2 = -\frac{95}{16}$$
$$x + \frac{1}{4} = \sqrt{-\frac{95}{16}}$$

No solutions

It is not possible to find the square root of negative values. The inference here is that no solutions are possible or there are no values of x that make $2x^2 + x + 12 = 0$

For the General Quadratic Equation $ax^2 + bx + c = 0$

$$ax^2 + bx + c = 0$$
$$ax^2 + bx = -c$$
$$a\left(x^2 + \frac{b}{a}x\right) = -c$$
$$a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) = -c + \frac{b^2}{4a}$$
$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a}$$
$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$
$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$
$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Now there is a general solution for quadratic equations, many people memorise this expression. The example below shows how the general quadratic formula is used.

As a summary, if $ax^2 + bx + c$ can be factorised easily, then use this method to obtain solutions to quadratic equations. If factorising is difficult or not possible, then the general quadratic equation is used.

Solve the equation $4x^2 - x - 9 = 0$ by using the general quadratic formula.

$$4x^2 - x - 9 = 0$$

$a = 4,\ b = -1$ and $c = -9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4 \times 4 \times (-9)}}{2 \times 4}$$

$$x = \frac{1 \pm \sqrt{145}}{8}$$

$x \approx 1.63$ or -1.38

Solve the equation $2x^2 + 3x + 5 = 0$ by using the general quadratic formula.

$$2x^2 + 3x + 5 = 0$$

$a = 2,\ b = 3$ and $c = 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times 5}}{2 \times 2}$$

$$x = \frac{-3 \pm \sqrt{9 - 40}}{4}$$

$$x = \frac{-3 \pm \sqrt{-33}}{4}$$

Remembering that the square root of a negative number is undefined, the inference here is that there are no solutions to the quadratic equation.

With the examples given so far, it can be seen that there can be two solutions or no solutions. Obtaining a solution is determined by the square root.

Therefore, it is the value of $b^2 - 4ac$ that will determine the nature of the solutions.

Because $b^2 - 4ac$ is so significant, it is given a special name, the **discriminant** and the symbol Δ, ($\Delta = b^2 - 4ac$). The solutions to the equation are sometimes called the **roots** of the equation.

The value of the Discriminant	**Nature of the roots**	
If $\Delta > 0$	2 different solutions (roots)	$x = \frac{-b + \sqrt{\Delta^+}}{2a}$ *and* $x = \frac{-b - \sqrt{\Delta^+}}{2a}$
If $\Delta = 0$	2 equal solutions (roots)	$x = \frac{-b + \sqrt{0}}{2a}$ *and* $x = \frac{-b - \sqrt{0}}{2a}$
If $\Delta < 0$	No real solutions (roots)	$x = \frac{-b + \sqrt{\Delta^-}}{2a}$ *and* $x = \frac{-b - \sqrt{\Delta^-}}{2a}$

A Word problem:

The sum of the squares of two numbers is 97. The sum of the numbers is 13, determine the numbers.

Let the 2 numbers be *a* and *b*. Then $a^2 + b^2 = 97$ and $a + b = 13$.
Rewriting $a + b = 13$ gives $b = 13 - a$
Now $a^2 + b^2 = 97$ becomes $a^2 + (13-a)^2 = 97$, One variable has been eliminated.

$$a^2 + (13-a)^2 = 97$$
$$a^2 + 169 - 26a + a^2 = 97$$
$$2a^2 - 26a + 169 = 97$$
$$2a^2 - 26a + 72 = 0$$
$$2(a^2 - 13a + 36) = 0$$
$$2(a-4)(a-9) = 0$$
$$a = 4 \text{ or } 9$$

If *a=9* then *b=13-9=4*, If *a=4* the b=*13-4=9*, the two numbers are 4 and 9

Video ‘Solving quadratic equations’

Activity

1. Solve these quadratic equations for x: (by factorising)

(a) $x^2 - x - 6 = 0$ (b) $2x^2 - 9x + 4 = 0$

(c) $4x^2 + 19x - 5 = 0$ (d) $2x^2 - 4x - 16 = 0$

(e) $0.5x^2 + x - 12 = 0$ (f) $4x^2 - 9 = 0$

(g) $2x^2 + 8x + 8 = 0$ (h) $x^2 - 4x = 0$

2. Solve these quadratic equations for x: (by any method)

(a) $x^2 + 12x + 2 = 0$ (b) $x^2 + 4x - 7 = 0$

(c) $3x^2 + x + 1 = 0$ (d) $4x^2 - 5x + 7 = 0$

(e) $0.1x^2 + 0.2x = 0$ (f) $10 - 5x - x^2 = 0$

(g) $3.5x^2 + 24x - 3.5 = 0$ (h) $3x^2 - x - 10 = 0$

3. Solve these quadratic equations for x: (by completing the square)

(a) $x^2 + 4x - 7 = 0$ (b) $2x^2 + 4x - 1 = 0$

4. Solve these quadratic equations for x: (by using the general quadratic equation)

(a) $3x^2 - 8x - 3 = 0$ (b) $x^2 - x + 4 = 0$

(c) $2x^2 - 7x + 3 = 0$ (d) $4x^2 - 9x + 4 = 0$

5. Calculate the discriminant for these quadratic equations and comment.

(a) $x^2 + 4x = 0$ (b) $2x^2 - 12x + 18 = 0$

(c) $2x^2 - 3x + 3 = 0$ (d) $-2x^2 + 5x - 4 = 0$

6. In a right angled triangle, the length of the hypotenuse is 20. The length of one side is 4 longer than the other. Calculate all the lengths of the triangle.

Topic 4: Solving logarithmic and exponential equations

Logarithmic equations

There are two major ideas required when solving Logarithmic Equations.

The first is the Definition of a Logarithm. You may recall from an earlier topic:

> If a number (N) is written as a power with a base (b) and an exponent (e), such as
>
> $N = b^e$
>
> then $\log_b N = e$.
>
> The base, b, must be a positive number ($b > 0$) and not equal to 1 ($b \neq 1$).
>
> The number, N, must also be positive ($N > 0$)

For the equation below, changing the Logarithm form to Exponential form will solve for the variable x. (These were covered in the Definition of a Log topic)

$$\begin{aligned} \log_{10} x &= 2 \\ x &= 10^2 \\ x &= 100 \end{aligned}$$

> Note:
>
> When solving any Logarithm equations, you should perform a check to see that the answer(s) obtained were consistent with the definition of a Log.

Similar to this, the question below is solved using the definition of a Log and a knowledge of equations containing exponents.

$$\log_x 25 = 2$$
$$25 = x^2$$
$$x = \pm 5$$

Check:
As the base of a Log must be *(b > 0)* and *(b ≠ 1),* the solution *x=-5* is not consistent with the definition.

The solution to this question is *x=5*

This type of question can become more complex depending on the numbers. In this example, a calculator is required. You may need to review Exponents – Equations with Exponents.

$$\log_x 22 = 2.1$$
$$22 = x^{2.1}$$
$$x = \sqrt[2.1]{22}$$
$$x = 4.358 \text{ to 3 d.p.}$$

A third type is where the variable becomes the exponent. For example:
$\log_4 5 = x$ can be rewritten using the definition of a log to be $5 = 4^x$ which can be solved as an exponential equation (see below). However, a much easier method of solution is to use the Change of Base Rule.

$$\log_4 5 = x$$
$$x = \frac{\log 5}{\log 4} \quad \left(\text{alternatively } \frac{\ln 5}{\ln 4}\right)$$
$$x = 1.161 \text{ to 3 d.p.}$$

The second major idea is based on equivalence. Put simply:

If ....

$$\log_b M = \log_b N$$
$$\text{then}\ldots$$
$$M = N$$

Note: There must be a single Log term on each side of the equals sign.

Example:

$$\log x + \log 5 = \log 20$$
$$\log 5x = \log 20 \quad \bullet\text{Using Log Rule 1 (Product Rule)}$$
$$5x = 20$$
$$x = 4$$

Example:

$$2\ln x = \ln 36$$
$$\ln x^2 = \ln 36 \quad \text{•Using Rule 3 (Power Rule)}$$
$$x^2 = 36$$
$$x = \pm 6$$

Check: The logarithm can only be found of positive values only,
so the solution is $x=6$.

Some equations will contain a mix of Log terms and numbers, there are two strategies to solve this;

(i) Rearrange to get Log terms on one side and number(s) on the other,
or
(ii) Replace the number with an equivalent Log (say) 1 is the same as $\log_4 4$

Example: Solve for x: $\log_4(2x+4)-2=\log_4 3$

$$\log_4(2x+4)-2=\log_4 3$$
$$\log_4(2x+4)-\log_4 3=2$$
$$\log_4\left(\frac{2x+4}{3}\right)=2$$
$$\left(\frac{2x+4}{3}\right)=4^2$$
$$\frac{2x+4}{3}=16$$
$$2x+4=48$$
$$2x=44$$
$$x=22$$

or

$$\log_4(2x+4)-2=\log_4 3$$
$$\log_4(2x+4)-2\log_4 4=\log_4 3$$
$$\log_4(2x+4)-\log_4 4^2=\log_4 3$$
$$\log_4\left(\frac{2x+4}{16}\right)=\log_4 3$$
$$\frac{2x+4}{16}=3$$
$$2x+4=48$$
$$2x=44$$
$$x=22$$

Mixed examples:

(i) Solve for x. $\log_9 x = 0.5$

$$\log_9 x = 0.5$$
$$x = 9^{0.5}$$
$$x = \sqrt{9}$$
$$x = 3$$

(ii) Express y in terms of x: $\log y = 2\log x + \log 8$

$$\log y = 2\log x + \log 8$$
$$\log y = \log x^2 + \log 8$$
$$\log y = \log 8x^2$$
$$y = 8x^2$$

(iii) Express y in terms of x: $2\ln y = 3 + 4\ln x$

$$
\begin{aligned}
2\ln y &= 3 + 4\ln x \\
\ln y^2 - \ln x^4 &= 3 \\
\ln \frac{y^2}{x^4} &= 3 \\
\frac{y^2}{x^4} &= e^3 \\
y^2 &= e^3 x^4 \\
y &= \sqrt{e^3 x^4}
\end{aligned}
$$

(iv) Solve for *x*: $\log_6 x + \log_6 (x+5) = 2$

$$
\begin{aligned}
\log_6 x + \log_6 (x+5) &= 2 \\
\log_6 (x^2 + 5x) &= 2 \\
x^2 + 5x &= 36 \\
x^2 + 5x - 36 &= 0 \\
(x-4)(x+9) &= 0 \\
x &= 4 \text{ or } -9
\end{aligned}
$$

Check: Because it is not possible to have the Log of a negative number, the solution is $x=4$.

Video 'Solving logarithmic equations'

Activity

1. Solve the following Logarithmic Equations.

(a)	$\log_{10} x = 2$	(b)	$\log_5 x = -2$
(c)	$\log_4 x = 0.5$	(d)	$\log_7 2x = 2$
(e)	$\log_2 (x+4) = 3$	(f)	$\log_9 \left(\frac{x}{4}\right) = -0.5$
(g)	$\log_3 x^2 = 2$	(h)	$\log_3 x = 2$
(i)	$\log_2 (x^2 + 2x) = 3$	(j)	$\log_x 4 = 4$
(k)	$\log_x 9 = 0.5$	(l)	$\log_x 64 = 2$
(m)	$\log_x (2x-1) = 2$	(n)	$\log_x 3 = 2$
(o)	$\log_x 10 = 3$	(p)	$\log_4 64 = x$
(q)	$10^{\log x} = 4$	(r)	$e^{\ln(x+4)} = 7$

2. Solve for x in the Logarithmic Equations.

(a)	$2\log x = \log 4$	(b)	$2\ln 2x = \ln 36$
(c)	$\log(x+5) = \log x + \log 6$	(d)	$3\log x = \log 4$
(e)	$2\log x = \log 2x + \log 3$	(f)	$2\log_3 2 - \log_3(x+1) = \log_3 5$
(g)	$\log_4 x + \log_4 6 = \log_4 12$	(h)	$3\log_3 2 + \log_3(x-2) = \log_3 5$
(i)	$\log_8(x+2) = 2 - \log_8 2$	(j)	$\log_2 x = 5 - \log_2(x+4)$
(k)	$\log x + \log 3 = \log 5$	(l)	$\log x - \log(x-1) = \log 4$
(m)	$\log(x-3) = 3$	(n)	$\ln(4-x) + \ln 2 = 2\ln x$
(o)	$\log_4(2x+4) - 3 = \log_4 3$	(p)	$\log_2 x + 3\log_2 2 = \log_2 \frac{2}{x}$

3. Solve for y in terms of the other variables present.

(a)	$\log y = \log x + \log 4$	(b)	$\log y = \frac{1}{2}\log 5 + \frac{1}{2}\log x$
(c)	$\log y = \log 4 - \log x$	(d)	$\log y + \log x = \log 4 + 2$
(e)	$\log_5 x + \log_5 y = \log_5 3 + 1$	(f)	$2\ln y = \ln e^2 - 3\ln x$
(g)	$\log_2 y + 2\log_2 x = 5 - \log_2 4$	(h)	$\log_7 y = 2\log_7 5 + \log_7 x + 2$
(i)	$3\ln y = 2 + 3\ln x$	(j)	$2(\log_4 y - 3\log_4 x) = 3$
(k)	$2^y = e^x$	(l)	$10^y = 3^{x+1}$

Exponential equations

An equation where the exponent (index) is a variable or contains a variable is called an exponential equation. An example is $3^x = 20$.

Let's consider the example below:

If an investor deposits \$20 000 in an account that compounds at 5% p.a., how long is it before this amount has increased to \$30 000? (Compounded annually)

Using the compound interest formula gives:

$$A = P(1+i)^n$$
$$30000 = 20000(1.05)^n$$

The first step is to rearrange the equation to get the form $a = b^x$ or $b^x = a$

$$\frac{30000}{20000} = 1.05^n$$
$$1.5 = 1.05^n$$

Then take the log of both sides. Because calculations will need to be performed, ordinary logarithms or natural logarithms should be used.

$1.5 = 1.05^n$	$1.5 = 1.05^n$
$\log 1.5 = \log 1.05^n$ taking the log of both sides	$\ln 1.5 = \ln 1.05^n$ taking the log of both sides
$\log 1.5 = n \log 1.05$ using the third log law	$\ln 1.5 = n \ln 1.05$ using the third log law
$\frac{\log 1.5}{\log 1.05} = n$ rearranging	$\frac{\ln 1.5}{\ln 1.05} = n$ rearranging
$n = 8.31$	$n = 8.31$

This means that it will take 8.31years (9 years in practice) for the growth in value to occur.

