Introduction to Exponents

When children study mathematics in primary school, they learn that repetitive addition, such as 3 + 3 + 3 + 3 + 3, is the same as the multiplication 5 x 3.

There are occasions when repetitive multiplication occurs. To write this in a simpler way, the concept of exponents was developed. A study of exponents also helps student have a greater understanding of the number system used.

A very large number which fascinates mathematicians is the Googol. The Googol is written as:

10 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000

This is equivalent to $10\times10\times10\times...............\times10\times10 \text{ for } 100 \text{ terms}$.

Using exponents, a short-hand way of writing this is 10^{100}.

While this number is beyond the comprehension of most people, other very large numbers such as the number of atoms in a quantity of material or the distance across the universe are still extremely large and very difficult to write in standard form. The same could be said extremely small numbers such as the diameter of an atom. An understanding of exponents gives us a more convenient way of writing both very large and very small numbers.

The first part of this module gives a thorough understanding of exponents leading to Scientific Notation which is used to represent numbers that are very large or extremely small.

Examples of repetitive multiplication are:

$3\times3\times3\times3\times3$ is written in exponential form as 3^5

$4\times4\times4\times4\times4\times4\times4\times4\times4$ is written in exponential form as 4^9

$x\times x\times x\times x\times x\times x$ is written in exponential form as x^6

$2a\times2a\times2a$ is written in exponential form as $(2a)^3$

It is important that brackets are used in this example to show that both the 2 and the a are being repeated three times.

$^{-}7 \times {}^{-}7 \times {}^{-}7 \times {}^{-}7 \times {}^{-}7$ is written in exponential form as $({}^{-}7)^5$

> It is important that brackets are used in this example to avoid confusion with a similar question ${}^{-}7^5$ (This will be covered below).

The Language of Exponents

The **power** a^n can be written in expanded form as: $a^n = a \times a \times a \times a \times a \ldots\ldots\ldots\ldots \times a$ $\quad[\text{for } n \text{ factors}]$

The **power** a^n consists of a **base** *a* and an **exponent** (or index) *n*.

Exponent (or index)

Base a 'The number of times it is repeated.'

'The number being repeated.'

Saying Powers

3^5 can be said as '3 to the power of 5', '3 to the 5th power', '3 to the 5th' or '3 raised to 5'.
4^9 can be said as '4 to the power of 9', '4 to the 9th power', '4 to the 9th' or '4 raised to 9'.

With common exponents such as 2 and 3, special language is used.

8^2 is most commonly said as 8 squared.
5^3 is most commonly said as 5 cubed.

$(2a)^3$ is said as '*2a* all cubed' to emphasize that both the *2* and the ***a*** are being cubed.

Powers containing numbers can be evaluated.

$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$

Watch out! Many students will perform the multiplication $3 \times 5 = 15$ which is incorrect.

$2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$

$(2.35)^4 = 2.35 \times 2.35 \times 2.35 \times 2.35 = 30.498$ (to 3 decimal places)

$\left(3a\right)^3 = 3a \times 3a \times 3a = 27a^3$

Be careful with negative numbers!

$\left({}^{-}4\right)^6 = {}^{-}4 \times {}^{-}4 \times {}^{-}4 \times {}^{-}4 \times {}^{-}4 \times {}^{-}4 = {}^{+}4096$

When solving ${}^{-}4^6$, use the order of operation rules where the powers are performed before the multiplication.

${}^{-}4^6 = {}^{-}1 \times 4^6 = {}^{-}1 \times 4096 = {}^{-}4096$

$\left({}^{-}2\right)^4 = {}^{-}2 \times {}^{-}2 \times {}^{-}2 \times {}^{-}2 = 16$

${}^{-}2^4 = {}^{-}1 \times 2^4 = {}^{-}1 \times 16 = {}^{-}16$

Powers containing variable bases can be expanded

$x^4 = x \times x \times x \times x$

$ab^3 = a \times b \times b \times b$

$(ab)^4 = ab \times ab \times ab \times ab = a \times a \times a \times a \times b \times b \times b \times b$

One last thing to note: when a variable does not appear to have an exponent, the exponent is 1.

