

## Introduction to decimals

Decimals are an alternative way of representing a part of a whole. While fractions are commonly used in everyday language, decimals are commonly used in calculations involving the metric system and are more calculator friendly. They are an extension of the base ten number system already covered in the Whole Numbers and Integers Module. A decimal point is used to separate the whole part from the fraction part.

	$\times 10$	$\times 10$	$\times 10$	$\times 10$	$\times 10$	$\times 10$	$\times 10$	$\times 10$	$\times 10$	$\times 10$	$\times 10$	$\times 10$	$\times 10$	$\times 10$
Hundred Thousands	Ten Thousands	Unit Thousands	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths	Ten-Thousandths	Hundred-Thousandths	Millionths		
					5	.	1	4						
Whole Number Part							Fractional Part							

Most decimals are either terminating decimals or recurring decimals.

**Terminating decimals** have a certain number of decimal places.  
For example: 0.34, 4.125, 3.2

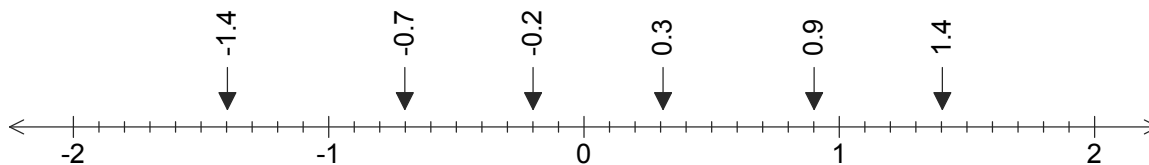
**Recurring decimals** have a digit or digits that keep repeating.  
For example:  $0.\overline{3} = 0.3333333\dots$  the 3 is repeated

$$0.\overline{16} = 0.16666\dots \text{ the 6 is repeated}$$

$$0.\overline{13} = 0.131313\dots \text{ the 13 is repeated}$$

There are decimals that are non-terminating and non-recurring. The best known example of this type of decimal is PI ( $\pi$ ). PI is a number used in circle measurements and is usually written as  $\pi = 3.141592654$  to 9 decimal places. The square root of some numbers also gives non-terminating and non-recurring decimals ( $\sqrt{2}$ ,  $\sqrt{3}$ ,  $\sqrt{5}\dots$ ).

It is easy to show decimals (to tenths) on a number line.



## Module contents

### Introduction

- Place value of decimals
- Changing between fractions and decimals
- Rounding decimals
- Adding and subtracting decimals
- Multiplying and dividing decimal

### Outcomes

- identify the type of decimal and name decimals
- convert between decimals and fractions.
- round decimals to different places
- perform all four operations using decimals.

## Check your skills

This module covers the following concepts, if you can successfully answer these questions, you do not need to do this module.

- Write in words and say the decimal 0.35
  - Write the decimal, one hundred and forty three thousandths, in digit form.
- Write the decimal, 0.135, as a simplified fraction.
  - Write the fraction,  $\frac{5}{8}$ , as a decimal.
- Round 2.15629 to 3 decimal places.
  - Round 45.15927 to the nearest tenth.
- Perform the operation indicated.
  - $1.451 + 0.92 + 5.6$
  - $3.59 - ^{-}1.628$
  - $1.34 \times 0.25$
  - $34.8 \div ^{-}0.7$  to 2 d.p.

Check your answers from the answer section at the end of the module.

## Topic 1: Place value of decimals

Hundred Thousands	Ten Thousands	Unit Thousands	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths	Ten-Thousandths	Hundred-Thousandths	Millionths
					0	.	3					
					0	.	0	7				

The top decimal is often said as 'point three' or 'zero point three', however, a better way of saying this is as three tenths because it reflects the place value of the number.

Having said this number as three tenths, it is easy to write as a fraction,  $0.3 = \frac{3}{10}$

The next example is often said as 'point zero seven or zero point zero seven', however a better way of saying this is as seven hundredths. From the place value diagram the seven is clearly in the hundredths place value.

The decimal seven hundredths can be written in decimal form as 0.07 or fraction form as  $\frac{7}{100}$

The number (as a decimal)	Written in its place value form (decimal and fraction)	Can be written and said as
0.05	$0.05 = \frac{5}{100}$	five hundredths
0.001	$0.001 = \frac{1}{1000}$	one thousandth
0.49	$\frac{4}{10} + \frac{9}{100}$ or $\frac{49}{100}$	four tenths and nine hundredths or (preferably) forty nine hundredths
0.065	$\frac{6}{100} + \frac{5}{1000}$ or $\frac{65}{1000}$	sixty five thousandths
3.87	$3 + \frac{8}{10} + \frac{7}{100}$ or $3\frac{87}{100}$	Three and eighty seven hundredths

The number....	Written in digit form is ....
twenty two hundredths	0.22
sixteen thousandths	0.016
four and seven tenths	4.7

The decimal  $0.\overline{3}$  is difficult to say using place values, so it is said as 'point three recurring' or 'zero point three recurring'. It is possible to write recurring decimals as a fraction but mostly it is difficult and not part of this module, although, in this case most people know that  $0.\overline{3} = \frac{1}{3}$ .

The decimal  $0.\overline{52}$  is said as 'zero point five two all recurring', the word 'all' indicates both the five and the two are recurring.



[Video 'Place Value'](#)

## Activity

- Write the decimals below in written form.
  - 0.5
  - 0.42
  - 1.06
  - 0.03
  - 0.125
  - 4.4
  - 0.007
  - 0.016
  - 72.94
- Write the decimals below in digit form.
  - Four tenths.
  - Thirteen hundredths.
  - Forty five thousandths.
  - Two and twenty five hundredths.
  - Five and eighty two hundredths.
  - Two hundred and seventy two hundredths.
- Answer True or False to the following statements.
  - The decimal  $0.\overline{4}$  is called a recurring decimal.
  - In the decimal  $0.\overline{21}$  only the 1 recurs.
  - The decimal thirty hundredths is the same as three tenths.
  - 0.65 can be written as sixty five tenths.

## Topic 2: Changing between fractions and decimals

### Decimals to fractions

Using the place value of decimals, it is easy to change decimals to fractions.

Already from Topic 1, we know that  $0.3 = \frac{3}{10}$  and  $0.07 = \frac{7}{100}$ .

The final thing to consider is whether the fraction will simplify. In this example the fraction does not simplify.