This can be checked by calculating:

$$1.05^{8.31}$$
$$= 1.5$$

This example contains a simple power.

$$\begin{aligned} 3^x &= 10 \\ \log 3^x &= \log 10 \\ x\log 3 &= \log 10 \\ x &= \frac{\log 10}{\log 3} \\ x &= \frac{1}{\log 3} \\ x &= 2.096 \end{aligned}$$

In this example, the 4 must be rearranged to obtain the power as the subject before taking the logarithm of both sides.

$$\begin{aligned} 4\times 7^x &= 20 \\ 7^x &= \frac{20}{4} \\ 7^x &= 5 \\ \log 7^x &= \log 5 \\ x\log 7 &= \log 5 \\ x &= \frac{\log 5}{\log 7} \\ x &= 0.8271 \end{aligned}$$

In this example, there is a power as the subject; however, the exponent is more complex than previous examples.

$$\begin{aligned} 12.5^{2n-1} &= 523.95 \\ \log 12.5^{2n-1} &= \log 523.95 \\ (2n-1)\log 12.5 &= \log 523.95 \\ 2n-1 &= \frac{\log 523.95}{\log 12.5} \\ 2n-1 &= 2.479 \\ 2n &= 3.4790 \\ n &= 1.7395 \end{aligned}$$

Example: Joan wants to retire when her superannuation fund reaches \$500 000. She invests \$1500 a month (after tax) into her superannuation fund. She assumes that the superannuation fund will return 6%pa or 0.5%pm. How long before her goal is reached?

This is a Future Value of an Annuity, with r = 0.005, R = 1500 and S = 500000.

$$S = R\frac{\left[(1+r)^n - 1\right]}{r}$$

$$500000 = 1500\frac{\left[(1+0.005)^n - 1\right]}{0.005}$$

$$2500 = 1500\left[(1+0.005)^n - 1\right] \quad \text{•mult both sides by 0.005}$$

$$1.666\overline{6} = 1.005^n - 1 \quad \text{•divide both sides by 1500}$$

$$2.666\overline{6} = 1.005^n \quad \text{•add 1 to both sides}$$

To solve this exponential equation, logs are required.

$$2.666\overline{6} = 1.005^n$$

$$\log 2.666\overline{6} = \log 1.005^n \quad \text{•take Log of both sides}$$

$$\log 2.666\overline{6} = n\log 1.005 \quad \text{•use Log Law 3}$$

$$\frac{\log 2.666\overline{6}}{\log 1.005} = n$$

$$n = 197 \text{ (rounded up)}$$

It will be 197 months or 16 years 5 months before Joan has enough money.

Video ‘Solving exponential equations’

Activity

1. Solve the following exponential equations.

(a) $4^x = 20$
(b) $1.6^x = 3.2$
(c) $3^{2x} = 0.125$
(d) $4.6^{0.5x} = 100$
(e) $6^{x+1} = 40$
(f) $11.2^{2-x} = 0.6$
(g) $2^x + 12 = 40$
(h) $3 \times 6^x - 10 = 12.5$
(i) $6^{x+4} = 6^{11}$
(j) $5^x = 2^{2x+1}$

2. If an investor deposits $250 000 in an account that compounds at 7% p.a., how long is it before this amount has increased to $400 000? (Compounded semi annually). Use the compound interest formula: $A = P(1+i)^n$

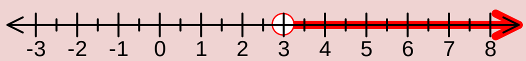
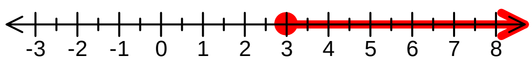
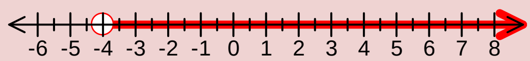
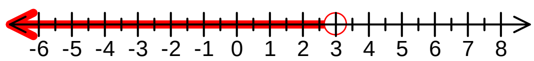
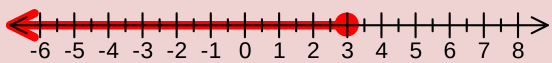
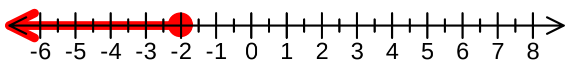
Topic 5: Inequalities

Inequalities are equations that contain an inequality sign instead of an equals sign.

The inequalities signs used are

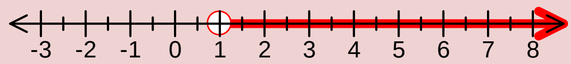
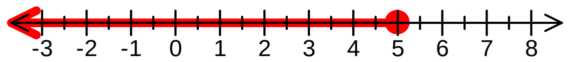
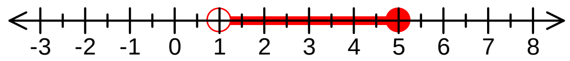
- Greater than >
- Greater than or equal to ≥
- Less than <
- Less than or equal to ≤

Sometimes it is useful to picture what is being communicated by the inequality by using number lines.

Example:		Number Line
$x > 3$	x is greater than 3	
$x \geq 3$	x is greater than or equal to 3	
$x > {}^{-}4$	x is greater than -4	
$x < 3$	x is less than 3	
$x \leq 3$	x is less than or equal to 3	
$x < {}^{-}2$	x is less than -2	

The appearance of the number lines for $x > 3$ and $x \geq 3$ is slightly different. If the value is included (≥) then number has a coloured in circle. If the value is not included (>) then the number has an empty circle.

Sometimes solutions to inequations can be expressed between two values. For example; if x is greater than 1 and less than or equal to 5, then this can be written as $1 < x \leq 5$. The use of the word AND means both conditions must be true.

Example:		**Number Line**
$x > 1$	x is greater than 1	
$x \leq 5$	x is less than or equal to 5	
Combined		
$1 < x \leq 5$	x is greater than 1 **and** x is less than or equal to 5	

How to read combined inequalities:

Start at the pronumeral – reading right.

x is less than or equal to 5

$$1 < x \leq 5$$

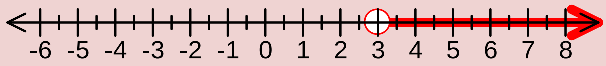
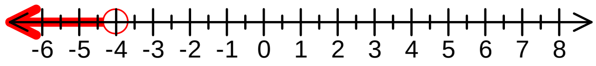
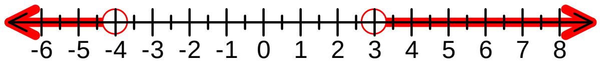
Start at the pronumeral – reading left.

x is greater than 1

(although the sign is a less than sign, it is read as greater than from the reverse direction)

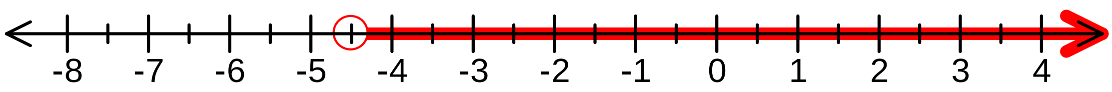
Altogether: x is less than or equal to 5 **and** x is greater than 1

When trying to describe solutions outside of two values, each section must be written separately joined by the word OR. If the solution is greater than 3 or less than -4, it is written as; $x > 3$ or $x < -4$. The use of the word OR is logical as the solution can only obey one condition.

Example:		**Number Line**
$x > 3$	x is greater than 3	
$x < -4$	x is less than -4	
Combined		
$x > 3$ or $x < -4$	x is greater than 3 **or** x is less than -4	

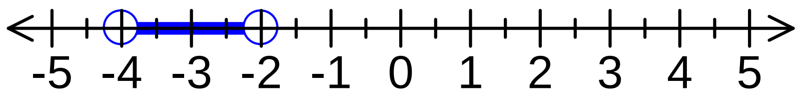
Example:
Write the inequality and draw the number line to describe the inequality: x is greater than – 4.5

$x > -4.5$



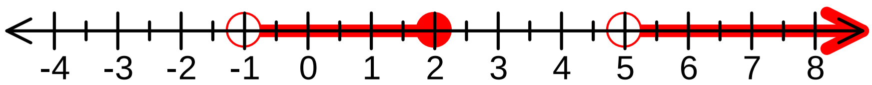
Example:

$-4 < x < -2$



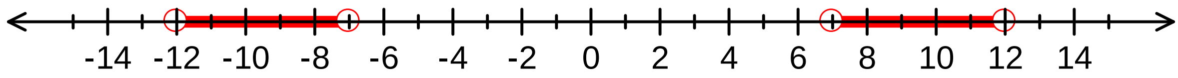
Example:

$-1 < x \leq 2$ or $x > 5$



Example:

7 < x < 12 or -12 < x < -7



Linear inequalities

A linear inequality contains a single variable with an exponent of 1. An example of a linear inequality is $3x-1<5$. There are some fundamental ideas that apply to inequalities.

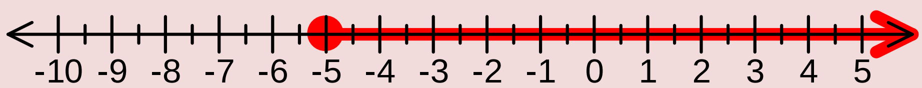
Addition and subtraction	If the same number is added or subtracted from both sides, the resulting inequality is the same as the initial inequality.	$x+3<6$ $x+3-3<6-3$ $x<3$	$x-7<6$ $x-7+7<6+7$ $x<13$
Multiplication and division by a **positive** number	If the same **positive** number multiplies or divides both sides, the resulting inequality is the same as the initial inequality.	$\frac{x}{2}>7$ $\frac{x}{2}\times 2>7\times 2$ $x>14$	$3x\le 9$ $\frac{3x}{3}\le\frac{9}{3}$ $x\le 3$
Multiplication and division by a negative number	If the same negative number multiplies or divides both sides, the resulting inequality is the reverse of the initial inequality.	$\frac{x}{-4}>6$ $\frac{x}{-4}\times -4<6\times -4$ $x<-24$	$-2x\ge 16$ $\frac{-2x}{-2}\le\frac{8}{-2}$ $x\le -4$

This last idea causes many errors when solving linear inequalities. The reasoning is poorly explained in books. In the box below, an attempt to give some meaning to this rule is given.

Consider the simple inequality $-x\le 5$.
What values of x will solve this inequality? Consider the following values of x:

If $x=-7$ Is $-(-7)\le 5$ $7\le 5$ *False*	If $x=-6$ Is $-(-6)\le 5$ $6\le 5$ *False*	If $x=-5$ Is $-(-5)\le 5$ $5\le 5$ *True*	If $x=-4$ Is $-(-4)\le 5$ $4\le 5$ *True*	If $x=-3$ Is $-(-3)\le 5$ $3\le 5$ *True*	....

On a number line, it is clear the solution is:



This can be described as $x\ge -5$. Both sides have been multiplied (or divided) by -1 and the inequality sign has reversed.

Example: Solve $2x-4\le 10$.

$$2x-4\le 10$$
$$2x-4+4\le 10+4$$
$$2x\le 14$$
$$\frac{2x}{2}\le\frac{14}{2}$$
$$x\le 7$$

Another way of expressing this answer is by **interval notation**. For this example, the answer written in interval notation is $(-\infty, 7]$. The set of answers is all numbers less than or equal to 7. The interval is reflecting this. The lower limit is given as $-\infty$ meaning the lower limit is very large negative values. The upper limit is 7.

Interval notation can use [ ] or ( ). [-3,5] is equivalent to $-3\le x\le 5$ where (-3,5) is equivalent to $-3<x<5$.

Example: Solve $3x-4>2(x-4)+5$.

$$3x-4>2(x-4)+5$$
$$3x-4>2x-8+5$$
$$3x-4>2x-3$$
$$3x-2x>4-3$$
$$x>1$$

Or in interval notation $(1,\infty)$

Solutions can be checked if required. Let's consider a number greater than 1, (say) 5. Substitute 5 into each side of the initial question;

$$3x-4 \qquad\qquad 2(x-4)+5$$
$$3(5)-4 \qquad\qquad 2(5-4)+5$$
$$11 \qquad\qquad 7$$

As 11 > 7, which contains the same inequality as the question, we are more confident that the solution is correct.

Example: Solve $9-2x>12$.

$$9-2x>12$$
$$-2x>12-9$$
$$-2x>3$$
$$\frac{-2x}{-2}<\frac{3}{-2}$$
$$x<-\frac{3}{2}$$

Or in interval notation $(-\infty,-\frac{3}{2})$

Checking: Consider $x = -2$

$$\begin{array}{cc} 9-2x & 12 \\ 9-2(-2) & 12 \\ 13 & 12 \end{array}$$

As 13 > 12, which contains the same inequality as the question, we are more confident that the solution is correct.

Example: Solve $\frac{2(3t-1)}{3} > \frac{4t-1}{2} + \frac{t}{4}$.

$$\frac{2(3t-1)}{3} > \frac{4t-1}{2} + \frac{t}{4}$$ •express with common denominator

$$\frac{8(3t-1)}{12} > \frac{6(4t-1)}{12} + \frac{3t}{12}$$ •multiply through by 12

$$24t-8 > 24t-6+3t$$
$$24t-8 > 27t-6$$

From here, the question can be solved by two different methods.

$$\begin{aligned} 24t-8 &> 27t-6 \\ 24t-27t &> 8-6 \\ -3t &> 2 \\ t &< -\frac{2}{3} \end{aligned}$$

or

$$\begin{aligned} 24t-8 &> 27t-6 \\ -8+6 &> 27t-24t \\ -2 &> 3t \\ -\frac{2}{3} &> t \\ t &< -\frac{2}{3} \end{aligned}$$

or in interval notation $(-\infty, -{}^{2}/_{3})$

Sometimes the variable is eliminated while solving inequalities. The example below shows the significance of the two possible types of solutions.

Example:

Solve $2(x-4)+(x-1) > 3x-7$

$$\begin{aligned} 2(x-4)+(x-1) &> 3x-7 \\ 2x-8+x-1 &> 3x-7 \\ 3x-9 &> 3x-7 \\ -9 &> -7 \end{aligned}$$

In this solution, it is impossible for -9 to be greater than -7, so there are no solutions.

In interval notation, the answer is given as $\emptyset$ (the empty set).

Solve $2(x-4)+(x-1) < 3x-7$

$$\begin{aligned} 2(x-4)+(x-1) &< 3x-7 \\ 2x-8+x-1 &< 3x-7 \\ 3x-9 &< 3x-7 \\ -9 &< -7 \end{aligned}$$

In this solution, -9 is always less than -7. That means that any value of x will solve the inequality.

In interval notation, the answer is given as $(-\infty, \infty)$

Solving a compound inequality

Earlier in this section, compound inequalities such as $2 < x \le 9$ were introduced. Now this idea is extended by solving compound inequalities.

Example: Solve $2 < 2x - 3 < 10$

This easier type can be done by performing the same operation on each term, to start with, adding 3 to each term, to give;

$$2 < 2x - 3 < 10$$
$$5 < 2x < 13$$

Now, dividing each term by 2;

$$5 < 2x < 13$$
$$\frac{5}{2} < x < \frac{13}{2}$$

A more complex compound inequality with have the variable in each part of the inequality. This type of inequality must be split into two separate parts. Each part is solved and the answers combined. The example below shows this process.