$a = a^1$

$x = x^1$

$de = \left(de\right)^1$ or d^1e^1

Common Powers and Roots

Squares	Square Roots	Cubes	Cube Roots	Other Powers	Other Roots
$1^2 = 1$	$\sqrt{1} = 1$	$1^3 = 1$	$\sqrt[3]{1} = 1$	$1^4 = 1$	$\sqrt[4]{1} = 1$
$2^2 = 4$	$\sqrt{4} = 2$	$2^3 = 8$	$\sqrt[3]{8} = 2$	$2^4 = 16$	$\sqrt[4]{16} = 2$
$3^2 = 9$	$\sqrt{9} = 3$	$3^3 = 27$	$\sqrt[3]{27} = 3$	$3^4 = 81$	$\sqrt[4]{81} = 3$
$4^2 = 16$	$\sqrt{16} = 4$	$4^3 = 64$	$\sqrt[3]{64} = 4$	$4^4 = 256$	$\sqrt[4]{256} = 4$
$5^2 = 25$	$\sqrt{25} = 5$	$5^3 = 125$	$\sqrt[3]{125} = 5$	$5^4 = 625$	$\sqrt[4]{625} = 5$
$6^2 = 36$	$\sqrt{36} = 6$	$6^3 = 216$	$\sqrt[3]{216} = 6$		
$7^2 = 49$	$\sqrt{49} = 7$			$1^5 = 1$	$\sqrt[5]{1} = 1$
$8^2 = 64$	$\sqrt{64} = 8$			$2^5 = 32$	$\sqrt[5]{32} = 2$
$9^2 = 81$	$\sqrt{81} = 9$			$3^5 = 243$	$\sqrt[5]{243} = 3$
$10^2 = 100$	$\sqrt{100} = 10$				

Calculators

Some calculators contain a ^ to calculate powers.

For example: 2.35^4=30.49800625

Other calculators use a w key

For example: 2.35w4=30.49800625

Other calculators use a $\boxed{\wedge}$ key

For example: 2.35 $\boxed{\wedge}$ 4=30.49800625

Module contents

Introduction

- **Exponent Rules – Whole Number Exponents**
- **Exponent Rules – Integer and Fraction Exponents**
- **Equations with Exponents**
- **Scientific Notation**
- **Operations with Numbers in Scientific Notation**

Answers to activity questions

Outcomes

- Name the parts of a number written in exponential form.
- Simplify exponents using rules.
- Solve equations containing exponents.
- Express numbers in scientific notation.
- Perform operations with numbers in scientific notations.

Check your skills

This module covers the following concepts, if you can successfully answer these questions, you do not need to do this module. Check your answers from the answer section at the end of the module.

1. (a) Simplify: $x^6 \times 4x^3 \times x$ (b) Simplify: $a^6 \div a^0$ (c) Simplify: $\left(x^3\right)^2 \times 4x^4 \div \left(2x^2\right)^4$

2. (a) Simplify: $\left(x^{-2}\right)^2 \times \left(x^{\frac{1}{2}}\right)^4 \times x^0$ (b) Simplify: $\sqrt[3]{512}$ (c) Simplify: $\left(27x^9y^6\right)^{\frac{2}{3}}$

3. (a) Solve for x: $x^4 = 81$ (b) Solve for x: $3\sqrt[4]{x} + 1 = 7$ (c) Solve for r: $V = \pi r^3$

4. (a) Write 634 000 000 in Scientific Notation
   (b) Write 9.12×10^{-5} in Standard Notation
   (c) Write 0.000 000 55 in Scientific Notation
   (d) Write 7.25×10^8 in Standard Notation

5. (a) Calculate 3.2×10^8 multiplied by 5.3×10^{-5}
   (b) Calculate 2.8×10^8 divided by 7.2×10^{-12}
   (c) Add 7.25×10^8 and 2.61×10^4

Topic 1: Exponent rules - whole number exponents

The product rule

To help develop the rule, let's consider the question below:

$$
\begin{aligned}
&2^5 \times 2^4 \\
&= \overbrace{(2\times 2\times 2\times 2\times 2)}^{\text{5 factors}} \times \overbrace{(2\times 2\times 2\times 2)}^{\text{4 factors}} \\
&= \overbrace{2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2}^{\text{altogether gives 9 factors}} \\
&= 2^9
\end{aligned}
$$

Generalising this:

$$\boxed{a^m \times a^n = a^{m+n}}$$

(When multiplying two powers with the same base, add the exponents.)