Decimal	Written as a fraction is .... and then simplified (if required) ....
0.05	$0.05 = \frac{5 \div 5}{100 \div 5} = \frac{1}{20}$
0.001	$0.001 = \frac{1}{1000}$
0.49	$0.49 = \frac{49}{100}$
0.65	$0.65 = \frac{65 \div 5}{100 \div 5} = \frac{13}{20}$
-0.44	$-0.44 = \frac{-44 \div 4}{100 \div 4} = \frac{-11}{25}$
2.75	$2.75 = 2 \frac{75 \div 25}{100 \div 25} = 2 \frac{3}{4}$
3.87	$3.87 = 3 \frac{87}{100}$



[Video 'Changing decimals to fractions'](#)

## Fractions to decimals

There are 2 methods of changing fractions to decimals.

The first method uses the place value of decimals ie: tenths, hundredths, thousandths etc. This method is a quick way of changing from a fraction to a decimal but is limited in what fractions it works for.

The second method involves division. The vinculum (bar) in a fraction means division so changing to a decimal involves dividing the numerator by the denominator.

### Method One:

Look at the fraction. If the fraction has a denominator of 10, 100, 1000 etc. or if an equivalent fraction has a denominator of 10, 100, 1000 etc. it can just be written as the decimal equivalent.

For example:

$$\frac{3}{10} \\ = 0.3$$

This is three tenths, as a decimal this is a 3 in the tenths place value.

$$\frac{2^{\times 2}}{5^{\times 2}} \\ = \frac{4}{10} \\ = 0.4$$

Two fifths is equivalent to four tenths, as a decimal this is a 4 in the tenths place value.

$$\frac{3^{\times 25}}{4^{\times 25}} \\ = \frac{75}{100} \\ = 0.75$$

Three quarters is equivalent to seventy five hundredths, as a decimal this is 70 hundredths or 7 tenths and 5 hundredths.

$$\frac{3^{\times 5}}{20^{\times 5}} \\ = \frac{15}{100} \\ = 0.15$$

Three twentieths is equivalent to fifteen hundredths, as a decimal this is 10 hundredths or 1 tenth and 5 hundredths.

$$\frac{7}{100} \\ = 0.07$$

This is seven hundredths, as a decimal this is 7 in the hundredths place value.

$$\frac{3^{\times 125}}{8^{\times 125}} \\ = \frac{375}{1000} \\ = 0.375$$

Three eights is equivalent to three hundred and seventy five thousandths, as a decimal this is 300 thousandths or 3 tenths, 70 thousandths or 7 hundredths and 5 thousandths

### Method Two:

Remember this method always works and is suitable for calculator use. An alternative way of writing a fraction is numerator  $\div$  denominator, performing this division results in a decimal answer.

For example:

$$\frac{3}{5} \quad \text{This is } 3 \div 5 \text{ calculated by } 5 \overline{)3.0}$$

To be able to divide, .0 is added to the three, the number 3 is not changed, and this enables tenths to be shared.

$$\frac{4}{9} \quad \text{This is } 4 \div 9 \text{ calculated by } 9 \overline{) 4.0^4 0^4 0}$$

In this question, a decimal point and many zeros were added. The answer is a recurring decimal.

$$\frac{2}{7} \quad \text{This is } 2 \div 7 \text{ calculated by } 7 \overline{) 2.0^6 0^4 0^5 0^1 0}$$

Continuing to divide by 7 will result in a recurring decimal. Usually answering to three or four decimal places is enough. This is 0.2857 to 4 decimal places.

$$\frac{9}{4} \quad \text{This is } 9 \div 4 \text{ calculated by } 4 \overline{) 9.10^2 0}$$

Fractions both proper and improper can be changed to a decimal this way.

$$\frac{-7}{8} \quad \text{This is } 7 \div 8 \text{ calculated by } 8 \overline{) 7.0^6 0^4 0}$$

As a  $\frac{-}{+} = \frac{-}{-}$ , the answer is  $-0.875$



[Video 'Changing fractions to decimals'](#)

The examples are done on a calculator are below.

	Using the $\div$ key	Using the fraction $\frac{\square}{\square}$ key
$\frac{3}{5}$	$3 \div 5 = 0.6$	$\frac{3}{5} = 0.6$
$\frac{4}{9}$	$4 \div 9 = 0.4444444444$	$\frac{4}{9} = 0.4444444444$
$\frac{2}{7}$	$2 \div 7 = 0.2857142857$	$\frac{2}{7} = 0.2857142857$
$\frac{9}{4}$	$9 \div 4 = 2.25$	$\frac{9}{4} = 2.25$
$\frac{-7}{8}$	$(-)\ 7 \div 8 = -0.875$	$(-)\ \frac{7}{8} = -0.875$

## Activity

1. Write the following decimals as simplified fractions.

- |     |      |     |       |     |       |
|-----|------|-----|-------|-----|-------|
| (a) | 0.08 | (b) | 0.6   | (c) | 0.14  |
| (d) | 0.64 | (e) | 4.25  | (f) | 6.2   |
| (g) | 12.5 | (h) | 3.125 | (i) | 0.075 |

2. Change the following fractions to decimals, check with your calculator.

- |     |                |     |                |     |                |
|-----|----------------|-----|----------------|-----|----------------|
| (a) | $\frac{7}{10}$ | (b) | $\frac{4}{5}$  | (c) | $\frac{3}{11}$ |
| (d) | $\frac{-2}{3}$ | (e) | $\frac{-7}{4}$ | (f) | $\frac{13}{9}$ |



### Topic 3: Rounding decimals

Rounding of decimals is an important skill to express an answer to a question. If an answer to a financial question is given to parts of a cent, then rounding is important to express the answer to the nearest cent. Often calculators will express decimal answers to many decimal places when only 2 or 3 decimal places are necessary.

When rounding a decimal, a place value or a specified number of decimal places to which the number must be rounded will be stated.

For example:

Round 0.4592 to the nearest hundredth.

The number 0.4592 is, in size, between 0.45 and 0.46 when the hundredths place (or 2 decimal places) is being considered. The purpose of rounding is to say that 0.4592 is closer to 0.46 than 0.45.

There are two steps to rounding to a stated place value. (Using the example above)

Step 1 Locate the digit in the place value being stated and underline it.

0.4592

Step 2 Locate the digit to the right of the place value stated. Now circle this digit.

0.45⑨2

If this digit is 'less than 5' then the digit in the stated place value stays the same.

If this digit is '5 or more' then the digit in the stated place value increases by one.

As the digit is ⑨, the 5 will increase to 6.

The answer is 0.4592  $\approx$  0.46 ( $\approx$  means approximately equal to)

Round 0.00572 to the nearest thousandth.

Step 1 Locate the digit in the place value being stated and underline it.

0.00572

Step 2 Locate the digit to the right of the place value stated. Now circle this digit.

0.005⑦2

As the digit is ⑦, which is '5 or more' the 5 will increase to 6.