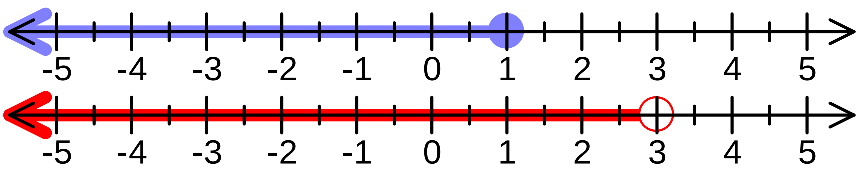
Example: Solve $4x - 7 < 2x - 1 \le 2 - x$

$$4x - 7 < 2x - 1 \le 2 - x$$

split into 2 parts

$$4x - 7 < 2x - 1 \quad \text{and} \quad 2x - 1 \le 2 - x$$
$$4x - 2x < 7 - 1 \qquad 2x + x \le 2 + 1$$
$$2x < 6 \qquad 3x \le 3$$
$$x < 3 \qquad x \le 1$$

Now these two answers need to be combined. Drawing the number lines is a good method for determining the overall solution.

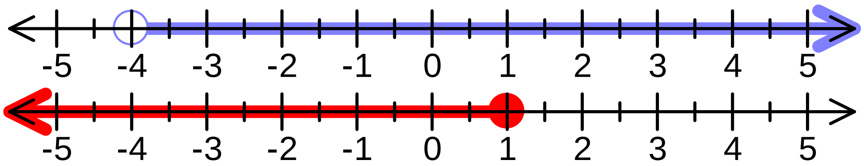


The solution is the 'common' part of the two individual parts. The overall solution is $x \le 1$.

Example: Solve $2(2x-3) \le 1-3x < 9-x$

$$2(2x-3) \le 1-3x < 9-x$$

$2(2x-3) \le 1-3x$	and	$1-3x < 9-x$
$4x-6 \le 1-3x$		$x-3x < 9-1$
$4x+3x \le 1+6$		$-2x < 8$
$7x \le 7$		$x > -4$
$x \le 1$		



The solution is $-4 < x \le 1$

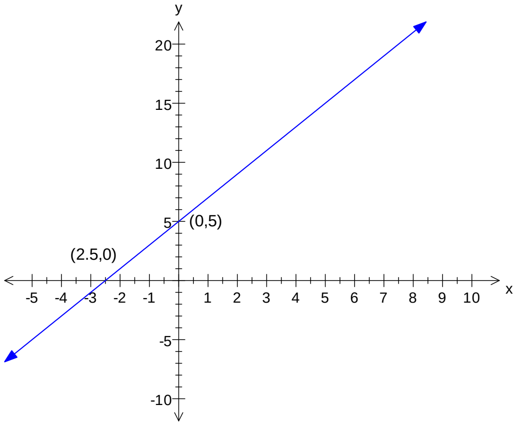
Linear inequalities (based on straight lines)

In this section, linear inequalities in the 2D (x-y) Cartesian plane will be covered. Examples of linear inequalities are $x > 5$, $y > 2x+5$ and $2x+3y > 12$. The starting point with these inequalities is to consider the straight line associated with the inequality;

for example; if asked to show $y > 2x+5$ on a Cartesian plane, the first step is to draw (sketch) the line of $y = 2x+5$.

y – intercept (when $x = 0$)	**x – intercept (when $y = 0$)**
$y = 2x+5$ $y = 2\times 0+5$ $y = 5 \Rightarrow (0,5)$	$y = 2x+5$ $0 = 2x+5$ $-5 = 2x$ $x = \frac{-5}{2} \Rightarrow (-2.5,0)$

The line of $y = 2x+5$ is sketched below.



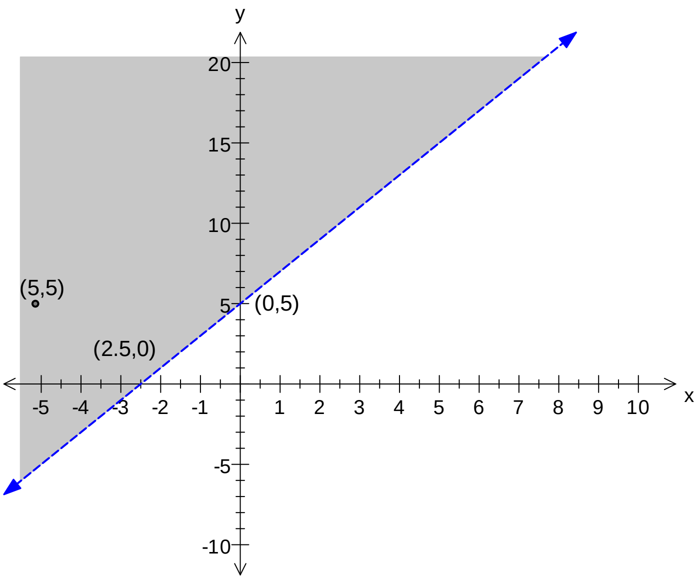
Because the inequality is (>), the line is not included. To show the line is not included is it shown as a **dashed** line. As the inequality is (>) the area above the line is shaded. This can often be difficult to determine so a check should be used. The point (-5,5) is clearly not on the line, so was selected as the check point.

$$y > 2x + 5$$

is $5 > 2 \times -5 + 5$?

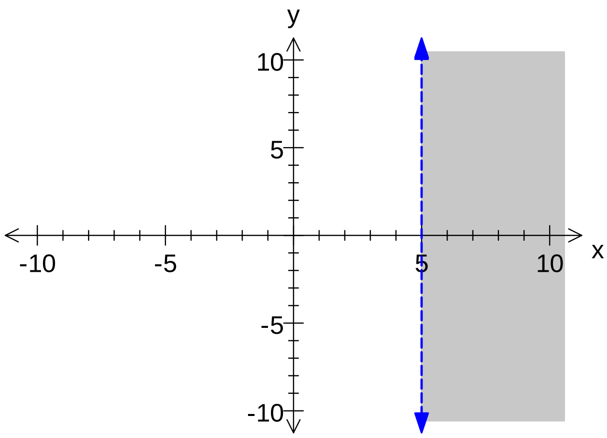
is $5 > -5$ True

This indicates that (-5, 5) is in the solution (shaded) area of the graph.



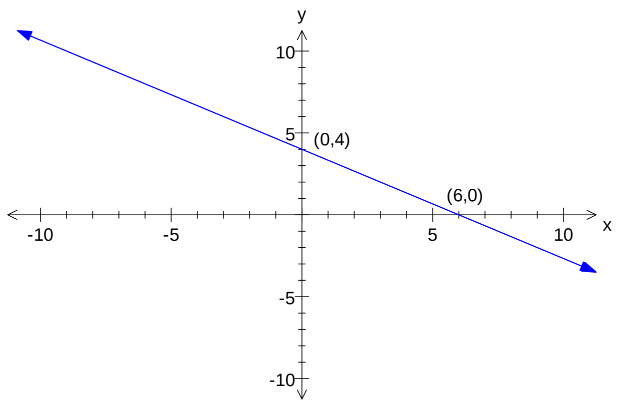
Now let's consider the example $x > 5$.

The graph of $x = 5$ is a vertical line, so the graph of $x > 5$ will be values greater than 5 with a dashed line on 5.



Now let's consider the example $2x+3y>12$.

y – intercept (when $x=0$)	**x – intercept (when $y=0$)**
$2x+3y=12$ $2\times 0+3y=12$ $3y=12$ $y=4 \Rightarrow (0,4)$	$2x+3y=12$ $2x+3\times 0=12$ $2x=12$ $x=6 \Rightarrow (6,0)$



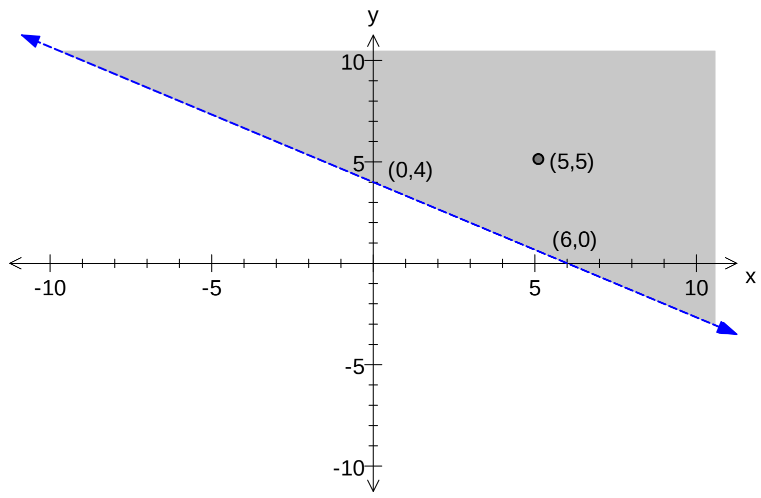
The line will be dashed due the absence of an equals sign.
The point (5,5) is suitable as a test point here.

$$2x+3y>12$$

is $2\times 5+3\times 5>12$?

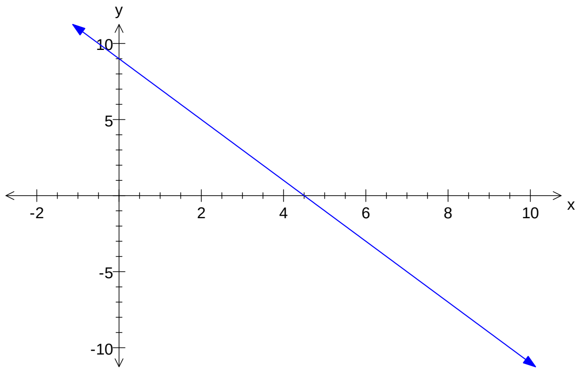
is $25>12$ True

The point (5,5) must be in the solution area. The shading is above the line. The final graph is:



Example $y \le 9 - 2x$.

y – intercept (when $x = 0$)	**x – intercept (when $y = 0$)**
$y = 9 - 2x$ $y = 9 - 2\times 0$ $y = 9 \Rightarrow (0,9)$	$y = 9 - 2x$ $0 = 9 - 2x$ $2x = 9$ $x = 4.5 \Rightarrow (4.5,0)$



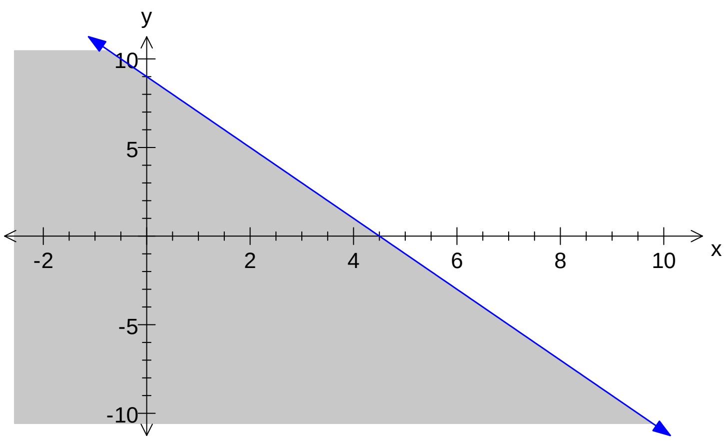
The line will be solid due the presence of an equals sign.
The point (5,5) is suitable as a test point here.

$$y \le 9 - 2x$$

$$\text{is } 5 \le 9 - 2\times 5?$$

$$\text{is } 5 \le -1 \text{ False}$$

The point (5,5) is not in the solution area. The shading is below the line. The final graph is:



[Video 'Inequalities']

Activity

1. Draw number lines to represent.

(a) $x > 3.5$

(b) $x < 9$

(c) $2 < x < 4$

(d) $-4 < x \leq 0$

(e) $x \geq -0.5$

(f) $-3.3 \leq x < 3$

2. Solve these Linear Inequalities

(a) $-x < 5$

(b) $-3x > 9$

(c) $\frac{3x}{5} > 1.2$

(d) $3x - 1 > 4$

(e) $2x + 3 \geq x - 3$

(f) $3(3x - 1) < 5x + 9$

(g) $\frac{x+2}{3} \geq \frac{2x-1}{6} + \frac{1}{2}$

(h) $\frac{a+2}{4} > \frac{2a-1}{3} + \frac{a}{6}$

(i) $\frac{2(3t-2)}{5} - \frac{2(4-t)}{3} \geq 4$

3. Solve these compound inequalities. Draw number lines to obtain (show) the solution.

(a) $0 < x - 7 < 5$

(b) $15 < 4x + 7 \leq 51$

(c) $5x < 2x + 3 \leq -x$

(d) $-3(x - 2) \leq 5 - 4x < 3 - x$

(e) $x - 1 < 2x + 2 < 3x + 1$

(f) $4x - 3 < 2x + 5 < 6x + 7$

4. Graph the following inequalities.

(a) $y > x + 4$

(b) $2x + y < 10$

(c) $y \geq 7 - 4x$

(d) $x + y < 9$

(e) Find the solutions common to $x + y > 6$ and $2y - 3x < 8$

Answers to activity questions

Topic 1: Equations

1. Solve the equations below for the given variable

(a)
$$7x = 49$$
$$\frac{\cancel{7}^1 x}{\cancel{7}^1} = \frac{49}{7}$$
$$x = 7$$
or
$$7x = 49$$
$$x = \frac{49}{7}$$
$$x = 7$$

(b)
$$x + 5 = 2$$
$$x + 5 - 5 = 2 - 5$$
$$x = {}^{-}3$$
or
$$x + 5 = 2$$
$$x = 2 - 5$$
$$x = {}^{-}3$$

(c)
$$\frac{w}{5} = {}^{-}3$$
$$\frac{\cancel{5}^1 \times w}{\cancel{5}^1} = {}^{-}3 \times 5$$
$$w = {}^{-}15$$
or
$$\frac{w}{5} = {}^{-}3$$
$$w = {}^{-}3 \times 5$$
$$w = {}^{-}15$$

(d)
$$6 + k = 4$$
$$6 - 6 + k = 4 - 6$$
$$k = {}^{-}2$$
or
$$6 + k = 4$$
$$k = 4 - 6$$
$$k = {}^{-}2$$

(e)
$$3x - 7 = 11$$
$$3x - 7 + 7 = 11 + 7$$
$$3x = 18$$
$$\frac{3x}{3} = \frac{18}{3}$$
$$x = 6$$

$$3x - 7 = 11$$
$$3x = 11 + 7$$
$$3x = 18$$
$$x = \frac{18}{3}$$
$$x = 6$$

(f)
$$p - 1.5 = 4.2$$
$$p - 1.5 + 1.5 = 4.2 + 1.5$$
$$p = 5.7$$
or
$$p - 1.5 = 4.2$$
$$p = 4.2 + 1.5$$
$$p = 5.7$$