Examples:

$5^4 \times 5^6$ $= 5^{4+6}$ $= 5^{10}$ or 9765625	$(^-3)^2 \times (^-3)^5$ $= (^-3)^{2+5}$ $= (^-3)^7$	$p^5 \times p^3 \times p$ $= p^5 \times p^3 \times p^1$ $= p^{5+3+1}$ $= p^9$
$2^5 \times 3^4$ cannot be done the bases are not the same	$3x^4 \times 8x^5$ $= 3\times 8\times x^{4+5}$ $= 24x^9$	$c^4 \times c^b$ $= c^{4+b}$
$2^7 + 2^2$ cannot be done the rule is for products	$(2b)^4 \times (2b)^8$ $= (2b)^{4+8}$ $= (2b)^{12}$	$^-2ab^3 \times 4a^5b^3$ $= {}^-2\times 4\times a^1 \times a^5 \times b^3 \times b^3$ $= {}^-8a^6b^6$

The quotient rule

To help develop the rule, let's consider the question below:

$$
\begin{aligned}
&2^7 \div 2^4 \\
&= \frac{2^7}{2^4} \quad \leftarrow \text{write in fraction form} \\
&= \frac{2 \times 2 \times 2 \times \cancel{2}^1 \times \cancel{2}^1 \times \cancel{2}^1 \times \cancel{2}^1}{\cancel{2}^1 \times \cancel{2}^1 \times \cancel{2}^1 \times \cancel{2}^1} \quad \leftarrow \text{simplify factors} \\
&= 2^3
\end{aligned}
$$

Generalising this:

$$\boxed{a^m \div a^n = a^{m-n}}$$

(When dividing two powers with the same base, subtract the exponents.)

Examples:

$\begin{aligned}&5^8 \div 5^6 \\ &= \frac{5^8}{5^6} \\ &= 5^{8-6} \\ &= 5^2 \quad \text{or} \quad 25\end{aligned}$	$\begin{aligned}&\left({}^-3\right)^9 \div \left({}^-3\right)^5 \\ &= \frac{\left({}^-3\right)^9}{\left({}^-3\right)^5} \\ &= \left({}^-3\right)^{9-5} \\ &= \left({}^-3\right)^4 \quad \text{or} \quad 81\end{aligned}$	$\begin{aligned}&p^{15} \div p^3 \\ &= \frac{p^{15}}{p^3} \\ &= p^{15-3} \\ &= p^{12}\end{aligned}$
$\begin{gathered}2^5 \div 3^4 \\ \text{cannot be done} \\ \text{the bases are not the same}\end{gathered}$	$\begin{aligned}&(2b)^{10} \div (2b)^5 \\ &= (2b)^{10-5} \\ &= (2b)^5\end{aligned}$	$\begin{aligned}&c^4 \div c^b \\ &= c^{4-b}\end{aligned}$
$\begin{gathered}2^7 - 2^2 \\ \text{cannot be done} \\ \text{the rule is for quotients}\end{gathered}$	$\begin{aligned}&4x^8 \div 8x^5 \\ &= \frac{{}^1\cancel{4}x^8}{{}^2\cancel{8}x^5} \\ &= \frac{x^{8-5}}{2} \\ &= \frac{x^3}{2}\end{aligned}$	$\begin{aligned}&12a^8b^3 \div 4a^5b^3 \\ &= \frac{{}^3\cancel{12}a^8b^3}{{}^1\cancel{4}a^5b^3} \\ &= \frac{3a^{8-5}b^{3-3}}{1} \\ &= 3a^3b^0 \\ &\text{As } b^0 = 1 \text{ (see next section)} \\ &= 3a^3\end{aligned}$

Zero exponent rule

To help develop the rule, let's consider the question below where the solution is obtained by exponents and by evaluating the exponents:

$$\begin{aligned}&2^5 \div 2^5\\&=\frac{2^5}{2^5}\\&=2^{5-5}\\&=2^0\end{aligned}$$

or alternatively

$$\begin{aligned}&2^5 \div 2^5\\&=\frac{2^5}{2^5}\\&=\frac{32}{32}\\&=1\end{aligned}$$

means that $2^0 = 1$

Generalising this for many different examples gives:

$$\boxed{a^0 = 1}$$

(A power with a zero exponent is equal to 1.)