The answer is 0.00572  $\approx$  0.006

Round  $0.\overline{3}$  to 2 d.p. (decimal places)

Step 1 Locate the digit in the place value being stated and underline it.

$0.\underline{3}33333$

Step 2 Locate the digit to the right of the place value stated. Now circle this digit.

$0.\underline{3}3\textcircled{3}33$

As the digit is  $\textcircled{3}$ , which is 'less than 5', the 3 will stay the same.

The answer is  $0.\overline{3} \approx 0.33$

Round 12.714 to 1 d.p.

Step 1 Locate the digit in the place value being stated and underline it.

$12.\underline{7}14$

Step 2 Locate the digit to the right of the place value stated. Now circle this digit.

$12.\underline{7}\textcircled{1}4$

As the digit is  $\textcircled{1}$ , which is 'less than 5', the 7 will stay the same.

The answer is  $12.714 \approx 12.7$

Round \$44.20658 to the nearest cent.

Step 1 Locate the digit in the place value being stated and underline it.

$\$44.\underline{2}0658$

Step 2 Locate the digit to the right of the place value stated. Now circle this digit.

$\$44.\underline{2}0\textcircled{6}58$

As the digit is  $\textcircled{6}$ , which is '5 or more', the 0 will increase to 1.

The answer is  $\$44.20658 \approx \$44.21$



[Video 'Rounding Decimals'](#)

## Activity

- Round the following decimals to the place value indicated.
    - 0.5946 to the nearest hundredth
    - 0.6076 to the nearest thousandth
    - 0.756 to the nearest tenth
    - $0.\overline{8}$  to the nearest hundredth
    - 3.1415926 to the nearest hundredth
    - 5.2007 to the nearest tenth
  
  - Round the following decimals to the number of decimal places indicated.
    - 0.667 to 2 decimal places
    - 0.7079 to 1 d.p.
    - 0.7804 to 3 d.p.
    - 4.2192 to 2 d.p.
    - $3.\overline{27}$  to 3 d.p.
    - 55.64 to 1 d.p.
  
  - Round the following amounts, to the nearest cent.

(a) \$0.5792	(b) \$1.6225	(c) \$10.52222
(d) \$15.766666	(e) \$78.245	(f) \$122.656565
-

## Topic 4: Adding and subtracting decimals

When adding or subtracting decimals, the numbers of the same place value must be lined up before the operation can be performed. To achieve this, the numbers are placed vertically. If a whole number is part of the question then the decimal point goes to the right of the units place value.

### For example:

Add 9.27, 3.6 and 12

Arrange numbers vertically, lining up numbers of the same place value and adding zeros to obtain the same number of decimal places.

Add the numbers in the hundredths column (=7), put the 7 in the hundredths column.  
 Add the numbers in the tenths column (=8), put the 8 in the tenths column.  
 Add the numbers in the units column(=14), put the 4 in the units column, and the carry in the tens column.  
 Add the numbers in the tens column and the carry(=2), put the 2 in the tens column.

$$\begin{array}{r}
 \phantom{0}^1 \\
 9.27 \\
 3.60 \\
 +12.00 \\
 \hline
 24.87
 \end{array}$$

By calculator

$$\boxed{9} \boxed{\cdot} \boxed{2} \boxed{7} \boxed{+} \boxed{3} \boxed{\cdot} \boxed{6} \boxed{+} \boxed{1} \boxed{2} \boxed{=} 24.87$$

### For example:

11.346 take 7.32.

Arrange numbers vertically, lining up numbers of the same place value and adding zeros to obtain the same number of decimal places.

In the thousandths column, 6-0=6  
 In the hundredths column, 4-2=2  
 In the tenths column, 3-3=0  
 In the units column, 1-7 cannot be done, so borrow 1 from the next column and pay back 10. Now 11-7=4

$$\begin{array}{r}
 \phantom{0}^0 \phantom{0}^{11} \\
 11.346 \\
 -7.320 \\
 \hline
 4.026
 \end{array}$$

By calculator

$$\boxed{1} \boxed{1} \boxed{\cdot} \boxed{3} \boxed{4} \boxed{6} \boxed{-} \boxed{7} \boxed{\cdot} \boxed{3} \boxed{2} \boxed{=} 4.026$$

### For example:

3 – 1.291

Arrange numbers vertically, lining up numbers of the same place value and adding zeros to obtain the same number of decimal places.

In the thousandths column, 0-1 cannot be done, so borrow from the units and payback each place value until the thousandths are done. Now 10-1=9  
 In the hundredths column, 9-9=0  
 In the tenths column, 9-2=7  
 In the units column, 2-1 =1

$$\begin{array}{r}
 \phantom{0}^2 \phantom{0}^9 \phantom{0}^9 \phantom{0}^{10} \\
 3.\phantom{0}0\phantom{0}0 \\
 -1.291 \\
 \hline
 1.709
 \end{array}$$

By calculator

$$\boxed{3} \boxed{-} \boxed{1} \boxed{\cdot} \boxed{2} \boxed{9} \boxed{1} \boxed{=} 1.709$$

**For example:**

$-8.42 + -7.87$  The method for signed number questions is fully explained in the Whole Numbers and Integers module.

Step 1	Replace the double sign $+^-$ with -	$\begin{aligned} & -8.42 + -7.87 \\ & = -8.42 - 7.87 \end{aligned}$
Step 2	The signs in front of both numbers are the same, so add the numbers $(8.42+7.87=16.29)$  The largest number has a - sign in front of it, so the answer has a - sign in front.	$\begin{array}{r} 1 \\ 8.42 \\ +7.87 \\ \hline 16.29 \\ = -16.29 \end{array}$

By calculator

$(-)$   $8$   $.$   $4$   $2$   $+$   $(-)$   $7$   $.$   $8$   $7$   $=$  -16.29

**For example:**

$-201.56 - -109.54$

Step 1	Replace the double sign $-^-$ with -	$\begin{aligned} & -201.56 - -109.54 \\ & = -201.56 + 109.54 \end{aligned}$
Step 2	The signs in front of both numbers are different, so subtract the numbers $(201.56-109.54=92.02)$  The largest number has a - sign in front of it, so the answer has a - sign in front.	$\begin{array}{r} 1 \quad 10 \quad 11 \\ 201.56 \\ -109.54 \\ \hline 92.02 \\ = -92.02 \end{array}$

By calculator

$(-)$   $2$   $0$   $1$   $.$   $5$   $6$   $-$   $(-)$   $1$   $0$   $9$   $.$   $5$   $4$   $=$  -92.02

**For example:**

$0.09172 + -0.05224$

Step 1	Replace the double sign $+^-$ with -	$\begin{aligned} & 0.09172 + -0.05224 \\ & = 0.09172 - 0.05224 \end{aligned}$
Step 2	The signs in front of both numbers are different, so subtract the numbers $(0.09172-0.05224)$  The largest number has an (implied) + sign in front of it, so the answer has a + sign in front.	$\begin{array}{r} 8 \quad 11 \quad 6 \quad 12 \\ 0.09172 \\ -0.05224 \\ \hline 0.03948 \\ = 0.03948 \end{array}$

By calculator

$0$   $.$   $0$   $9$   $1$   $7$   $2$   $+$   $(-)$   $0$   $.$   $0$   $5$   $2$   $2$   $4$   $=$  0.03948



[Video 'Adding and subtracting decimals'](#)

## Activity

1. Perform the following operations.

(a) Add 123.5, 17.92 and 14.