(g)
$${}^{-}3h = 21$$
$$\frac{\cancel{{}^{-}3}^1 h}{\cancel{{}^{-}3}^1} = \frac{21}{{}^{-}3}$$
$$h = {}^{-}7$$
or
$${}^{-}3h = 21$$
$$h = \frac{21}{{}^{-}3}$$
$$h = {}^{-}7$$

(h)

$$\frac{g}{5} = \frac{2}{3}$$
$$\frac{\cancel{5}^1 \times g}{\cancel{5}^1} = \frac{2 \times 5}{3}$$
$$g = \frac{10}{3} = 3\frac{1}{3}$$

or

$$\frac{g}{5} = \frac{2}{3}$$
$$g = \frac{2}{3} \times 5$$
$$g = \frac{10}{3} = 3\frac{1}{3}$$

(i)

$$5 - x = 7$$
$$5 - x + x = 7 + x$$
$$5 = 7 + x$$
$$5 - 7 = 7 - 7 + x$$
$${}^{-}2 = x$$

or

$$5 - x = 7$$
$$5 - 5 - x = 7 - 5$$
$${}^{-}x = 2$$
$$x = {}^{-}2$$

or

$$5 - x = 7$$
$${}^{-}x = 7 - 5$$
$${}^{-}x = 2$$
$$x = {}^{-}2$$

or

$$5 - x = 7$$
$$5 = 7 + x$$
$$5 - 7 = x$$
$$x = {}^{-}2$$

(j)

$$6 = 2t - 7$$
$$6 + 7 = 2t - 7 + 7$$
$$13 = 2t$$
$$\frac{13}{2} = \frac{\cancel{2}^1 t}{\cancel{2}^1}$$
$$6\frac{1}{2} = t$$

or

$$6 = 2t - 7$$
$$6 + 7 = 2t$$
$$13 = 2t$$
$$\frac{13}{2} = t$$
$$t = 6\frac{1}{2}$$

(k)

$$\frac{x}{3} + 2 = 5$$
$$\frac{x}{3} + 2 - 2 = 5 - 2$$
$$\frac{x}{3} = 3$$
$$\frac{\cancel{3}^1 \times x}{\cancel{3}^1} = 3 \times 3$$
$$x = 9$$

or

$$\frac{x}{3} + 2 = 5$$
$$\frac{x}{3} = 5 - 2$$
$$\frac{x}{3} = 3$$
$$x = 3 \times 3$$
$$x = 9$$

(l)

$$\frac{x - 4}{2} = 7$$
$$\frac{\cancel{2}^1 \times (x - 4)}{\cancel{2}^1} = 7 \times 2$$
$$x - 4 = 14$$
$$x - 4 + 4 = 14 + 4$$
$$x = 18$$

or

$$\frac{x - 4}{2} = 7$$
$$x - 4 = 7 \times 2$$
$$x - 4 = 14$$
$$x = 14 + 4$$
$$x = 18$$

(m)

$$\frac{4}{x} = \frac{5}{7}$$
$$\frac{x}{4} = \frac{7}{5}$$
$$\frac{\cancel{4}^1 \times x}{\cancel{4}^1} = \frac{4 \times 7}{5}$$
$$x = \frac{28}{5} = 5\frac{3}{5} \text{ or } 5.6$$

or

$$\frac{4}{x} = \frac{5}{7}$$
$$\frac{x}{4} = \frac{7}{5}$$
$$x = \frac{7 \times 4}{5}$$
$$x = \frac{28}{5} = 5\frac{3}{5} \text{ or } 5.6$$

(n)

$$
\begin{aligned}
{}^{-}x+1&=6\\
{}^{-}x+1-1&=6-1\\
{}^{-}x&=5\\
x&={}^{-}5
\end{aligned}
$$

or

$$
\begin{aligned}
{}^{-}x+1&=6\\
{}^{-}x&=6-1\\
{}^{-}x&=5\\
x&={}^{-}5
\end{aligned}
$$

(o)

$$
\begin{aligned}
{}^{-}4+3x&=15\\
{}^{-}4+4+3x&=15+4\\
3x&=19\\
\frac{3x}{3}&=\frac{19}{3}\\
x&=6\frac{1}{3}
\end{aligned}
$$

or

$$
\begin{aligned}
{}^{-}4+3x&=15\\
3x&=15+4\\
3x&=19\\
x&=\frac{19}{3}\\
x&=6\frac{1}{3}
\end{aligned}
$$

(p)

$$
\begin{aligned}
\frac{3r}{4}-5&=1\\
\frac{3r}{4}-5+5&=1+5\\
\frac{3r}{4}&=6\\
\frac{\cancel{4}^1\times 3r}{\cancel{4}^1}&=6\times 4\\
3r&=24\\
\frac{\cancel{3}^1 r}{\cancel{3}^1}&=\frac{24}{3}\\
r&=8
\end{aligned}
$$

or

$$
\begin{aligned}
\frac{3r}{4}-5&=1\\
\frac{3r}{4}&=1+5\\
\frac{3r}{4}&=6\\
3r&=6\times 4\\
3r&=24\\
r&=\frac{24}{3}\\
r&=8
\end{aligned}
$$

(q)

$$
\begin{aligned}
\frac{5-2x}{3}&=2\\
\frac{\cancel{3}^1\times(5-2x)}{\cancel{3}^1}&=2\times 3\\
5-2x&=6\\
5-5-2x&=6-5\\
{}^{-}2x&=1\\
\frac{{}^{-}\cancel{2}^1 x}{{}^{-}\cancel{2}^1}&=\frac{1}{{}^{-}2}\\
x&=-\frac{1}{2}
\end{aligned}
$$

or

$$
\begin{aligned}
\frac{5-2x}{3}&=2\\
5-2x&=2\times 3\\
5-2x&=6\\
{}^{-}2x&=6-5\\
{}^{-}2x&=1\\
x&=\frac{1}{{}^{-}2}\text{ or }-\frac{1}{2}
\end{aligned}
$$

(r)

$$
\begin{aligned}
\frac{2x-5}{4}&={}^{-}7\\
\frac{\cancel{4}^1\times(2x-5)}{\cancel{4}^1}&={}^{-}7\times 4\\
2x-5&={}^{-}28\\
2x-5+5&={}^{-}28+5\\
2x&={}^{-}23\\
\frac{\cancel{2}^1 x}{\cancel{2}^1}&=\frac{{}^{-}23}{2}\\
x&={}^{-}11\frac{1}{2}
\end{aligned}
$$

$$
\begin{aligned}
\frac{2x-5}{4}&={}^{-}7\\
2x-5&={}^{-}7\times 4\\
2x-5&={}^{-}28\\
2x&={}^{-}28+5\\
2x&={}^{-}23\\
x&=\frac{{}^{-}23}{2}\\
x&={}^{-}11\frac{1}{2}
\end{aligned}
$$

2. Solve the equations below for t.

(a)

$$\begin{aligned} 7(t-1) &= 9 \\ 7t-7 &= 9 \\ 7t-7+7 &= 9+7 \\ 7t &= 16 \\ \frac{\cancel{7}^1 t}{\cancel{7}^1} &= \frac{16}{7} \\ t &= 2\frac{2}{7} \end{aligned}$$

or

$$\begin{aligned} 7(t-1) &= 9 \\ 7t-7 &= 9 \\ 7t &= 9+7 \\ 7t &= 16 \\ t &= \frac{16}{7} = 2\frac{2}{7} \end{aligned}$$

(b)

$$\begin{aligned} 2t+a &= 2 \\ 2t+a-a &= 2-a \\ 2t &= 2-a \\ \frac{\cancel{2}^1 t}{\cancel{2}^1} &= \frac{2-a}{2} \\ t &= \frac{2-a}{2} \end{aligned}$$

or

$$\begin{aligned} 2t+a &= 2 \\ 2t &= 2-a \\ t &= \frac{2-a}{2} \end{aligned}$$

(c)

$$\begin{aligned} \frac{w}{t} &= {}^{-}3 \\ \frac{\cancel{t}^1 \times w}{\cancel{t}^1} &= {}^{-}3 \times t \\ w &= {}^{-}3t \\ -\frac{w}{3} &= t \end{aligned}$$

or

$$\begin{aligned} \frac{w}{t} &= {}^{-}3 \\ w &= {}^{-}3t \\ -\frac{w}{3} &= t \end{aligned}$$

(d)

$$\begin{aligned} 3(2t-a) &= 4 \\ 6t-3a &= 4 \\ 6t-3a+3a &= 4+3a \\ 6t &= 4+3a \\ \frac{6t}{6} &= \frac{4+3a}{6} \\ t &= \frac{4+3a}{6} \end{aligned}$$

or

$$\begin{aligned} 3(2t-a) &= 4 \\ 6t-3a &= 4 \\ 6t &= 4+3a \\ t &= \frac{4+3a}{6} \end{aligned}$$

(e)

$$\begin{aligned} 3t-x+y &= {}^{-}7 \\ 3t-x+y-y &= {}^{-}7-y \\ 3t-x &= {}^{-}7-y \\ 3t-x+x &= {}^{-}7-y+x \\ 3t &= {}^{-}7-y+x \\ \frac{\cancel{3}^1 t}{\cancel{3}^1} &= \frac{{}^{-}7-y+x}{3} \\ t &= \frac{{}^{-}7-y+x}{3} \\ t &= \frac{x-7-y}{3} \end{aligned}$$

or

$$\begin{aligned} 3t-x+y &= {}^{-}7 \\ 3t-x &= {}^{-}7-y \\ 3t &= x-7-y \\ t &= \frac{x-7-y}{3} \end{aligned}$$

(f)

$$\begin{aligned} 4-3t &= 7 \\ 4-4-3t &= 7-4 \\ {}^{-}3t &= 3 \\ \frac{{}^{-}\cancel{3}^1 t}{{}^{-}\cancel{3}^1} &= \frac{3}{{}^{-}3} \\ t &= {}^{-}1 \end{aligned}$$

or

$$\begin{aligned} 4-3t &= 7 \\ {}^{-}3t &= 7-4 \\ {}^{-}3t &= 3 \\ t &= \frac{3}{{}^{-}3} = {}^{-}1 \end{aligned}$$

3. Solve the equations below for the variable used.

(a)

$$\begin{aligned}7t-1&=5t+5\\ 7t-1+1&=5t+5+1\\ 7t&=5t+6\\ 7t-5t&=5t-5t+6\\ 2t&=6\\ \frac{\cancel{2}^1t}{\cancel{2}^1}&=\frac{6}{2}\\ t&=3\end{aligned}$$

or

$$\begin{aligned}7t-1&=5t+5\\ 7t&=5t+5+1\\ 7t&=5t+6\\ 7t-5t&=6\\ 2t&=6\\ t&=\frac{6}{2}\\ t&=3\end{aligned}$$

(b)

$$\begin{aligned}3x+11&=3-x\\ 3x+11-11&=3-11-x\\ 3x&={}^-8-x\\ 3x+x&={}^-8-x+x\\ 4x&={}^-8\\ \frac{\cancel{4}^1x}{\cancel{4}^1}&=\frac{{}^-8}{4}\\ x&={}^-2\end{aligned}$$

or

$$\begin{aligned}3x+11&=3-x\\ 3x&=3-11-x\\ 3x&={}^-8-x\\ 3x+x&={}^-8\\ 4x&={}^-8\\ x&=\frac{{}^-8}{4}\\ x&={}^-2\end{aligned}$$

(c)

$$\begin{aligned}15-p&=2p\\ 15-p+p&=2p+p\\ 15&=3p\\ \frac{15}{3}&=\frac{\cancel{3}^1p}{\cancel{3}^1}\\ 5&=p\end{aligned}$$

or

$$\begin{aligned}15-p&=2p\\ 15&=2p+p\\ 15&=3p\\ \frac{15}{3}&=p\\ 5&=p\end{aligned}$$

(d)

$$\begin{aligned}2(a-3)&=4a-6\\ 2a-6&=4a-6\\ 2a-6+6&=4a-6+6\\ 2a&=4a\\ 2a-2a&=4a-2a\\ 0&=2a\\ \frac{0}{2}&=\frac{\cancel{2}^1a}{\cancel{2}^1}\\ 0&=a\end{aligned}$$

or

$$\begin{aligned}2(a-3)&=4a-6\\ 2a-6&=4a-6\\ 2a&=4a-6+6\\ 2a&=4a\\ 0&=4a-2a\\ 0&=2a\\ \frac{0}{2}&=a\\ 0&=a\end{aligned}$$

(e)

$$\begin{aligned}3(2s-7)&=2(3s-1)\\ 6s-21&=6s-2\\ 6s-21+21&=6s-2+21\\ 6s&=6s+19\\ 6s-6s&=6s-6s+19\\ 0&=19!\end{aligned}$$

There is no solution

or

$$\begin{aligned}3(2s-7)&=2(3s-1)\\ 6s-21&=6s-2\\ 6s&=6s-2+21\\ 6s&=6s+19\\ 6s-6s&=19\\ 0&=19!\end{aligned}$$

No solution

(f)

$$\begin{aligned}5(y-2) &= 3(4-y)\\ 5y-10 &= 12-3y\\ 5y-10+10 &= 12+10-3y\\ 5y &= 22-3y\\ 5y+3y &= 22-3y+3y\\ 8y &= 22\\ \frac{\cancel{8}^1 y}{\cancel{8}^1} &= \frac{\cancel{22}^{11}}{\cancel{8}^4}\\ y &= \frac{11}{4} = 2\frac{3}{4}\end{aligned}$$

or

$$\begin{aligned}5(y-2) &= 3(4-y)\\ 5y-10 &= 12-3y\\ 5y &= 12+10-3y\\ 5y &= 22-3y\\ 5y+3y &= 22\\ 8y &= 22\\ y &= \frac{22}{8} = 2\frac{6}{8} = 2\frac{3}{4}\end{aligned}$$

(g)

$$\begin{aligned}0.6(3y-1) &= 0.4y-1\\ 1.8y-0.6 &= 0.4y-1\\ 1.8y-0.6+0.6 &= 0.4y-1+0.6\\ 1.8y &= 0.4y-0.4\\ 1.8y-0.4y &= 0.4y-0.4y-0.4\\ 1.4y &= {}^{-}0.4\\ \frac{1.4y}{1.4} &= \frac{{}^{-}0.4}{1.4}\\ y &= \frac{{}^{-}0.4}{1.4} = \frac{{}^{-}4}{14} = \frac{{}^{-}2}{7}\end{aligned}$$

or

$$\begin{aligned}0.6(3y-1) &= 0.4y-1\\ 1.8y-0.6 &= 0.4y-1\\ 1.8y &= 0.4y-1+0.6\\ 1.8y &= 0.4y-0.4\\ 1.8y-0.4y &= {}^{-}0.4\\ 1.4y &= {}^{-}0.4\\ y &= \frac{{}^{-}0.4}{1.4} = \frac{{}^{-}4}{14} = \frac{{}^{-}2}{7}\end{aligned}$$