Examples:

$5^0 = 1$	$\left({}^{-}3\right)^0 = 1$	$p^5 \times p^0$ $= p^5 \times 1$ $= p^5$
$(5a)^0 = 1$	$5a^0$ $= 5 \times 1$ $= 5$	$\left(4a^2c^8\right)^0$ $= 1$
$16x^6y^3 \div 4x^6y$ $= \dfrac{{}^{4}\cancel{16}x^6y^3}{{}^{1}\cancel{4}x^6y^1}$ $= 4x^{6-6}y^{3-1}$ $= 4x^0y^2$ $= 4y^2$	$1\,234\,567^0 = 1$	$4ab^3 \div 4ab^3$ $= \dfrac{\cancel{4}^1a^1b^3}{\cancel{4}^1a^1b^3}$ $= a^{1-1}b^{3-3}$ $= a^0b^0$ $= 1$

The power rule

To help develop the rule, let's consider the question below:

$$\left(2^5\right)^3$$
$$= \overbrace{(2\times2\times2\times2\times2)}^{5 \text{ factors}} \times \overbrace{(2\times2\times2\times2\times2)}^{5 \text{ factors}} \times \overbrace{(2\times2\times2\times2\times2)}^{5 \text{ factors}}$$
$$= \overbrace{2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2}^{\text{altogether gives } 3\times5=15 \text{ factors}}$$
$$= 2^{15}$$

Generalising this:

$$\boxed{\left(a^m\right)^n = a^{m\times n}}$$

(When a power is raised to a power, multiply the exponents.)

Examples:

$\left(3^4\right)^3$ $= 3^{4\times3}$ $= 3^{12}$ or 531441	$\left[\left({}^-3\right)^2\right]^5$ $= \left({}^-3\right)^{2\times5}$ $= \left({}^-3\right)^{10}$ or 59049	$\left(p^5\right)^3$ $= p^{5\times3}$ $= p^{15}$
$\left(x^0\right)^4$ $= x^{0\times4}$ $= x^0$ $= 1$ or $\left(x^0\right)^4$ $= 1^4$ $= 1$	$\left[\left(2x\right)^4\right]^4$ $= \left(2x\right)^{4\times4}$ $= \left(2x\right)^{16}$	$\left(c^4\right)^b$ $= c^{4\times b}$ $= c^{4b}$
$\left(x^6\right)^{\frac{1}{3}}$ $= x^{6\times\frac{1}{3}}$ $= x^2$	$\left[\left(b^2\right)^4\right]^a$ $= b^{2\times4\times a}$ $= b^{8a}$	$\left(c^{-4}\right)^{-5}$ $= c^{-4\times-5}$ $= c^{20}$

Two more useful power rules are

$(ab)^m = a^m b^m$ or $a^m b^m = (ab)^m$

$\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$ or $\dfrac{a^m}{b^m} = \left(\dfrac{a}{b}\right)^m$

Examples:

$\begin{aligned}&(3x)^3\\&=3^3x^3\\&=27x^3\end{aligned}$	$\begin{aligned}&4(xy)^2\\&=4x^2y^2\end{aligned}$	$\begin{aligned}&\left(4p^5\right)^3\\&=4^3p^{5\times3}\\&=64p^{15}\end{aligned}$
$\begin{aligned}&\left(\frac{5}{a}\right)^4\\&=\frac{5^4}{a^4}\\&=\frac{625}{a^4}\end{aligned}$	$\begin{aligned}&\left(\frac{a}{b}\right)^3\\&=\frac{a^3}{b^3}\end{aligned}$	$\begin{aligned}&\left(\frac{2x^2}{y^3}\right)^4\\&=\frac{2^4x^{2\times4}}{y^{3\times4}}\\&=\frac{16x^8}{y^{12}}\end{aligned}$
$\begin{aligned}&5\left(2x^6y^2\right)^2\\&=5\times2^2x^{6\times2}y^{2\times2}\\&=5\times4x^{12}y^4\\&=20x^{12}y^4\end{aligned}$	$\begin{aligned}&\left(\frac{3a^4}{b^3}\right)^2\times\frac{4a^3}{b}\\&=\frac{3^2a^{4\times2}}{b^{3\times2}}\times\frac{4a^3}{b}\\&=\frac{9a^8\times4a^3}{b^6\times b^1}\\&=\frac{36a^{8+3}}{b^{6+1}}\\&=\frac{36a^{11}}{b^7}\end{aligned}$	$\begin{aligned}&\left(3x^2y^3\right)^2\times4\left(xy^4\right)^3\\&=3^2x^{2\times2}y^{3\times2}\times4x^3y^{4\times3}\\&=9x^4y^6\times4x^3y^{12}\\&=36x^{4+3}y^{6+12}\\&=36x^7y^{18}\end{aligned}$

Video ‘Exponent Rules – Whole Number Exponents’

Activity

1. Evaluate the following powers.

(a)	3^5	(b)	5^3	(c)	7^3
(d)	2^9	(e)	14^3	(f)	5^5
(g)	3^0	(h)	$\left(2^7\right)^0$	(i)	$\left(3^2\right)^3$
(j)	$2^4 \times 2^6$	(k)	$4^7 \div 4^5$	(l)	-4^3
(m)	$(-5)^3$	(n)	0.85^6	(o)	$4.5^{1.25}$
(p)	$(1.006)^{240}$	(q)	$(1.05)^{20}$		

2. Simplify the following (leave your answer in power form).

(a)	$5^2 \times 5^5$	(b)	$m^4 \times m^5$	(c)	$a^4 \times a^3 \times a$
(d)	$10^4 \times 10 \times 10^0$	(e)	$\frac{r^4}{2} \times \frac{r}{4}$	(f)	$5c^2 \times 3c^4$
(g)	$a^2b^4 \times a^3bc$	(h)	$4x^3y^4z^2 \times 5x^2y$	(i)	$5^2 e f^3 g^5 \times 5^3 e^2 f^2 g^0$
(j)	$4^9 \div 4^5$	(k)	$x^8 \div x^5$	(l)	$h^5 \div h$
(m)	$3^x \div 3^4$	(n)	$\frac{16a^5}{4a^2}$	(o)	$\frac{5a^2b^4}{25a^2b}$
(p)	$a^{\frac{1}{2}} \div a^{\frac{1}{4}}$	(q)	$\left(2^4\right)^3$	(r)	$\left(5^4\right)^{\frac{1}{2}}$
(s)	$\left(b^4\right)^5$	(t)	$(3t)^4$	(u)	$\left(2a^4b^3\right)^3$
(v)	$\left(\frac{4}{y}\right)^3$	(w)	$\frac{\left(2y^4\right)^2}{\left(2x^2\right)^4}$	(x)	$\left(2^5\right)^b$

3. Simplify the following.

(a)	$\left(a^2\right)^4 \times a^3$	(b)	$\frac{\left(3y^4\right)^2}{2y^5}$	(c)	$\frac{2x^5 \times x^4}{\left(x^2\right)^3}$
(d)	$3b^0 + (3b)^0$	(e)	$\frac{(3n)^2 \times 3n^2}{9n^4}$	(f)	$5(mn)^2 \times \left(mn^2\right)^3$
(g)	$m^2n^5p^7 \times m^3p^4 \times n^2p$	(h)	$-4(a^2b^5)^0$	(i)	$2a^2b^3 \times 3ab^4$
(j)	$\frac{2x^5y^4}{10x^2y^3}$	(k)	$\left(\frac{2x^3}{y^4}\right)^4 \times \left(\frac{y}{x}\right)^2$	(l)	$4a^0 - \left(\frac{a}{4}\right)^0$
(m)	$\frac{25p^6}{36q^8} = \left(\frac{\quad}{\quad}\right)^2$	(n)	$27a^6b^9c^3 = (\quad)$	(o)	$\frac{\left(3xy^2\right)^3}{2x(2y)^2} \div \frac{6xy^0}{4\left(xy^2\right)^3}$

Topic 2: Exponent rules – integers and fraction exponents

Negative exponents

To help develop the rule, let's consider the question below:

$\frac{4}{32}$ - both the 4 and the 32 are powers of 2.