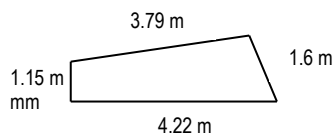
(b)  $-5.652 + -16.21$

(c) Subtract 14.725 from 28.14

(d)  $-47.292 - +8.67$

(e) A length of metal rod was 1.524m long at  $20^{\circ}\text{C}$ , after heating to  $180^{\circ}\text{C}$ , the length increased to 1.541m. By how much did the metal rod expand?

(f) Find the perimeter (distance around) of the following shape.



(g) Jane bought the following items for lunch: a salad roll \$4.55, apple juice \$3.75 and a chocolate cookie \$1.35. How much change will Jane get from her \$10 lunch allowance?

## Topic 5: Multiplying and dividing decimals

### Multiplying by multiples of 10

Multiplying decimals by 10, 100, 1000 etc is slightly different to whole numbers because of the presence of the decimal point. This is achieved by moving the decimal point to the right by the number of places equal to the number of zeros. Moving the decimal point to the right increases the size of the number, each place by a factor of 10.

Example:  $3.418 \times 100$

100 contains 2 zeros, move the decimal point 2 places to the right.

  
 $3.418 \times 100 = 341.8$


Example:  $14.25 \times 1000$

1000 contains 3 zeros, move the decimal point 3 places to the right.

  
 $14.250 \times 1000 = 14250$  Sometimes zero(s) need to be inserted.

Example:  $0.08725 \times 10\,000$

10 000 contains 4 zeros, move the decimal point 4 places to the right.

  
 $0.08725 \times 10\,000 = 872.5$

Example:  $81.753 \times 200$

200 contains 2 zeros, move the decimal point 2 places to the right and then multiply by 2.

  
 $81.753 \times 100 = 8175.3$  then  $8175.3 \times 2 = 16350.6$



[Video 'Multiplying by Multiples of 10'](#)

## Multiplying decimals

When multiplying decimals, multiply the numbers without the decimal points. When the answer is obtained, count the number of digits after the decimal points in the question, then locate the decimal point in the answer such that there are the same number of digits after the decimal point.

Example:  $1.24 \times 0.6$

Multiply the numbers ignoring decimal points, that is  $124 \times 6$

$$\begin{array}{r} 12 \\ 124 \\ \times 6 \\ \hline 744 \end{array}$$

Do the multiplication.

In the question, there are 3 digits after the decimal points, so the answer should have 3 digits after the decimal point.

The answer is  $0.774$

Example:  $71.2 \times 2.5$

Multiply the numbers ignoring decimal points, that is  $712 \times 25$

$$\begin{array}{r} 1 \\ 712 \\ \times 25 \\ \hline 3560 \end{array}$$

Do the multiplication.

In the question, there are 2 digits after the decimal points, so the answer should have 2 digits after the decimal point.

The answer is  $178.00$  or simply  $178$

$$\begin{array}{r} 14240 \\ +14240 \\ \hline 17800 \end{array}$$

Example:  $-3.14 \times +0.8$

Multiply the numbers ignoring decimal points and signs, that is  $314 \times 8$

$$\begin{array}{r} 13 \\ 314 \\ \times 8 \\ \hline 2512 \end{array}$$

Do the multiplication.

In the question, there are 3 digits after the decimal points, so the answer should have 3 digits after the decimal point.

Now think: a negative  $\times$  a positive  $\rightarrow$  negative

The answer is  $-2.512$



Example:  $-5.72 \times -2000$

Multiply the numbers ignoring signs, that is  $5.72 \times 2000$

2000 contains 3 zeros, move the decimal point 3 places to the right and then multiply by 2.

$5.720 \times 1000 = 5720$  then  $5720 \times 2 = 11440$

Now think: a negative  $\times$  a negative  $\rightarrow$  positive

The answer is  $+11\ 440$  or  $11\ 440$

$$\begin{array}{r} 1 \\ 5720 \\ \times 2 \\ \hline 11440 \end{array}$$



[Video 'Multiplying Decimals'](#)

## Dividing by multiples of 10

Dividing decimals by 10, 100, 1000 etc. is slightly different to whole numbers because of the presence of the decimal point. This is achieved by moving the decimal point to the left by the number of places equal to the number of zeros. Moving the decimal point to the left decreases the size of the number, each place by a factor of 10.

Example:  $367.8 \div 100$

100 contains 2 zeros, move the decimal point 2 places to the left.

$367.8 \div 100 = 3.678$

Example:  $0.573 \div 1000$

1000 contains 3 zeros, move the decimal point 3 places to the left.

$0000.573 \div 1000 = 0.000573$  Sometimes zero(s) need to be inserted.

Example:  $-15.25 \div 50$

50 contains 1 zero, move the decimal point 1 place to the left and then divide by 5.

$15.25 \div 10 = 1.525$  then  $1.525 \div 5 = 0.305$

Now think: a negative  $\div$  a positive = a negative

Answer is  $-0.305$



[Video 'Dividing by Multiples of 10'](#)

## Dividing decimals

When dividing decimals, the divisor (number being divided by) should be a whole number. To achieve this, the divisor must be multiplied by 10, 100, 1000 etc. to make the divisor a whole number. The dividend (number being divided into) also needs to be multiplied by 10,100, 1000 etc.

In the question,  $27 \div 0.9$ , both the dividend, 27, and the divisor, 0.9, must be multiplied by 10. The question becomes  $27 \times 10 \div 0.9 \times 10 = 270 \div 9 = 30$

In the question,  $0.225 \div 0.005$ , both the dividend and divisor must be multiplied by 1000. The question becomes  $225 \div 5 = 45$ .