(h)

$$\begin{aligned}4-7a &= 3(1-3a)\\ 4-7a &= 3-9a\\ 4-4-7a &= 3-4-9a\\ {}^{-}7a &= {}^{-}1-9a\\ {}^{-}7a+9a &= {}^{-}1-9a+9a\\ 2a &= {}^{-}1\\ \frac{2a}{2} &= \frac{{}^{-}1}{2}\\ a &= \frac{{}^{-}1}{2}\end{aligned}$$

or

$$\begin{aligned}4-7a &= 3(1-3a)\\ 4-7a &= 3-9a\\ {}^{-}7a &= 3-4-9a\\ {}^{-}7a &= {}^{-}1-9a\\ {}^{-}7a+9a &= {}^{-}1\\ 2a &= {}^{-}1\\ a &= \frac{{}^{-}1}{2}\end{aligned}$$

(i)

$$\begin{aligned}2-(3x+9) &= 4(5-3x)\\ 2-3x-9 &= 20-12x\\ {}^{-}3x-7 &= 20-12x\\ {}^{-}3x-7+7 &= 20+7-12x\\ {}^{-}3x &= 27-12x\\ {}^{-}3x+12x &= 27-12x+12x\\ 9x &= 27\\ \frac{\cancel{9}^1 x}{\cancel{9}^1} &= \frac{27}{9}\\ x &= 3\end{aligned}$$

or

$$\begin{aligned}2-(3x+9) &= 4(5-3x)\\ 2-3x-9 &= 20-12x\\ {}^{-}3x-7 &= 20-12x\\ {}^{-}3x &= 20+7-12x\\ {}^{-}3x &= 27-12x\\ {}^{-}3x+12x &= 27\\ 9x &= 27\\ x &= \frac{27}{9}\\ x &= 3\end{aligned}$$

4. Transpose the following equations for the subject given in the brackets.

(a)

$$P = \frac{W}{t} \quad (W)$$
$$Pt = \frac{\cancel{t}^{1} W}{\cancel{t}^{1}}$$
$$Pt = W$$

or

$$P = \frac{W}{t} \quad (W)$$
$$Pt = W$$

(b)

$$P = \frac{W}{t} \quad (t)$$
$$\frac{P}{1} = \frac{W}{t}$$
$$\frac{1}{P} = \frac{t}{W}$$
$$\frac{1 \times W}{P} = \frac{t \times \cancel{W}^{1}}{\cancel{W}^{1}}$$
$$\frac{W}{P} = t$$

or

$$P = \frac{W}{t} \quad (t)$$
$$Pt = W$$
$$t = \frac{W}{P}$$

(c)

$$A = l \times b \quad (l)$$
$$\frac{A}{b} = \frac{l \times \cancel{b}^{1}}{\cancel{b}^{1}}$$
$$\frac{A}{b} = l$$

or

$$A = l \times b \quad (l)$$
$$\frac{A}{b} = l$$

(d)

$$C = Vq + F \quad (q)$$
$$C - F = Vq + F - F$$
$$C - F = Vq$$
$$\frac{C - F}{V} = \frac{\cancel{V}^{1} q}{\cancel{V}^{1}}$$
$$\frac{C - F}{V} = q$$

or

$$C = Vq + F \quad (q)$$
$$C - F = Vq$$
$$\frac{C - F}{V} = q$$

(e)

$$V = \frac{1}{3}\pi r^2 h \quad (h)$$
$$3 \times V = 3 \times \frac{1}{3}\pi r^2 h$$
$$3V = \pi r^2 h$$
$$\frac{3V}{\pi r^2} = \frac{\cancel{\pi r^2}^{1} h}{\cancel{\pi r^2}^{1}}$$
$$\frac{3V}{\pi r^2} = h$$

or

$$V = \frac{1}{3}\pi r^2 h \quad (h)$$
$$3V = \pi r^2 h$$
$$\frac{3V}{\pi r^2} = h$$

(f)

$$V = \frac{1}{3}\pi r^2 h \quad (r)$$
$$3V = \pi r^2 h$$
$$\frac{3V}{\pi h} = \frac{\cancel{\pi}^1 r^2 \cancel{h}^1}{\cancel{\pi}^1 \cancel{h}^1}$$
$$\frac{3V}{\pi h} = r^2$$
$$\sqrt{\frac{3V}{\pi h}} = \sqrt{r^2}$$
$$\sqrt{\frac{3V}{\pi h}} = r$$

or

$$V = \frac{1}{3}\pi r^2 h \quad (r)$$
$$3V = \pi r^2 h$$
$$\frac{3V}{\pi h} = r^2$$
$$\sqrt{\frac{3V}{\pi h} = r}$$

(g)

$$R = \frac{l}{\mu a} \quad (l)$$
$$R \times \mu a = \frac{l \times \cancel{\mu a}^1}{\cancel{\mu a}^1}$$
$$R\mu a = l$$

or

$$R = \frac{l}{\mu a} \quad (l)$$
$$R\mu a = l$$

(h)

$$R = \frac{l}{\mu a} \quad (a)$$
$$R \times \mu a = \frac{l \times \cancel{\mu a}^1}{\cancel{\mu a}^1}$$
$$R\mu a = l$$
$$\frac{\cancel{R\mu}^1 a}{\cancel{R\mu}^1} = \frac{l}{R\mu}$$
$$a = \frac{l}{R\mu}$$

or

$$R = \frac{l}{\mu a} \quad (a)$$
$$R\mu a = l$$
$$a = \frac{l}{R\mu}$$

(i)

$$A = \frac{1}{2}bh \quad (b)$$
$$2 \times A = 2 \times \frac{1}{2}bh$$
$$2A = bh$$
$$\frac{2A}{h} = \frac{b\cancel{h}^1}{\cancel{h}^1}$$
$$\frac{2A}{h} = b$$

or

$$A = \frac{1}{2}bh \quad (b)$$
$$2A = bh$$
$$\frac{2A}{h} = b$$

(j)

$$C = 2\pi r \quad (r)$$
$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$$
$$\frac{C}{2\pi} = r$$

or

$$C = 2\pi r \quad (r)$$
$$\frac{C}{2\pi} = r$$

(k)

$$c^2 = a^2 + b^2 \quad (a)$$
$$c^2 - b^2 = a^2 + b^2 - b^2$$
$$c^2 - b^2 = a^2$$
$$\sqrt{c^2 - b^2} = \sqrt{a^2}$$
$$\sqrt{c^2 - b^2} = a$$

or

$$c^2 = a^2 + b^2 \quad (a)$$
$$c^2 - b^2 = a^2$$
$$\sqrt{c^2 - b^2} = a$$

(l)

$$PV = nRT \quad (P)$$
$$\frac{P\cancel{V}^1}{\cancel{V}^1} = \frac{nRT}{V}$$
$$P = \frac{nRT}{V}$$

or

$$PV = nRT \quad (P)$$
$$P = \frac{nRT}{V}$$

(m)

$$PV = nRT \quad (n)$$
$$\frac{PV}{RT} = \frac{n\cancel{RT}^1}{\cancel{RT}^1}$$
$$\frac{PV}{RT} = n$$

or

$$PV = nRT \quad (n)$$
$$\frac{PV}{RT} = n$$

(n)

$$Q_1 = P(Q_2 - Q_1) \quad (P)$$
$$\frac{Q_1}{(Q_2 - Q_1)} = \frac{P\,\cancel{(Q_2 - Q_1)}^1}{\cancel{(Q_2 - Q_1)}^1}$$
$$\frac{Q_1}{(Q_2 - Q_1)} = P$$

or

$$Q_1 = P(Q_2 - Q_1) \quad (P)$$
$$\frac{Q_1}{(Q_2 - Q_1)} = P$$

(o)

$$Q_1 = P(Q_2 - Q_1) \quad (Q_2)$$
$$Q_1 = PQ_2 - PQ_1$$
$$Q_1 + PQ_1 = PQ_2 - PQ_1 + PQ_1$$
$$Q_1 + PQ_1 = PQ_2$$
$$\frac{Q_1 + PQ_1}{P} = \frac{PQ_2}{P}$$
$$\frac{Q_1 + PQ_1}{P} = Q_2$$

or

$$Q_1 = P(Q_2 - Q_1) \quad (Q_2)$$
$$Q_1 = PQ_2 - PQ_1$$
$$Q_1 + PQ_1 = PQ_2$$
$$\frac{Q_1 + PQ_1}{P} = Q_2$$

(p)

$$Q_1 = P(Q_2 - Q_1) \quad (Q_1)$$
$$Q_1 = PQ_2 - PQ_1$$
$$Q_1 + PQ_1 = PQ_2 - PQ_1 + PQ_1$$
$$Q_1 + PQ_1 = PQ_2$$
$$Q_1(1 + P) = PQ_2$$
$$\frac{Q_1(1 + P)}{(1 + P)} = \frac{PQ_2}{(1 + P)}$$
$$Q_1 = \frac{PQ_2}{(1 + P)}$$

or

$$Q_1 = P(Q_2 - Q_1) \quad (Q_1)$$
$$Q_1 = PQ_2 - PQ_1$$
$$Q_1 + PQ_1 = PQ_2$$
$$Q_1(1 + P) = PQ_2$$
$$Q_1 = \frac{PQ_2}{(1 + P)}$$

(q)

$$R_2 = R_1(1+\alpha t) \quad (t)$$
$$R_2 = R_1 + R_1\alpha t$$
$$R_2 - R_1 = R_1 - R_1 + R_1\alpha t$$
$$R_2 - R_1 = R_1\alpha t$$
$$\frac{R_2 - R_1}{R_1\alpha} = \frac{\cancel{R_1\alpha}^1 t}{\cancel{R_1\alpha}^1}$$
$$\frac{R_2 - R_1}{R_1\alpha} = t$$

or

$$R_2 = R_1(1+\alpha t) \quad (t)$$
$$R_2 = R_1 + R_1\alpha t$$
$$R_2 - R_1 = R_1\alpha t$$
$$\frac{R_2 - R_1}{R_1\alpha} = t$$

(r)

$$E = mc^2 \quad (c)$$
$$\frac{E}{m} = \frac{\cancel{m}^1 c^2}{\cancel{m}^1}$$
$$\frac{E}{m} = c^2$$
$$\sqrt{\frac{E}{m}} = \sqrt{c^2}$$
$$\sqrt{\frac{E}{m}} = c$$

or

$$E = mc^2 \quad (c)$$
$$\frac{E}{m} = c^2$$
$$\sqrt{\frac{E}{m}} = c$$

(s)

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \quad (v)$$
$$\frac{1}{f} - \frac{1}{u} = \frac{1}{u} - \frac{1}{u} + \frac{1}{v}$$
$$\frac{1}{f} - \frac{1}{u} = \frac{1}{v}$$
$$\frac{u-f}{fu} = \frac{1}{v}$$
$$\frac{v}{1} = \frac{fu}{u-f}$$
$$v = \frac{fu}{u-f}$$

or

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \quad (v)$$
$$\frac{1}{f} - \frac{1}{u} = +\frac{1}{v}$$
$$\frac{u-f}{fu} = \frac{1}{v}$$
$$v = \frac{fu}{u-f}$$

(t)

$$T = mg + m\omega^2 r \quad (r)$$
$$T - mg = mg - mg + m\omega^2 r$$
$$T - mg = m\omega^2 r$$
$$\frac{T - mg}{m\omega^2} = \frac{\cancel{m\omega^2}^1 r}{\cancel{m\omega^2}^1}$$
$$\frac{T - mg}{m\omega^2} = r$$

or

$$T = mg + m\omega^2 r \quad (r)$$
$$T - mg = m\omega^2 r$$
$$\frac{T - mg}{m\omega^2} = r$$

(u)

$$P=\frac{V_1(V_2-V_1)}{gJ}\quad (V_2)$$
$$P=\frac{V_1V_2-(V_1)^2}{gJ}$$
$$P\times gJ=\frac{V_1V_2-(V_1)^2}{\cancel{gJ}^1}\times\frac{\cancel{gJ}^1}{1}$$
$$PgJ=V_1V_2-(V_1)^2$$
$$PgJ+(V_1)^2=V_1V_2-(V_1)^2+(V_1)^2$$
$$PgJ+(V_1)^2=V_1V_2$$
$$\frac{PgJ+(V_1)^2}{V_1}=\frac{\cancel{V_1}^1V_2}{\cancel{V_1}^1}$$
$$\frac{PgJ+(V_1)^2}{V_1}=V_2$$

or

$$P=\frac{V_1(V_2-V_1)}{gJ}\quad (V_2)$$
$$P=\frac{V_1V_2-(V_1)^2}{gJ}$$
$$PgJ=V_1V_2-(V_1)^2$$
$$PgJ+(V_1)^2=V_1V_2$$
$$\frac{PgJ+(V_1)^2}{V_1}=V_2$$

(v)

$$Z=\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}\quad (\omega)$$
$$Z^2=\left(\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}\right)^2$$
$$Z^2=R^2+\left(\frac{1}{\omega C}\right)^2$$
$$Z^2-R^2=R^2-R^2+\left(\frac{1}{\omega C}\right)^2$$
$$Z^2-R^2=\left(\frac{1}{\omega C}\right)^2$$
$$\sqrt{Z^2-R^2}=\sqrt{\left(\frac{1}{\omega C}\right)^2}$$
$$\sqrt{Z^2-R^2}=\frac{1}{\omega C}$$
$$\omega\times\sqrt{Z^2-R^2}=\frac{1\times\cancel{\omega}^1}{\cancel{\omega}^1C}$$
$$\omega\sqrt{Z^2-R^2}=\frac{1}{C}$$
$$\frac{\omega\cancel{\sqrt{Z^2-R^2}}^1}{\cancel{\sqrt{Z^2-R^2}}^1}=\frac{1}{C\sqrt{Z^2-R^2}}$$
$$\omega=\frac{1}{C\sqrt{Z^2-R^2}}$$

or

$$Z=\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}\quad (\omega)$$
$$Z^2=R^2+\left(\frac{1}{\omega C}\right)^2$$
$$Z^2-R^2=\left(\frac{1}{\omega C}\right)^2$$
$$\sqrt{Z^2-R^2}=\frac{1}{\omega C}$$
$$\omega\sqrt{Z^2-R^2}=\frac{1}{C}$$
$$\omega=\frac{1}{C\sqrt{Z^2-R^2}}$$

Topic 2: Equations with exponents

1. Solve for x:

(a)
$$x^3 = 27$$
$$x = \sqrt[3]{27}$$
$$x = 3$$

(b)
$$x^5 + 1 = 244$$
$$x^5 + 1 - 1 = 244 - 1$$
$$x^5 = 243$$
$$\sqrt[5]{\left(x^5\right)} = \sqrt[5]{243}$$
$$x = 3$$

(c)
$$3\sqrt[4]{x} = 6$$
$$\frac{3\sqrt[4]{x}}{3} = \frac{6}{3}$$
$$\sqrt[4]{x} = 2$$
$$\left(\sqrt[4]{x}\right)^4 = 2^4$$
$$x = 16$$