This question can be solved two ways: by normal fraction methods and using exponents. Because the starting question is the same, the answers must also be the same.

$$\frac{\cancel{4}^1}{\cancel{32}^8} \xleftrightarrow{\text{Question the same}} \frac{4}{32}$$

$$=\frac{1}{8} \qquad\qquad =\frac{2^2}{2^5}$$

$$=\frac{1}{2^3} \xleftrightarrow{\text{Answers must be the same}} =2^{2-5} =2^{-3}$$

This means that: $\frac{1}{2^3}=2^{-3}$

Generalising this:

$$\boxed{a^{-m}=\frac{1}{a^m} \quad \text{or} \quad \frac{1}{a^{-m}}=a^m}$$

One good way to remember this rule is to say

'Take the reciprocal and change the sign of the exponent'

Note:

There is a convention in mathematics that answers to questions should be expressed with positive exponents only.

When solving a question, use the rules in the previous section first (if possible), if an answer still contains a negative exponent then use the rule above to express with positive exponents.

Examples:

3^{-4} $=\dfrac{3^{-4}}{1}$ $=\dfrac{1}{3^4}$ or $\dfrac{1}{81}$	$\dfrac{1}{5^{-3}}$ $=\dfrac{5^3}{1}$ $=5^3$ or 125	$\left(\dfrac{2}{7}\right)^{-2}$ $=\left(\dfrac{7}{2}\right)^2$ $=\dfrac{49}{4}$
$\dfrac{2^{-3}}{3^{-2}}$ $=\dfrac{3^2}{2^3}$ $=\dfrac{9}{8}$ or $1\dfrac{1}{8}$	$4^{-1}e^2f^{-3}$ $=\dfrac{1}{4^1}e^2\dfrac{1}{f^3}$ $=\dfrac{e^2}{4f^3}$	$\left(\dfrac{a^3}{4}\right)^{-2}$ $=\left(\dfrac{4}{a^3}\right)^2$ $=\dfrac{4^2}{a^6}$
$2\left(p^{-3}\right)^2q^4\times\left(4p^2q^{-3}\right)^{-4}$ $=2p^{-3\times2}q^4\times\left(2^2\right)^{-4}p^{2\times-4}q^{-3\times-4}$ $=2^1p^{-6}q^4\times2^{-8}p^{-8}q^{12}$ $=2^{1+{}^{-}8}p^{-6+{}^{-}8}q^{4+12}$ $=2^{-7}p^{-14}q^{16}$ $=\dfrac{q^{16}}{2^7p^{14}}$ or $\dfrac{q^{16}}{128p^{14}}$	$\dfrac{2x^{-1}y^3}{3x^3y}\div\dfrac{xy^{-2}}{x^{-2}y}$ $=\dfrac{2x^{-1}y^3}{3x^3y^1}\times\dfrac{x^{-2}y^1}{x^1y^{-2}}$ $=\dfrac{2x^{-1+{}^{-}2}y^{3+1}}{3x^{3+1}y^{1+{}^{-}2}}$ $=\dfrac{2x^{-3}y^4}{3x^4y^{-1}}$ $=\dfrac{2}{3}x^{-3-4}y^{4-{}^{-}1}$ $=\dfrac{2}{3}x^{-7}y^5$ $=\dfrac{2y^5}{3x^7}$	$\dfrac{\left(x^2y\right)^{-2}}{x^{-4}y}$ $=\dfrac{x^{2\times-2}y^{1\times-2}}{x^{-4}y}$ $=\dfrac{x^{-4}y^{-2}}{x^{-4}y^1}$ $=x^0y^{-2-1}$ $=y^{-3}$ $=\dfrac{1}{y^3}$

Fractional exponents

To help develop the rule, let's consider the information below:

We know using exponents that: $x^{\frac{1}{3}} \times x^{\frac{1}{3}} \times x^{\frac{1}{3}} = x^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = x^1 = x$

From the ideas about roots of numbers: $\sqrt[3]{x} \times \sqrt[3]{x} \times \sqrt[3]{x} = x$

That means that $x^{\frac{1}{3}} = \sqrt[3]{x}$

Generalising this:

$$\boxed{a^{\frac{1}{n}} = \sqrt[n]{a}}$$

(This is for unit fractions)

Before proceeding with some examples, let's have a look at the meaning of different roots.