In the question,  $0.0081 \div 0.9$ , both the dividend and divisor must be multiplied by 10. Only the divisor must become whole, the dividend can stay as a fraction. The question becomes  $0.081 \div 9 = 0.009$

Question	Working	Answer
$12.78 \div 0.6$ $= 127.8 \div 6 \text{ (}\times 10\text{)}$	$\begin{array}{r} 21.3 \\ 6 \overline{)127.18} \end{array}$	21.3
$0.0538 \div 7$	$\begin{array}{r} 0.007685 \\ 7 \overline{)0.053860405} \end{array}$	0.00769 to 5 d.p.
$0.3993 \div 0.11$ $= 39.93 \div 11 \text{ (}\times 100\text{)}$	$\begin{array}{r} 3.63 \\ 11 \overline{)39.933} \end{array}$	3.63
$-150.15 \div -0.7$ $= -1501.5 \div -7$	Ignore signs $\begin{array}{r} 214.5 \\ 7 \overline{)151031.35} \end{array}$	Think: negative $\div$ negative = positive +214.5 or 214.5
$0.01488 \div -0.14$ $= 1.488 \div -14$	Ignore signs $\begin{array}{r} 0.10628 \\ 14 \overline{)1.4880120} \end{array}$	Think: positive $\div$ negative = negative -0.1063 to 4 d.p.



[Video 'Dividing Decimals'](#)

## Activity

1. Perform the operation indicated. (Answer to 4 d.p. if required)
    - (a)  $8.342 \times 10000$
    - (b)  $-0.85 \times 1000$
    - (c)  $0.47 \times 0.4$
    - (d)  $-4.5 \times 1.2$
    - (e)  $363.425 \div 1000$
    - (f)  $0.4795 \div 100$
    - (g)  $6036 \div 0.3$
    - (h)  $-143.5 \div +0.7$
    - (i)  $8801.9 \div -0.06$
  
  2. Jim bought 4.25m of canvas at \$23 p.m. How much did he pay?
  
  3. Jan bought a TV with a deposit of \$125 and 12 monthly payments of \$65.42. How much did Jan pay at the end of 12 months?
  
  4. John earned \$456.90 for 30 hours work, what was his hourly rate?
  
  5. Floor tiles are 0.3m square. How many tiles are required to tile a room 5.35m long and 4.75m wide?
-

## Topic 6: Significant figures and measurements

Measurement has limitations, even when there are comparisons to recognised standards. For example; when people use a measuring tape to measure their height, there is a limitation in the precision or accuracy possible. Is it possible, for example, to say that a given height is precisely 178 cm, or 177.9 cm or even 177.88 cm? This topic, deals with issues related to precision or accuracy in measurements.

### Significant figures

Every recorded measurement has a number of significant figures. Using the height measurement above as an example, 178 cm has 3 significant figures, 177.9 cm has 4 significant figures and 177.88 cm has 5 significant figures. As a rule of thumb, the more accurate the measuring instrument, the higher the number of significant figures the measurement will contain. A measuring tape that uses centimetres and/or millimetres could only measure someone's height to (perhaps) 3 significant figures independent of units, 1.78m, 178 cm or 1780 mm.

### Determining the number of significant figures

In the decimal 0.0421 there are 3 significant figures. The zeroes before the digits are merely for the purpose of assigning place value to the digits 421. If this measurement is in metres, then this is equivalent to 42.1 millimetres – the measurement still having 3 significant figures.

The decimal 4.005 has 4 significant figures, the zeroes in the middle are all significant.

The decimal 5.560 is an interesting example. In ordinary Mathematics the zero at the end is often removed and the number remains the same. But in the context of these numbers being recorded from use of some kind of measuring device, the zero on the end is highly significant. The presence of this zero is indicating that the digit in the third decimal place is known and it is a zero. This number has, therefore, 4 significant figures. A measurement recorded as 9.500, therefore, has 4 significant figures.

In general, there may be no unit of measurement that can assist in any decision about the significance of the number. Where a whole number has no zeroes on the end, however, the number of significant digits is equal to the number of digits in the number. For example; 34 085 has 5 significant figures.

Where a whole number has zeroes on the end, the number of significant figures is unclear. For example; 45 600 may have 3, 4 or 5 significant figures. The zeroes on the number must be present for the purpose of place value; however, they may also be significant. When a measurement is taken, it would be helpful for the number of significant figures to be noted also. If no information is available then it is sometimes assumed that it has only 3 significant figures and that the zeroes are not significant.

The context may help in identifying the number of significant figures; the temperature 1400°K may only have 2 significant figures, but the amount of money \$1400 has 4 significant figures. If there is any doubt, always choose the minimum number of significant figures.

For whole numbers ending with zeroes, some conventions may be used to communicate the number of significant figures, examples include:

- 45 600 (to 3 sig. figs) stating the number of significant figures in brackets
- 45 600 a bar is placed under or over the last significant number
- If the number was presented as 45 600. where the presence of the decimal point indicates all digits present are significant.

Units may also give an indication of significant figures, for example; a measurement of 45 600mm expressed as 45.6m makes it clear that there are 3 significant figures. Expressing measurements in larger units as decimals is more helpful in this context.

Examples:

Number	Sig. Figs.		Reasoning
4.01 V	3	<sup>1 2 3</sup> 4.01	Zeroes within the number are all significant
0.006 A	1	<sup>1</sup> 0.006	Zeroes before the digits are for place value and are not significant.
13.50 m	4	<sup>1 2 3 4</sup> 13.50	Any zero(es) on the end after a decimal point are not required for place value so they must indicate significance.
-6.25 msec <sup>-1</sup>	3	<sup>1 2 3</sup> -6.25	Every digit is significant.
4200 mm	2	<sup>1 2</sup> 4200	As there is no information about significant places, must assume only 2 sig. figs.
120 mL = 1.20 L	3	<sup>1 2 3</sup> 1.20	From the unit conversion included it is clear there are 3 sig. figs.

When numbers are written in scientific notation, the number of significant figures can be determined from the number part (1 – 9.9999...)

For example:

$1.6 \times 10^{-19}$  contains two significant figures.

$6.352 \times 10^{12}$  contains four significant figures.

## Rounding a number to a number of significant figures

Later in this topic, it is necessary to express numbers to a certain number of significant figures. The skill of rounding numbers is part of this process.

If two measured quantities are multiplied, for example  $7.26 \times 0.45$ , the calculator sequence will be:

**7** **.** **2** **6** **X** **0** **.** **4** **5** **=** 3.267

The 7.26 has 3 significant figures and 0.45 has 2 significant figures, so should all the digits in the answer be used? Does the answer contain 4 significant figures?