(d)
$$\sqrt[3]{x+1} = 5$$
$$\left(\sqrt[3]{x+1}\right)^3 = 5^3$$
$$x + 1 = 125$$
$$x + 1 - 1 = 125 - 1$$
$$x = 124$$

(e)
$$\sqrt[12]{x} = 2$$
$$\left(\sqrt[12]{x}\right)^{12} = 2^{12}$$
$$x = 4096$$

(f)
$$4\sqrt{x} + 31 = 63$$
$$4\sqrt{x} + 31 - 31 = 63 - 31$$
$$4\sqrt{x} = 32$$
$$\frac{4\sqrt{x}}{4} = \frac{32}{4}$$
$$\sqrt{x} = 8$$
$$\left(\sqrt{x}\right)^2 = 8^2$$
$$x = 64$$

(g)
$$5x^2 + 4 = 9$$
$$5x^2 + 4 - 4 = 9 - 4$$
$$5x^2 = 5$$
$$\frac{5x^2}{5} = \frac{5}{5}$$
$$x^2 = 1$$
$$x = \pm 1$$

(h)
$$7(x^4 + 1) = 119$$
$$\frac{7(x^4 + 1)}{7} = \frac{119}{7}$$
$$x^4 + 1 - 1 = 17 - 1$$
$$x^4 = 16$$
$$x = \sqrt[4]{16}$$
$$x = \pm 2$$

(i)
$$x^{-4} = 20$$
$$\frac{1}{x^4} = \frac{20}{1}$$
$$x^4 = \frac{1}{20}$$
$$\sqrt[4]{x^4} = \sqrt[4]{\frac{1}{20}}$$
$$x \approx 0.47287$$

(j)
$$\frac{x^{-1}}{4} = 11$$
$$\frac{x^{-1}}{4} \times 4 = 11 \times 4$$
$$x^{-1} = 44$$
$$x = \frac{1}{44}$$

(k)
$$4x^{-5} = 5$$
$$\frac{4x^{-5}}{4} = \frac{5}{4}$$
$$x^{-5} = 1.25$$
$$x^5 = \frac{1}{1.25}$$
$$x^5 = 0.8$$
$$\sqrt[5]{x^5} = \sqrt[5]{0.8}$$
$$x \approx 0.9565$$

(l)
$$3x^{\frac{1}{2}} + 7 = 19$$
$$3x^{\frac{1}{2}} + 7 - 7 = 19 - 7$$
$$3x^{\frac{1}{2}} = 12$$
$$\frac{3x^{\frac{1}{2}}}{3} = \frac{12}{3}$$
$$x^{\frac{1}{2}} = 4$$
$$\left(x^{\frac{1}{2}}\right)^2 = 4^2$$
$$x = 16$$

(m)
$$3x^4 + 256 = 1024$$
$$3x^4 + 256 - 256 = 1024 - 256$$
$$\frac{3x^4}{3} = \frac{768}{3}$$
$$x^4 = 256$$
$$\sqrt[4]{x^4} = \sqrt[4]{256}$$
$$x = 4$$

2. Rearrange these formulae for the pronumeral in brackets

(a)

$$A = 4\pi r^2 \quad [r]$$
$$\frac{A}{4\pi} = \frac{4\pi r^2}{4\pi}$$
$$\frac{A}{4\pi} = r^2$$
$$\sqrt{\frac{A}{4\pi}} = \sqrt{r^2}$$
$$\sqrt{\frac{A}{4\pi}} = r$$

(b)

$$V = \pi r^2 h \quad [r]$$
$$\frac{V}{\pi h} = \frac{\pi r^2 h}{\pi h}$$
$$\frac{V}{\pi h} = r^2$$
$$\sqrt{\frac{V}{\pi h}} = \sqrt{r^2}$$
$$\sqrt{\frac{V}{\pi h}} = r$$

(c)

$$c^2 = a^2 + b^2 \quad [b]$$
$$c^2 - a^2 = a^2 + b^2 - a^2$$
$$c^2 - a^2 = b^2$$
$$\sqrt{c^2 - a^2} = \sqrt{b^2}$$
$$\sqrt{c^2 - a^2} = b$$

(d)

$$F = \frac{1}{2}mv^2 \quad [v]$$
$$2 \times F = 2 \times \frac{1}{2}mv^2$$
$$2F = mv^2$$
$$\frac{2F}{m} = \frac{mv^2}{m}$$
$$\frac{2F}{m} = v^2$$
$$\sqrt{\frac{2F}{m}} = \sqrt{v^2}$$
$$\sqrt{\frac{2F}{m}} = v$$

(e)

$$Q = SLd^2 \quad [d]$$
$$\frac{Q}{SL} = \frac{SLd^2}{SL}$$
$$\frac{Q}{SL} = d^2$$
$$\sqrt{\frac{Q}{SL}} = \sqrt{d^2}$$
$$\sqrt{\frac{Q}{SL}} = d$$

(f)

$$v^2 = u^2 + 2as \quad [u]$$
$$v^2 - 2as = u^2 + 2as - 2as$$
$$v^2 - 2as = u^2$$
$$\sqrt{v^2 - 2as} = \sqrt{u^2}$$
$$\sqrt{v^2 - 2as} = u$$

(g)

$$Z = \sqrt{R^2 + \omega^2 L^2} \quad [L]$$
$$Z^2 = \left(\sqrt{R^2 + \omega^2 L^2}\right)^2$$
$$Z^2 = R^2 + \omega^2 L^2$$
$$Z^2 - R^2 = R^2 + \omega^2 L^2 - R^2$$
$$Z^2 - R^2 = \omega^2 L^2$$
$$\frac{Z^2 - R^2}{\omega^2} = \frac{\omega^2 L^2}{\omega^2}$$
$$\frac{Z^2 - R^2}{\omega^2} = L^2$$
$$\sqrt{\frac{Z^2 - R^2}{\omega^2}} = \sqrt{L^2}$$
$$\sqrt{\frac{Z^2 - R^2}{\omega^2}} = L$$

(h)

$$R = \frac{2GM}{c^2} \quad [c]$$
$$c^2 R = \frac{2GM}{c^2} \times c^2$$
$$c^2 R = 2GM$$
$$\frac{c^2 R}{R} = \frac{2GM}{R}$$
$$c^2 = \frac{2GM}{R}$$
$$\sqrt{c^2} = \sqrt{\frac{2GM}{R}}$$
$$c = \sqrt{\frac{2GM}{R}}$$

(i)

$$P = \frac{\pi^2 EI}{L^2} \quad [L]$$
$$L^2 \times P = \frac{\pi^2 EI}{L^2} \times L^2$$
$$L^2 P = \pi^2 EI$$
$$\frac{L^2 P}{P} = \frac{\pi^2 EI}{P}$$
$$L^2 = \frac{\pi^2 EI}{P}$$
$$\sqrt{L^2} = \sqrt{\frac{\pi^2 EI}{P}}$$
$$L = \sqrt{\frac{\pi^2 EI}{P}}$$

The next questions are challenging, take care!

(j)
$$T = 2\pi\sqrt{\frac{r^3}{GM}} \quad [r]$$
$$\frac{T}{2\pi} = \frac{2\pi\sqrt{\frac{r^3}{GM}}}{2\pi}$$
$$\frac{T}{2\pi} = \sqrt{\frac{r^3}{GM}}$$
$$\left(\frac{T}{2\pi}\right)^2 = \left(\sqrt{\frac{r^3}{GM}}\right)^2$$
$$\left(\frac{T}{2\pi}\right)^2 = \frac{r^3}{GM}$$
$$GM \times \left(\frac{T}{2\pi}\right)^2 = \frac{r^3}{GM} \times GM$$
$$GM\left(\frac{T}{2\pi}\right)^2 = r^3$$
$$\sqrt[3]{GM\left(\frac{T}{2\pi}\right)^2} = \sqrt[3]{r^3}$$
$$\sqrt[3]{GM\left(\frac{T}{2\pi}\right)^2} = r$$

(k)
$$F = m\left(g - \frac{v^2}{R}\right) \quad [v]$$
$$\frac{F}{m} = \frac{m\left(g - \frac{v^2}{R}\right)}{m}$$
$$\frac{F}{m} = g - \frac{v^2}{R}$$
$$\frac{F}{m} - g = g - \frac{v^2}{R} - g$$
$$\frac{F}{m} - g = -\frac{v^2}{R}$$
$$g - \frac{F}{m} = \frac{v^2}{R}$$
$$R\left(g - \frac{F}{m}\right) = \frac{v^2}{R} \times R$$
$$R\left(g - \frac{F}{m}\right) = v^2$$
$$\sqrt{R\left(g - \frac{F}{m}\right)} = \sqrt{v^2}$$
$$\sqrt{R\left(g - \frac{F}{m}\right)} = v$$

(l)
$$R = \frac{\pi r^4 (p_2 - p_1)}{8nL} \quad [r]$$
$$8nL \times R = \frac{\pi r^4 (p_2 - p_1)}{8nL} \times 8nL$$
$$8nLR = \pi r^4 (p_2 - p_1)$$
$$\frac{8nLR}{\pi(p_2 - p_1)} = r^4$$
$$\sqrt[4]{\frac{8nLR}{\pi(p_2 - p_1)}} = \sqrt[4]{r^4}$$
$$\sqrt[4]{\frac{8nLR}{\pi(p_2 - p_1)}} = r$$

(m)
$$A = P(1+i)^n \quad [i]$$
$$\frac{A}{P} = \frac{P(1+i)^n}{P}$$
$$\frac{A}{P} = (1+i)^n$$
$$\sqrt[n]{\frac{A}{P}} = \sqrt[n]{(1+i)^n}$$
$$\sqrt[n]{\frac{A}{P}} = 1+i$$
$$\sqrt[n]{\frac{A}{P}} - 1 = 1+i-1$$
$$\sqrt[n]{\frac{A}{P}} - 1 = i$$

Topic 3: Quadratic equations

1. Solve these quadratic equations for x: (by factorising)

(a)
$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$x - 3 = 0 \text{ or } x + 2 = 0$$
$$x = 3 \qquad x = -2$$

(b)
$$2x^2 - 9x + 4 = 0$$
$$(2x-1)(x-2) = 0$$
$$2x - 1 = 0 \text{ or } x - 2 = 0$$
$$x = \frac{1}{2} \qquad x = 2$$

(c)
$$4x^2 + 19x - 5 = 0$$
$$(4x-1)(x+5) = 0$$
$$4x - 1 = 0 \text{ or } x + 5 = 0$$
$$x = \frac{1}{4} \qquad x = -5$$

(d)
$$2x^2 - 4x - 16 = 0$$
$$2(x^2 - 2x - 8) = 0$$
$$2(x-4)(x+2) = 0$$
$$x - 4 = 0 \text{ or } x + 2 = 0$$
$$x = 4 \qquad x = -2$$

(e)
$$0.5x^2 + x - 12 = 0$$
$$x^2 + 2x - 24 = 0$$
$$(x+6)(x-4) = 0$$
$$x + 6 = 0 \quad \text{or} \quad x - 4 = 0$$
$$x = -6 \qquad x = 4$$

(f)
$$4x^2 - 9 = 0$$
$$(2x-3)(2x+3) = 0$$
$$2x - 3 = 0 \quad \text{or} \quad 2x + 3 = 0$$
$$x = \frac{3}{2} \qquad x = -\frac{3}{2}$$

(g)
$$2x^2 + 8x + 8 = 0$$
$$2\left(x^2 + 4x + 4\right) = 0$$
$$2(x+2)^2 = 0$$
$$x = -2$$

(h)
$$x^2 - 4x = 0$$
$$x(x-4) = 0$$
$$x - 4 = 0 \quad \text{or} \quad x = 0$$
$$x = 4$$

2. Solve these quadratic equations for x: (by any method)

(a)
$$x^2 + 12x + 2 = 0$$
$a = 1,\ b = 12$ and $c = 2$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-12 \pm \sqrt{12^2 - 4\times1\times2}}{2\times1}$$
$$x = \frac{-12 \pm \sqrt{136}}{2}$$
$$x = -6 \pm \sqrt{34} \approx -11.83 or -0.17$$

(b)
$$x^2 + 4x - 7 = 0$$
$a = 1,\ b = 4$ and $c = -7$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4\times1\times(-7)}}{2\times1}$$
$$x = \frac{-4 \pm \sqrt{44}}{2} = -2 \pm \sqrt{11}$$
$x \approx -5.32$ or 1.32

(c)
$$3x^2 + x + 1 = 0$$
$a = 3,\ b = 1$ and $c = 1$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-1 \pm \sqrt{1^2 - 4\times3\times1}}{2\times3}$$
$$x = \frac{-1 \pm \sqrt{-11}}{6}$$
No roots

(d)
$$4x^2 - 5x + 7 = 0$$
$a = 4,\ b = -5$ and $c = 7$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{5 \pm \sqrt{(-5)^2 - 4\times4\times7}}{2\times4}$$
$$x = \frac{5 \pm \sqrt{-87}}{8}$$
No roots

(e)
$$0.1x^2 + 0.2x = 0 \ (\times \text{ by } 10)$$
$$x^2 + 2x = 0$$
$$x(x+2) = 0$$
$$x + 2 = 0 \quad \text{or} \quad x = 0$$
$$x = -2$$

(f)
$$10 - 5x - x^2 = 0$$
$a = -1,\ b = -5$ and $c = 10$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{5 \pm \sqrt{(-5)^2 - 4\times(-1)\times10}}{2\times(-1)}$$
$$x = \frac{5 \pm \sqrt{65}}{-2}$$
$x \approx -6.53$ or 1.53

(g)

$$3.5x^2 + 24x - 3.5 = 0 \quad (\times 2)$$
$$7x^2 + 48x - 7 = 0$$
$$(7x-1)(x+7) = 0$$
$$7x - 1 = 0 \text{ or } x + 7 = 0$$
$$x = \frac{1}{7} \qquad x = -7$$

(h)

$$3x^2 - x - 10 = 0$$
$$(x-2)(3x+5) = 0$$
$$x - 2 = 0 \text{ or } 3x + 5 = 0$$
$$x = 2 \qquad x = -\frac{5}{3}$$

3. Solve these quadratic equations for *x*: (by completing the square)

(a)

$$x^2 + 4x - 7 = 0$$
$$x^2 + 4x + 4 = 7 + 4$$
$$(x+2)^2 = 11$$
$$x + 2 = \pm\sqrt{11}$$
$$x = -2 \pm \sqrt{11}$$

(b)

$$2x^2 + 4x - 1 = 0$$
$$2\left(x^2 + 2x + 1\right) = 1 + 2$$
$$2(x+1)^2 = 3$$
$$(x+1)^2 = \frac{3}{2}$$
$$x + 1 = \pm\sqrt{1.5}$$
$$x = -1 \pm \sqrt{1.5}$$

4. Solve these quadratic equations for x: (by using the general quadratic equation)

(a)
$$3x^2 - 8x - 3 = 0$$
$$a = 3,\ b = -8 \text{ and } c = -3$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{8 \pm \sqrt{(-8)^2 - 4 \times 3 \times (-3)}}{2 \times 3}$$
$$x = \frac{8 \pm \sqrt{100}}{6}$$
$$x = -\frac{1}{3} \text{ or } 3$$

(b)
$$x^2 - x + 4 = 0$$
$$a = 1,\ b = -1 \text{ and } c = 4$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{1 \pm \sqrt{(-1)^2 - 4 \times 1 \times 4}}{2 \times 1}$$
$$x = \frac{1 \pm \sqrt{-15}}{2}$$
No roots

(c)
$$2x^2 - 7x + 3 = 0$$
$$a = 2,\ b = -7 \text{ and } c = 3$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{7 \pm \sqrt{(-7)^2 - 4 \times 2 \times 3}}{2 \times 2}$$
$$x = \frac{7 \pm \sqrt{25}}{6}$$
$$x = \frac{1}{3} \text{ or } 2$$

(d)
$$4x^2 - 9x + 4 = 0$$
$$a = 4,\ b = -9 \text{ and } c = 4$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{9 \pm \sqrt{(-9)^2 - 4 \times 4 \times 4}}{2 \times 4}$$
$$x = \frac{9 \pm \sqrt{17}}{8}$$
$$x \approx 0.61 \text{ or } 1.64$$

5. Calculate the discriminant for these quadratic equations and comment.

(a)
$$x^2 + 4x = 0$$
$$a = 1, b = 4, c = 0$$
$$\Delta = b^2 - 4ac = 16 - 4 \times 1 \times 0 = 16$$
$$\therefore 2\ \ different\ \ roots$$

(b)
$$2x^2 - 12x + 18 = 0$$
$$a = 2, b = -12, c = 18$$
$$\Delta = b^2 - 4ac = 144 - 4 \times 2 \times 18 = 0$$
$$2\ Equal\ \ roots\ (1\ root)$$

(c)
$$2x^2 - 3x + 3 = 0$$
$$a = 2, b = -3, c = 3$$
$$\Delta = b^2 - 4ac = 9 - 4 \times 2 \times 3 = -15$$
No roots

(d)
$$-2x^2 + 5x - 4 = 0$$
$$a = -2, b = 5, c = -4$$
$$\Delta = b^2 - 4ac = 25 - 4 \times (-2) \times (-4) = -7$$
No roots

6. In a right angled triangle, the length of the hypotenuse is 20. The length of one side is 4 longer than the other. Calculate all the lengths of the triangle.

Let the one side be *x*, then the other side is *x+4*.
Using Pythagoras' Theorem:

$$x^2+(x+4)^2=20^2$$
$$x^2+x^2+8x+16=400$$
$$2x^2+8x-384=0$$
$$2(x^2+4x-192)=0$$
$$2(x+16)(x-12)=0$$
$$x+16=0 \quad \text{or} \quad x-12=0$$
$$x=-16 \qquad x=12$$

x = 12 is a viable solution, the right angled triangle has sides (12, 16, 20)

Topic 4: Solving logarithmic and exponential equations

1. Solve the following Logarithmic Equations.

(a)

$$\log_{10} x = 2$$
$$x = 10^2$$
$$x = 100$$

(b)

$$\log_5 x = -2$$
$$x = 5^{-2}$$
$$x = \frac{1}{25}$$

(c)

$$\log_4 x = 0.5$$
$$x = 4^{0.5} \quad \bullet\text{or } \sqrt{4}$$
$$x = 2$$

(d)

$$\log_7 2x = 2$$
$$2x = 7^2$$
$$2x = 49$$
$$x = \frac{49}{2} = 24.5$$

(e)

$$\log_2 (x+4) = 3$$
$$x + 4 = 2^3$$
$$x + 4 = 8$$
$$x = 4$$

(f)

$$\log_9 \left(\frac{x}{4}\right) = -0.5$$
$$\frac{x}{4} = 9^{-0.5}$$
$$\frac{x}{4} = \frac{1}{\sqrt{9}} \quad \bullet 9^{0.5} = \sqrt{9}$$
$$\frac{x}{4} = \frac{1}{3}$$
$$x = \frac{4}{3}$$

(g)

$$\log_3 x^2 = 2$$
$$x^2 = 3^2$$
$$x^2 = 9$$
$$x = \pm\sqrt{9} = \pm 3$$

(h)

$$\log_3 x = 2$$
$$x = 3^2$$
$$x = 9$$

(i)

$$\log_2 \left(x^2 + 2x\right) = 3$$
$$x^2 + 2x = 2^3$$
$$x^2 + 2x - 8 = 0$$
$$(x+4)(x-2) = 0$$
$$x = -4 \text{ or } 2$$

Checking; when *x=-4*, *log₂8=3*

when *x=2*, *log₂8=3*

(j)

$$\log_x 4 = 4$$
$$4 = x^4$$
$$x = \sqrt[4]{4}$$
$$x = 1.4142 \text{ to 4 d.p.}$$

(k)

$$\log_x 9 = 0.5$$
$$9 = x^{0.5}$$
$$x = 9^2$$
$$x = 81$$

(l)

$$\log_x 64 = 2$$
$$64 = x^2$$
$$x = \pm\sqrt{64}$$
$$x = 8$$

Bases cannot be negative

(m)

$$\log_x (2x-1) = 2$$
$$2x - 1 = x^2$$
$$0 = x^2 - 2x + 1$$
$$0 = (x-1)^2$$
$$x - 1 = 0$$
$$x = 1$$

(n)

$$\log_x 3 = 2$$
$$3 = x^2$$
$$x = \pm\sqrt{3}$$
$$x = 1.732$$

Bases cannot be negative

There are two answers of $x=1$

(o)

$$\log_x 10 = 3$$
$$10 = x^3$$
$$x = \sqrt[3]{10}$$
$$x = 2.154$$

(p)

$$\log_4 64 = x$$
$$x = \frac{\log 64}{\log 4} \quad \text{•change of base}$$
$$x = 3$$

(q)

$$10^{\log x} = 4$$
$$x = 4$$

or

$$10^{\log x} = 4$$
$$\log 4 = \log x \quad \text{•Definition}$$
$$x = 4$$

(r)

$$e^{\ln(x+4)} = 7$$
$$x + 4 = 7$$
$$x = 3$$

2. Solve for x in the Logarithmic Equations.

(a)

$$2\log x = \log 4$$
$$\log x^2 = \log 4$$
$$x^2 = 4$$
$$x = 2 \text{ (-2 is not possible)}$$

(b)

$$2\ln 2x = \ln 36$$
$$\ln(2x)^2 = \ln 36$$
$$4x^2 = 36$$
$$x^2 = 9$$
$$x = 3 \text{ (-3 is not possible)}$$

(c)

$$\log(x+5) = \log x + \log 6$$
$$\log(x+5) = \log 6x$$
$$x + 5 = 6x$$
$$5 = 5x$$
$$x = 1$$

(d)

$$3\log x = \log 4$$
$$\log x^3 = \log 4$$
$$x^3 = 4$$
$$x = \sqrt[3]{4} \approx 1.587 \text{ to 3 d.p.}$$

(e)

$$2\log x = \log 2x + \log 3$$
$$\log x^2 = \log 6x$$
$$x^2 = 6x$$
$$x^2 - 6x = 0$$
$$x(x-6) = 0$$
$$x = 6 \text{ (0 is not possible)}$$

(f)

$$2\log_3 2 - \log_3(x+1) = \log_3 5$$
$$\log_3 4 - \log_3(x+1) = \log_3 5$$
$$\log_3\left(\frac{4}{x+1}\right) = \log_3 5$$
$$\frac{4}{x+1} = 5$$
$$4 = 5x + 5$$
$$-1 = 5x$$
$$x = \frac{-1}{5}$$

(g)

$$\log_4 x + \log_4 6 = \log_4 12$$
$$\log_4 6x = \log_4 12$$
$$6x = 12$$
$$x = 2$$

(h)

$$3\log_3 2 + \log_3(x-2) = \log_3 5$$
$$\log_3 2^3 + \log_3(x-2) = \log_3 5$$
$$\log_3 8(x-2) = \log_3 5$$
$$8x - 16 = 5$$
$$8x = 21$$
$$x = \frac{21}{8} \text{ or } 2\frac{5}{8}$$

(i)
$$\log_8(x+2)=2-\log_8 2$$
$$\log_8(x+2)+\log_8 2=2$$
$$\log_8 2(x+2)=2$$
$$2x+4=64$$
$$2x=60$$
$$x=30$$

(j)
$$\log_2 x=5-\log_2(x+4)$$
$$\log_2 x+\log_2(x+4)=5$$
$$\log_2 x(x+4)=5$$
$$x^2+4x=2^5$$
$$x^2+4x-32=0$$
$$(x-4)(x+8)=0$$
$$x=4$$
(-8 is not possible)

(k)
$$\log x+\log 3=\log 5$$
$$\log 3x=\log 5$$
$$3x=5$$
$$x=\frac{5}{3}$$

(l)
$$\log x-\log(x-1)=\log 4$$
$$\log\left(\frac{x}{x-1}\right)=\log 4$$
$$\frac{x}{x-1}=4$$
$$x=4x-4$$
$$4=3x$$
$$x=\frac{4}{3}$$

(m)
$$\log(x-3)=3$$
$$x-3=10^3$$
$$x-3=1000$$
$$x=1003$$

(n)
$$\ln(4-x)+\ln 2=2\ln x$$
$$\ln 2(4-x)=\ln x^2$$
$$8-2x=x^2$$
$$0=x^2+2x-8$$
$$0=(x+4)(x-2)$$
$x=2$ (-4 is not possible)

(o)
$$\log_4(2x+4)-3=\log_4 3$$
$$\log_4(2x+4)-\log_4 3=3$$
$$\log_4\left(\frac{2x+4}{3}\right)=3$$
$$\frac{2x+4}{3}=64$$
$$2x+4=192$$
$$2x=188$$
$$x=94$$

(p)
$$\log_2 x+3\log_2 2=\log_2\frac{2}{x}$$
$$\log_2 x+\log_2 2^3=\log_2\frac{2}{x}$$
$$\log_2 8x=\log_2\frac{2}{x}$$
$$8x=\frac{2}{x}$$
$$8x^2=2$$
$$x^2=\frac{1}{4}$$
$x=\frac{1}{2}$ ($-\frac{1}{2}$ is not possible)

3. Solve for y in terms of the other variables present.

(a)
$$\log y = \log x + \log 4$$
$$\log y = \log 4x$$
$$y = 4x$$

(b)
$$\log y = \frac{1}{2}[\log 5 + \log x]$$
$$\log y = \frac{1}{2}\log 5x$$
$$\log y = \log\sqrt{5x} \qquad \bullet(5x)^{\frac{1}{2}} = \sqrt{5x}$$
$$y = \sqrt{5x}$$

(c)
$$\log y = \log 4 - \log x$$
$$\log y = \log\frac{4}{x}$$
$$y = \frac{4}{x}$$

(d)
$$\log y + \log x = \log 4 + 2$$
$$\log xy - \log 4 = 2$$
$$\log\left(\frac{xy}{4}\right) = 2$$
$$\frac{xy}{4} = 10^2$$
$$xy = 400$$
$$y = \frac{400}{x}$$

(e)
$$\log_5 x + \log_5 y = \log_5 3 + 1$$
$$\log_5 x + \log_5 y - \log_5 3 = 1$$
$$\log_5\left(\frac{xy}{3}\right) = 1$$
$$\frac{xy}{3} = 5$$
$$xy = 15$$
$$y = \frac{15}{x}$$

(f)
$$2\ln y = \ln e^2 - 3\ln x$$
$$\ln y^2 = \ln e^2 - \ln x^3$$
$$\ln y^2 = \ln\frac{e^2}{x^3}$$
$$y^2 = \frac{e^2}{x^3}$$
$$y = \sqrt{\frac{e^2}{x^3}}$$

(g)
$$\log_2 y + 2\log_2 x = 5 - \log_2 4$$
$$\log_2 y + \log_2 x^2 = 5 - 2$$
$$\log_2 x^2 y = 3$$
$$x^2 y = 2^3$$
$$y = \frac{8}{x^2}$$

(h)
$$\log_7 y = 2\log_7 5 + \log_7 x + 2$$
$$\log_7 y - \log_7 5^2 - \log_7 x = 2$$
$$\log_7\frac{y}{25x} = 2$$
$$\frac{y}{25x} = 7^2$$
$$y = 1225x$$

(i)
$$3\ln y = 2 + 3\ln x$$
$$\ln y^3 = 2 + \ln x^3$$
$$\ln y^3 - \ln x^3 = 2$$
$$\ln\frac{y^3}{x^3} = 2$$
$$\frac{y^3}{x^3} = e^2$$
$$y^3 = e^2 x^3$$
$$y = \sqrt[3]{e^2 x^3}$$

(j)
$$2(\log_4 y - 3\log_4 x) = 3$$
$$2(\log_4 y - \log_4 x^3) = 3$$
$$\log_4\left(\frac{y}{x^3}\right)^2 = 3$$
$$\frac{y^2}{x^6} = 64$$
$$y^2 = 64x^6$$
$$y = 8x^3$$

(k)

$$2^y = e^x$$
$$\ln 2^y = x$$
$$y \ln 2 = x$$
$$y = \frac{x}{\ln 2}$$

(l)

$$10^y = 3^{x+1}$$
$$\log 10^y = \log 3^{x+1}$$
$$y \log 10 = \log 3^{x+1}$$
$$y = (x+1)\log 3$$

1. Solve the following exponential equations.

(a)

$4^x = 20$

$\log 4^x = \log 20$ Take log of both sides

$x \log 4 = \log 20$ Using the third log law

$$x = \frac{\log 20}{\log 4}$$
$$x = 2.161$$

$\boxed{\text{or}}$

$4^x = 20$

$\ln 4^x = \ln 20$ Take natural log of both sides

$x \ln 4 = \ln 20$ Using the third log law

$$x = \frac{\ln 20}{\ln 4}$$
$$x = 2.161$$

(b)

$$1.6^x = 3.2$$
$$\log 1.6^x = \log 3.2$$
$$x \log 1.6 = \log 3.2$$
$$x = \frac{\log 3.2}{\log 1.6}$$
$$x = 2.475$$

$$1.6^x = 3.2$$
$$\ln 1.6^x = \ln 3.2$$
$$x \ln 1.6 = \ln 3.2$$
$$x = \frac{\ln 3.2}{\ln 1.6}$$
$$x = 2.475$$

(c)

$$3^{2x} = 0.125$$
$$\log 3^{2x} = \log 0.125$$
$$2x \log 3 = \log 0.125$$
$$2x = \frac{\log 0.125}{\log 3}$$
$$2x = -1.8928$$
$$x = \frac{-1.8928}{2}$$
$$x = -0.946$$

$$3^{2x} = 0.125$$
$$\ln 3^{2x} = \ln 0.125$$
$$2x \ln 3 = \ln 0.125$$
$$2x = \frac{\ln 0.125}{\ln 3}$$
$$2x = -1.8928$$
$$x = \frac{-1.8928}{2}$$
$$x = -0.946$$