Basically, an answer cannot be any more accurate than the numbers used to calculate it. The answer is limited by the number 0.45 which contains 2 significant figures. The answer should be expressed to 2 significant figures. The process of rounding is used to decide the value of the second significant figure.

$3.\overset{1}{2}\overset{2}{6}7$  - 2 is the second significant figure, the next digit is 6, so the 2 will round up to 3

$$7.26 \times 0.45 = 3.3$$

Example:

Express 39 757 to 3 sig. figs.

$39\overset{1}{7}\overset{2}{5}\overset{3}{7}$  - 7 is the third significant figure, the next digit is 5, so the 7 will round up to 8

$$39\ 757 \text{ to 3 sig. figs.} = 39\ 800$$

Express 0.0324 to 2 sig. figs.

$0.\overset{1}{0}.\overset{2}{3}\overset{3}{2}4$  - 2 is the second significant figure, the next digit is 4, so the 2 will remain as 2

$$0.0324 \text{ to 2 sig. figs.} = 0.032$$

Express 251 000 to 3 sig. figs.

$251\overset{1}{0}\overset{2}{0}\overset{3}{0}$  - 1 is the third significant figure, the next digit is 0, so the 1 will remain as 1

$$251\ 000 \text{ to 3 sig. figs.} = 251\ 000$$

## Errors in measurement

When a quantity is measured there is always an amount of uncertainty in the measurement. In some instances this is stated on the instrument (eg Voltmeter).

The distance between two fence posts is 5.4m accurate to 0.1m. In this case the distance can be written as  $5.4 \pm 0.05\text{m}$ , that is, the actual distance is between 5.35 and 5.45m. The figure  $\pm 0.05$  is called the absolute error.

From this the relative error can be calculated. If this is expressed as a percentage, it is often called the percentage error.

The measurement  $5.4 \pm 0.05\text{m}$  has a relative error of  $\frac{0.05}{5.4} \times 100\% = 0.926\%$  .

A voltmeter has a reading of  $1.3 \pm 0.02\text{V}$

- The absolute error is  $\pm 0.02$
- The percentage error is  $\pm 1.54\%$
- The value contains 2 sig. figs.

## Accuracy and precision

Two groups of students are given the task of calculating the acceleration due gravity.

Group A Value obtained: $9.15 \pm 0.02 \text{ m/sec}^2$ Percentage error of $\frac{0.02}{9.15} \times 100\% = 0.22\%$ $9.15 \pm 0.22\% \text{ m/sec}^2$	Group B Value obtained: $9.9 \pm 0.1 \text{ m/sec}^2$ Percentage error of $\frac{0.1}{9.9} \times 100\% = 1.0\%$ $9.9 \pm 1.0\% \text{ m/sec}^2$
--	---

Group A has obtained a value with a lower percentage error, so Group A's result is more **precise**. Precision is a measure of certainty, the more certainty – the greater the precision. The lower the percentage error, the greater the certainty and hence the greater the precision.

Group B has obtained a value of  $9.9 \pm 0.1 \text{ m/sec}^2$  which is closer to the accepted value of  $9.81 \text{ m/sec}^2$ . This measurement is more **accurate** because it is closer to the accepted value. Accuracy is about achieving a result close to the actual or accepted value.

## Calculations with errors

If the calculations involve **adding and subtracting**, then the measurements are added or subtracted but the absolute errors are added.

For example:

The potential difference across two resistors in series is measured. The total potential difference is the sum of the two resistor's potential differences ( $V_1 = 3.22 \pm 0.02V$ ,  $V_2 = 6.39 \pm 0.02V$ ).

$$\begin{aligned}V_T &= V_1 + V_2 \\V_T &= 3.22 \pm 0.02 + 6.39 \pm 0.02 \\V_T &= 9.61 \pm 0.04\end{aligned}$$

In this example, both potential difference measurements are given as 3 significant figures. The answer should be expressed with 3 sig. figs. If the measurements are to a different number of significant figures, then the lower number of significant figures should be used.

If the calculations involve **multiplying and dividing**, then the measurements are multiplied or divided and the percentage errors are added.

For example:

To calculate the resistance of a resistor the potential difference across the resistor and the current through the resistor is measured ( $V = 4.72 \pm 0.05V$ ,  $I = 0.57 \pm 0.02A$ ).

The resistance is calculated using Ohm's Law:

$$V = 4.72 \pm 0.05V \text{ becomes } 4.72 \pm 1.06\%$$

$$I = 0.57 \pm 0.02A \text{ becomes } 0.57 \pm 3.51\%$$

$$R = \frac{V}{I}$$

$$R = \frac{4.72 \pm 1.06\%}{0.57 \pm 3.51\%}$$

$$R = 8.28070\Omega \pm 4.57\%$$

The percentage error needs to be changed back to an absolute error. The answer is rounded to 2 sig. figs. due to the fact that the current,  $I$ , is expressed to 2 sig. figs. The absolute error is then rounded to the same place value as the answer.

$$R = 8.3 \pm 0.378$$

$$R = 8.3 \pm 0.4\Omega$$



## Activity

- 1) State the number of significant figures in the numbers below.
  - a) 34.8
  - b) 90.12
  - c) 0.00521
  - d) 125 000
  - e) -8.40
  - f) 40.06
  - g) 450mL (0.450L)
  - h)  $6.02 \times 10^{23}$
- 2) Write the numbers below to the number of significant figures indicated.
  - a) 0.002341 to 2 significant figures
  - b) 39.50 to 2 significant figures
  - c) 56 750 to 3 significant figures
  - d) 5.2391 to 4 significant figures
  - e) 755 826 to 3 significant figures
- 3) For the following instrument readings, state (i) the reading to 2 sig. figs, (ii) the absolute error and (iii) the percentage error.
  - a)  $1.44 \pm 0.02$
  - b)  $36.55 \pm 0.05$
  - c)  $3\ 675 \pm 5$
  - d)  $3.6 \pm 0.05$
  - e)  $450 \pm 15$
- 4) Perform the following calculations using the measurements given
  - a)  $(4.55 \pm 0.02) + (2.51 \pm 0.05)$
  - b)  $(26.7 \pm 0.05) - (5.7 \pm 0.05)$
- 5) A steel can is measured for radius and height. The radius is  $2.5 \pm 0.1$  cm and the height is  $12.5 \pm 0.1$  cm
  - a) Calculate the area of the base using  $A = \pi r^2$  where  $\pi = 3.1415926$
  - b) Using the result of a) calculate the volume of the can using  $V = Ah$ .

(Note: In the formula  $C = 2\pi r$  - 2 is a constant, not a measurement, so error analysis doesn't apply to it,  $\pi$  must be expressed to more significant places than the measurements used.
- 6) The stream discharge rate  $Q$  ( $m^3 / \text{sec}$ ) is calculated by the equation  $Q = Av$  where  $A$  is the cross-sectional area of the stream and  $v$  is the velocity of the water.