(d)

$$4.6^{0.5x} = 100$$
$$\log 4.6^{0.5x} = \log 100$$
$$0.5x \log 4.6 = \log 100$$
$$0.5x = \frac{\log 100}{\log 4.6}$$
$$0.5x = 3.018$$
$$x = \frac{3.0177}{0.5}$$
$$x = 6.035$$

$$4.6^{0.5x} = 100$$
$$\ln 4.6^{0.5x} = \ln 100$$
$$0.5x \ln 4.6 = \ln 100$$
$$0.5x = \frac{\ln 100}{\ln 4.6}$$
$$0.5x = 3.018$$
$$x = \frac{3.0177}{0.5}$$
$$x = 6.035$$

(e)

$$\begin{aligned} 6^{x+1} &= 40 \\ \log 6^{x+1} &= \log 40 \\ (x+1)\log 6 &= \log 40 \\ x+1 &= \frac{\log 40}{\log 6} \\ x+1 &= 2.0588 \\ x &= 2.0588-1 \\ x &= 1.0588 \end{aligned}$$

$$\begin{aligned} 6^{x+1} &= 40 \\ \ln 6^{x+1} &= \ln 40 \\ (x+1)\ln 6 &= \ln 40 \\ x+1 &= \frac{\ln 40}{\ln 6} \\ x+1 &= 2.0588 \\ x &= 2.0588-1 \\ x &= 1.0588 \end{aligned}$$

(f)

$$\begin{aligned} 11.2^{2-x} &= 0.6 \\ \log 11.2^{2-x} &= \log 0.6 \\ (2-x)\log 11.2 &= \log 0.6 \\ 2-x &= \frac{\log 0.6}{\log 11.2} \\ 2-x &= -0.2114 \\ 2+0.2114 &= x \\ x &= 2.2114 \end{aligned}$$

$$\begin{aligned} 11.2^{2-x} &= 0.6 \\ \ln 11.2^{2-x} &= \ln 0.6 \\ (2-x)\ln 11.2 &= \ln 0.6 \\ 2-x &= \frac{\ln 0.6}{\ln 11.2} \\ 2-x &= -0.2114 \\ 2+0.2114 &= x \\ x &= 2.2114 \end{aligned}$$

(g)

$2^x + 12 = 40$ rearrange to get 2^x as the subject

$$\begin{aligned} 2^x &= 40-12 \\ 2^x &= 28 \\ \log 2^x &= \log 28 \\ x\log 2 &= \log 28 \\ x &= \frac{\log 28}{\log 2} \\ x &= 4.8074 \end{aligned}$$

$2^x + 12 = 40$ rearrange to get 2^x as the subject

$$\begin{aligned} 2^x &= 40-12 \\ 2^x &= 28 \\ \ln 2^x &= \ln 28 \\ x\ln 2 &= \ln 28 \\ x &= \frac{\ln 28}{\ln 2} \\ x &= 4.8074 \end{aligned}$$

(h)

$3\times 6^x - 10 = 12.5$ rearrange to make 6^x the subject

$$\begin{aligned} 3\times 6^x &= 12.5+10 \\ 3\times 6^x &= 22.5 \\ 6^x &= \frac{22.5}{3} \\ 6^x &= 7.5 \\ \log 6^x &= \log 7.5 \\ x\log 6 &= \log 7.5 \\ x &= \frac{\log 7.5}{\log 6} \\ x &= 1.1245 \end{aligned}$$

$3\times 6^x - 10 = 12.5$ rearrange to make 6^x the subject

$$\begin{aligned} 3\times 6^x &= 12.5+10 \\ 3\times 6^x &= 22.5 \\ 6^x &= \frac{22.5}{3} \\ 6^x &= 7.5 \\ \ln 6^x &= \ln 7.5 \\ x\ln 6 &= \ln 7.5 \\ x &= \frac{\ln 7.5}{\ln 6} \\ x &= 1.1245 \end{aligned}$$

(i)

$$6^{x+4} = 6^{11}$$

as the bases are the same

the exponents can be equated

$$\begin{aligned} x+4 &= 11 \\ x &= 7 \end{aligned}$$

(j)

$$\begin{aligned}
5^x &= 2^{2x+1} \\
\log 5^x &= \log 2^{2x+1} \\
x\log 5 &= (2x+1)\log 2 \\
x &= (2x+1)\frac{\log 2}{\log 5} \\
x &= 0.4307(2x+1) \\
x &= 0.8614x+0.4307 \\
1x-0.8614x &= 0.4307 \\
0.1386x &= 0.4307 \\
x &= \frac{0.4307}{0.1386} \\
x &= 3.108
\end{aligned}$$

$$\begin{aligned}
5^x &= 2^{2x+1} \\
\ln 5^x &= \ln 2^{2x+1} \\
x\ln 5 &= (2x+1)\ln 2 \\
x &= (2x+1)\frac{\ln 2}{\ln 5} \\
x &= 0.4307(2x+1) \\
x &= 0.8614x+0.4307 \\
1x-0.8614x &= 0.4307 \\
0.1386x &= 0.4307 \\
x &= \frac{0.4307}{0.1386} \\
x &= 3.108
\end{aligned}$$

2. If an investor deposits \$250 000 in an account that compounds at 7% p.a., how long is it before this amount has increased to \$400 000? (Compounded semi annually – half yearly). Use the compound interest formula: $A = P(1+i)^n$

7% p.a. = 3.5% per half year

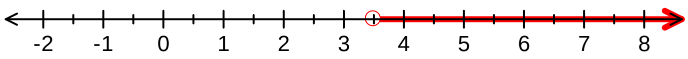
$$\begin{aligned}
A &= P(1+i)^n \\
400000 &= 250000(1+0.035)^n \\
\frac{400000}{250000} &= 1.035^n \\
1.6 &= 1.035^n \\
\log 1.6 &= \log 1.035^n \\
\frac{\log 1.6}{\log 1.035} &= n \\
n &= 13.66
\end{aligned}$$

It will take 13.66 half years for \$250 000 to grow to \$400 000. This is 6.83 years or, in reality, 7 years to the end of the period.

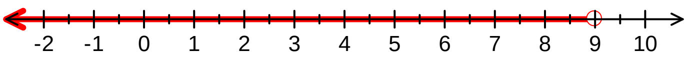
Topic 5: Inequalities

1. Draw number lines to represent.

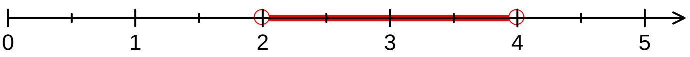
(a) $x > 3.5$



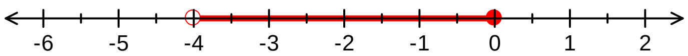
(b) $x < 9$



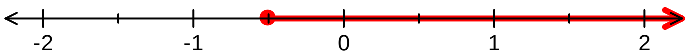
(c) $2 < x < 4$



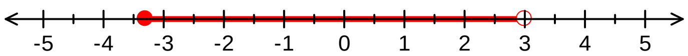
(d) $-4 < x \leq 0$



(e) $x \geq -0.5$



(f) $-3.3 \leq x < 3$



2. Solve these Linear Inequalities

(a)
$$\begin{aligned} -x &< 5 \\ x &> -5 \end{aligned}$$

(b)
$$\begin{aligned} -3x &> 9 \\ x &< -3 \end{aligned}$$

(c)
$$\begin{aligned} \frac{3x}{5} &> 1.2 \\ 3x &> 6 \\ x &> 2 \end{aligned}$$

(d)
$$\begin{aligned} 3x - 1 &> 4 \\ 3x &> 5 \\ x &> \frac{5}{3} \end{aligned}$$

(e)
$$\begin{aligned} 2x + 3 &\geq x - 3 \\ 2x - x &\geq -3 - 3 \\ x &\geq -6 \end{aligned}$$

(f)
$$\begin{aligned} 3(3x - 1) &< 5x + 9 \\ 9x - 3 &< 5x + 9 \\ 9x - 5x &< 9 + 3 \\ 4x &< 12 \\ x &< 3 \end{aligned}$$

(g)
$$\begin{aligned} \frac{x+2}{3} &\geq \frac{2x-1}{6} + \frac{1}{2} \\ \frac{2(x+2)}{6} &\geq \frac{2x+2}{6} \\ 2x + 4 &\geq 2x + 2 \\ 4 &\geq 2 \end{aligned}$$

this is always true;
answer is all real x.

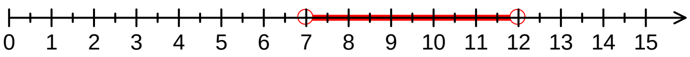
(h)
$$\begin{aligned} \frac{a+2}{4} &> \frac{2a-1}{3} + \frac{a}{6} \\ \frac{a+2}{4} &> \frac{5a-2}{6} \\ \frac{3a+6}{12} &> \frac{10a-4}{12} \\ 3a + 6 &> 10a - 4 \\ -7a &> -10 \\ a &< \frac{10}{7} \end{aligned}$$

(i) $$\frac{2(3t-2)}{5}-\frac{2(4-t)}{3}\geq 4$$
$$\frac{6(3t-2)}{15}-\frac{10(4-t)}{15}\geq \frac{60}{15}$$
$$18t-12-40+10t\geq 60$$
$$28t-52\geq 60$$
$$28t\geq 112$$
$$t\geq 4$$

3. Solve these compound inequalities. Draw number lines to obtain (show) the solution.

(a) $0<x-7<5$

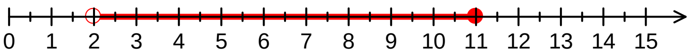
$7<x<12$



(b) $15<4x+7\leq 51$

$8<4x\leq 44$

$2<x\leq 11$



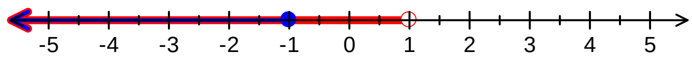
(c) $5x<2x+3\leq -x$

$5x<2x+3$ and $2x+3\leq -x$

$3x<3$ $\quad$ $3x\leq -3$

$x<1$ $\quad$ $x\leq -1$

combining $x\leq -1$



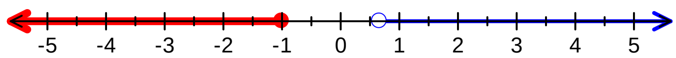
Overall: $x\leq 1$

(d) $-3(x-2)\leq 5-4x<3-x$

$-3x+6\leq 5-4x$ and $5-4x<3-x$

$x\leq -1$ $\quad$ $-3x<-2$

$x\leq -1$ $\quad$ $x>\frac{2}{3}$



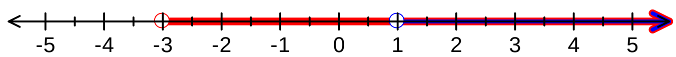
Overall: no solution

(e) $x-1<2x+2<3x+1$

$x-1<2x+2$ and $2x+2<3x+1$

$-x<3$ $\quad$ $-x<-1$

$x>-3$ $\quad$ $x>1$



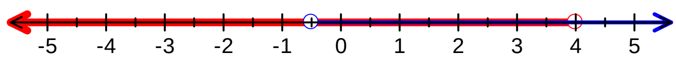
Overall: $x\geq 1$

(f) $4x-3<2x+5<6x+7$

$4x-3<2x+5$ and $2x+5<6x+7$

$2x<8$ $\quad$ $-4x<2$

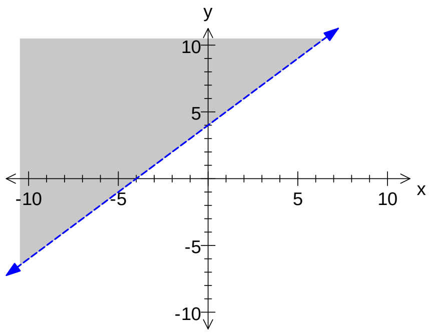
$x<4$ $\quad$ $x>-\frac{1}{2}$



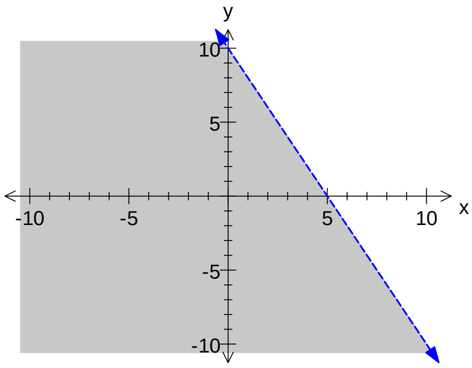
Overall: $-\frac{1}{2}<x<4$

4. Graph the following inequalities.

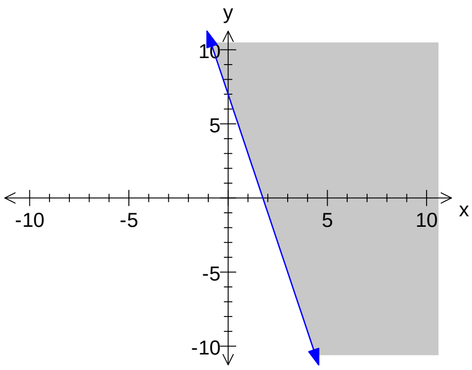
(a) $y > x + 4$



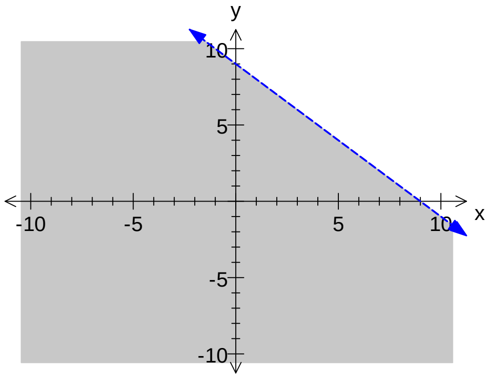
(b) $2x + y < 10$



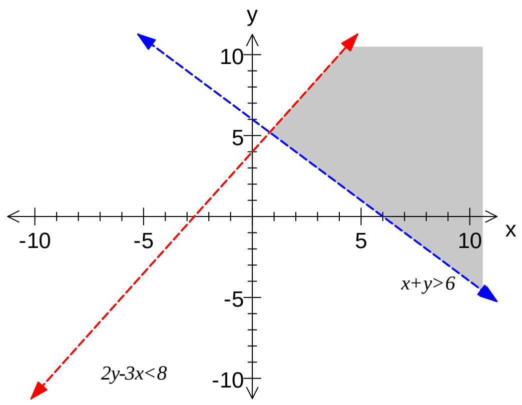
(c) $y \geq 7 - 4x$



(d) $x + y < 9$



(e) Find the solutions common to $x + y > 6$ and $2y - 3x < 8$



y
x
10
5
-5
-10
-10
-5
5
10
x+y>6
2y-3x<8