The measured value of  $A$  is  $4.6 \pm 0.2$  m<sup>2</sup> and flows at  $1.6 \pm 0.2$  m/sec.

Calculate the stream discharge rate.



## Changing between fractions and decimals

1. (a)  $0.08 = \frac{8^{+4}}{100^{+4}} = \frac{2}{25}$
- (b)  $0.6 = \frac{6^{+2}}{10^{+2}} = \frac{3}{5}$
- (c)  $0.14 = \frac{14^{+2}}{100^{+2}} = \frac{7}{50}$
- (d)  $0.64 = \frac{64^{+4}}{100^{+4}} = \frac{16}{25}$
- (e)  $4.25 = 4 \frac{25^{+25}}{100^{+25}} = 4 \frac{1}{4}$
- (f)  $6.2 = 6 \frac{2^{+2}}{10^{+2}} = 6 \frac{1}{5}$
- (g)  $12.5 = 12 \frac{5^{+5}}{10^{+5}} = 12 \frac{1}{2}$
- (h)  $3.125 = 3 \frac{125^{+125}}{1000^{+125}} = 3 \frac{1}{8}$
- (i)  $0.075 = \frac{75^{+25}}{1000^{+25}} = \frac{3}{40}$
2. (a)  $\frac{7}{10} = 0.7$
- (b)  $\frac{4}{5} = \frac{8}{10} = 0.8$
- (c)  $\frac{3}{11} = 0.273$  (to 3 d.p.)  

$$\begin{array}{r} 0.2727 \\ 11 \overline{)3.0^8 0^3 0^8 0} \end{array}$$
- (d)  $\frac{-2}{3} = -0.\dot{6}$   
 or  $-0.6667$  (to 4 d.p.)  

$$\begin{array}{r} 0.6666 \\ 3 \overline{)2.0^2 0^2 0^2 0} \end{array}$$
- (e)  $\frac{-7}{4} = -1.75$   

$$\begin{array}{r} 1.75 \\ 4 \overline{)7.3^0 2^0} \end{array}$$
- (f)  $\frac{13}{9} = 1.\dot{4} = 1.444$  (to 3 d.p.)  

$$\begin{array}{r} 1.444 \\ 9 \overline{)13.4^0 4^0 4^0} \end{array}$$

## Rounding decimals

1. (a) 0.5946 to the nearest hundredth is 0.59
- (b) 0.6076 to the nearest thousandth is 0.608
- (c) 0.756 to the nearest tenth is 0.8
- (d)  $0.\overline{8}$  to the nearest hundredth is 0.89
- (e) 3.1415926 to the nearest hundredth is 3.14
- (f) 5.2007 to the nearest hundredth is 5.20
2. (a) 0.667 to 2 decimal places is 0.67
- (b) 0.7079 to 1 d.p. is 0.7
- (c) 0.7804 to 3 d.p. is 0.780

- (d) 4.2192 to 2 d.p. is 4.22  
 (e)  $\overline{3.27}$  to 3 d.p. is 3.273  
 (f) 55.64 to 1 d.p. is 55.6
3. (a)  $\$0.5792 \approx \$0.58$     (b)  $\$1.6225 \approx \$1.62$     (c)  $\$10.52222 \approx \$10.52$   
 (d)  $\$15.76666 \approx \$15.77$     (e)  $\$78.245 \approx \$78.25$     (f)  $\$122.6565 \approx \$122.66$

## Adding and subtracting decimals

1. (a) 
$$\begin{array}{r} \overset{1}{1} \\ 123.50 \\ 17.92 \\ +14.00 \\ \hline 155.42 \end{array}$$

(b)  $-5.652 + -16.21$

$= -5.652 - 16.21$

The signs in front of both numbers are the same, so add the numbers

$$\begin{array}{r} \overset{1}{5.652} \\ +16.210 \\ \hline 21.862 \end{array}$$

The largest number (16.21) has a – sign in front of it, so the answer has a – sign in front.

Answer is  $-21.862$

(c) 
$$\begin{array}{r} \overset{7}{2} \overset{11}{\cancel{8}} \overset{3}{\cancel{1}} \overset{10}{\cancel{0}} \\ -14.725 \\ \hline 13.415 \end{array}$$

(d)  $-47.292 - +8.67$

$= -47.292 - 8.67$

The signs in front of both numbers are the same, so add the numbers

$$\begin{array}{r} \overset{1}{47.292} \\ + \overset{1}{8.670} \\ \hline 55.962 \end{array}$$

The largest number (47.292) has a – sign in front of it, so the answer has a – sign in front.

Answer is  $-55.962$

(e) 
$$\begin{array}{r} \overset{3}{1} \overset{11}{\cancel{5}} \overset{11}{\cancel{1}} \\ -1.524 \\ \hline 0.017 \end{array}$$
 The metal rod expanded by 0.017m (or 17 mm)

(f) Perimeter =  $3.79 + 1.6 + 4.22 + 1.15$

$$\begin{array}{r} \phantom{0}^1 \phantom{0}^1 \\ 3.79 \\ 1.60 \\ 4.22 \\ +1.15 \\ \hline 10.76 \end{array}$$

It is 10.76m around the shape.

(g) Cost = \$4.55+\$3.75+\$1.35

$$\begin{array}{r} \phantom{0}^1 \phantom{0}^1 \\ 4.55 \\ 3.75 \\ +1.35 \\ \hline 9.65 \end{array}$$

Change will be \$10 - \$9.65

$$\begin{array}{r} \phantom{0}^9 \phantom{0}^9 \\ 10.00 \\ -9.65 \\ \hline 0.35 \end{array}$$

Change is \$0.35 or 35 cents

## Multiplying and dividing decimals

1. (a) Add a zero, move the decimal point 4 places to the right.

$$8.3420 \times 10000 = 83420$$

(b) Add a zero, move the decimal point 3 places to the right.

$$-0.850 \times 1000 = -850$$

(c) Remove the decimal points.

$$\begin{array}{r} \phantom{0}^2 \\ 47 \\ \times 4 \\ \hline 188 \end{array}$$

There are 3 decimal places in answer.  
Answer is 0.188

(d) Remove decimal points and signs.

$$\begin{array}{r} \phantom{0}^2 \\ 45 \\ \times 12 \\ \hline 90 \\ +450 \\ \hline \phantom{0}^1 \\ 540 \end{array}$$

There are 2 decimal places in answer. A negative x a positive → a negative  
Answer is -5.40

(e) Move the decimal point 3 places to the left.

$$363.425 \div 1000 = 0.363425$$

(f) Move the decimal point 2 places to the left.

$$000.4795 \div 100 = 0.004795$$

(g) Multiply both numbers by 10 to become

$$60360 \div 3$$

$$\begin{array}{r} 20120 \\ 3 \overline{)60360} \end{array}$$

Answer is 20 120

(h) Remove signs and multiply both numbers by 10 to become

$$1435 \div 7$$

$$\begin{array}{r} 205 \\ 7 \overline{)1435} \end{array}$$

A negative ÷ a positive → a negative  
Answer is -205

(i) Remove signs and multiply both numbers by 100 to become

$$880190 \div 6$$

$$\begin{array}{r} 146698.\bar{3} \\ 6 \overline{)880190} \end{array}$$

A positive ÷ a negative → a negative.  
Answer is -146698. $\bar{3}$  or -146698.3333 to 4d.p.

2.

$$\begin{array}{r} \overset{1}{4}25 \\ \times 23 \\ \hline 1275 \\ +8500 \\ \hline 9775 \end{array}$$

There are 2 digits after the decimal point, the answer is \$97.75

3. Jan paid \$125 + 12 x \$65.42

$$\begin{array}{r} \overset{1}{6}542 \\ \times 12 \\ \hline 13084 \\ +65420 \\ \hline 78504 \end{array}$$

Answer = \$785.04 + \$125

$$\begin{array}{r} \overset{1}{7}85.04 \\ +125.00 \\ \hline 910.04 \end{array}$$

Jan paid \$910.04

4. John earned \$456.90 for 30 hours work, what was his hourly rate?  
This becomes  $456.90 \div 30$ , first  $\div 10$  and then  $\div 3$

First  $456.90 \div 10 = 45.69$  then  $3 \overline{)45.69} \begin{array}{r} 15.23 \\ \hline \end{array}$  Answer is \$15.23 per hour.

5. Method 1:

Along the length of the room, there is  $5.35 \div 0.3$  tiles, which is 18 tiles (with some waste).

Along the width of the room, there is  $4.75 \div 0.3$  tiles, which is 16 tiles (with some waste).

Number of tiles required is  $18 \times 16 = 288$

Method 2:

Area of 1 tile =  $0.3 \times 0.3 = 0.09 \text{ m}^2$

Area of the floor =  $5.35 \times 4.75 = 25.4215 \text{ m}^2$  (Calc.)

Number of tiles =  $25.4215 \div 0.09 = 282.46 \approx 283$

Working:

$$\begin{array}{r} \overset{1}{1}7.8\bar{3} \\ 3 \overline{)5^2 3.^2 5^1 0} \end{array}$$

$$\begin{array}{r} \overset{1}{1}5.8\bar{3} \\ 3 \overline{)4^1 7.^2 5^1 0} \end{array}$$

$$\begin{array}{r} \overset{4}{1}8 \\ \times 16 \\ \hline 108 \\ +180 \\ \hline 288 \end{array}$$

Which answer is better?

Method 1 takes into account the waste when tiles are cut, so Method 1 is the best answer. In a practical situation a tiler would probably order 300 tiles to allow for breakages and pattern matching.

## Significant figures

1) State the number of significant figures in the numbers below.

- a) 34.8                      3 significant figures  
b) 90.12                    4 significant figures

- c) 0.00521                      3 significant figures
- d) 125 000                      3 significant figures
- e) -8.40                         3 significant figures
- f) 40.06                         4 significant figures
- g) 450mL (0.450L)            3 significant figures
- h)  $6.02 \times 10^{23}$               3 significant figures

2) Write the numbers below to the number of significant figures indicated.

- a) 0.002341 to 2 significant figures                      0.0023
- b) 39.50 to 2 significant figures                              40
- c) 56 750 to 3 significant figures                            56 700
- d) 5.2391 to 4 significant figures                         5.239
- e) 755 826 to 3 significant figures                        756 000

3) For the following instrument readings, state (i) the reading to 2 sig. figs, (ii) the absolute error and (iii) the percentage error.

instrument reading	reading to 2 sig. figs	absolute error	the percentage error
$1.44 \pm 0.02$	1.4	0.02	1.4%
$36.55 \pm 0.05$	37	0.05	0.14%
$3\ 675 \pm 5$	3 700	5	0.14%
$3.6 \pm 0.05$	3.6	0.05	1.4%
$450 \pm 15$	450	15	3.3%

4) Perform the following calculations using the measurements given

a)  $(4.55 \pm 0.02) + (2.51 \pm 0.05) = 7.06 \pm 0.07$

b)  $(26.7 \pm 0.05) - (5.7 \pm 0.05) = 21 \pm 0.1$

5) A steel can is measured for radius and height. The radius is  $2.5 \pm 0.1$  cm and the height is  $12.5 \pm 0.1$  cm

a) Calculate the area of the base using  $A = \pi r^2$  where  $\pi = 3.1415926$

$$A = \pi r^2$$

$$= 3.1415926 \times (2.5 \pm 4\%)^2 \quad \bullet \pm 0.1 = 4\%$$

$$= 19.63 \pm 8\% \quad \bullet \text{error doubled due to squaring}$$

$$= 20 \pm 1.6 \text{ cm}^2 \quad \bullet \text{radius is expressed to 2 sf, answer expressed to 2 sf.}$$

b) Using the result of a) calculate the volume of the can using  $V = Ah$ .

$$V = Ah$$

$$= 20 \pm 1.6 \times 12.5 \pm 0.1 \quad \bullet \text{express each error as a \%}$$

$$= 20 \pm 8\% \times 12.5 \pm 0.8\% \quad \bullet \text{multiply measurements, add \% errors}$$

$$= 250 \pm 8.8\% \quad \bullet \text{express answer to 2 sf, change \% error to error}$$

$$= 250 \pm 22 \text{ cm}^3$$

6) The stream discharge rate  $Q$  ( $\text{m}^3 / \text{sec}$ ) is calculated by the equation  $Q = Av$  where  $A$  is the cross-sectional area of the stream and  $v$  is the velocity of the water.

The measured value of  $A$  is  $4.6 \pm 0.2 \text{ m}^2$  and flows at a velocity of  $1.6 \pm 0.2 \text{ m/sec}$ .

Calculate the stream discharge rate.

$$Q = Av$$

$$= 4.6 \pm 0.2 \times 1.6 \pm 0.2$$

$$= 4.6 \pm 4.3\% \times 1.6 \pm 12.5\%$$

$$= 7.36 \pm 16.8\%$$

$$= 7.4 \pm 1.24 \text{ m}^3 / \text{sec}$$