Square root: the square root of a number is usually written as $\sqrt{n}$ but more correctly should be written as $\sqrt[2]{n}$, the 2 indicating a relationship to squaring.

Square root of a number: for example $\sqrt{36} = 6$, because $6 \times 6 = 6^2 = 36$.

Cube root of a number: for example $\sqrt[3]{125} = 5$, because $5 \times 5 \times 5 = 5^3 = 125$.

Forth root of a number: for example $\sqrt[4]{16} = 2$, because $2 \times 2 \times 2 \times 2 = 2^4 = 16$.

Fifth root of a number: for example $\sqrt[5]{243} = 3$, because $3 \times 3 \times 3 \times 3 \times 3 = 3^5 = 243$.

etc.

Using Calculators

Calculators vary in the way roots are calculated.

Most calculators find square roots by: [√■] [2] [5] [=] 5

Most calculators find cube roots by: ($\sqrt[3]{■}$) [8] [=] 2

To find any root, most calculators use [$\sqrt[x]{\ }$]

To find $\sqrt[4]{625}$, do [4] ($\sqrt[■]{□}$) [6] [2] [5] [=] 5

Another option is to use the 'raising to an exponent' key ([$x^■$] or w) using the fraction or changing the fraction to a decimal: $\sqrt[4]{625} = (625)^{\frac{1}{4}} = (625)^{0.25}$, do [6] [2] [5] [$x^■$] [0] [•] [2] [5] [=] 5 or

[6] [2] [5] [$x^■$] [$\frac{■}{□}$] [1]  [4] [=] 5

Examples:

$25^{\frac{1}{2}}$ $=\sqrt{25}$ $=5$	$256^{\frac{1}{4}}$ $=\sqrt[4]{256}$ $=4$	$256^{\frac{1}{4}}$ $=\left(2^8\right)^{\frac{1}{4}}$ $=2^{8\times\frac{1}{4}}$ $=2^2$ or 4
$(20)^{\frac{1}{4}}$ $=2.115$ (to 3 d.p.) By calculator	$\left(p^6\right)^{\frac{1}{3}}$ $=p^{6\times\frac{1}{3}}$ $=p^2$	$\left(1.515\overline{3}\right)^{\frac{1}{52}}$ $=\sqrt[52]{1.515\overline{3}}$ $=1.008$ to 3 d.p. By calculator
$\left(\frac{9}{16}\right)^{\frac{1}{2}}$ $=\sqrt{\frac{9}{16}}$ $=\frac{\sqrt{9}}{\sqrt{16}}$ $=\frac{3}{4}$	$\left(9x^4\right)^{\frac{1}{2}}$ $=9^{\frac{1}{2}}x^{4\times\frac{1}{2}}$ $=\sqrt{9}x^2$ $=3x^2$	$\sqrt{64x^{64}}$ $=\sqrt{64}\sqrt{x^{64}}$ $=8x^{64\times\frac{1}{2}}$ $=8x^{32}$
$100^{\frac{-1}{2}}$ $=\frac{1}{100^{\frac{1}{2}}}$ $=\frac{1}{\sqrt{100}}$ $=\frac{1}{10}$	$\left(\frac{16x^4}{81y^8}\right)^{\frac{1}{4}}$ $=\left(\frac{2^4x^4}{3^4y^8}\right)^{\frac{1}{4}}$ $=\frac{2^{4\times\frac{1}{4}}x^{4\times\frac{1}{4}}}{3^{4\times\frac{1}{4}}y^{8\times\frac{1}{4}}}$ $=\frac{2^1x^1}{3^1y^2}$ $=\frac{2x}{3y^2}$	$\left(\frac{16x^4}{81y^8}\right)^{\frac{1}{4}}$ $=\frac{\sqrt[4]{16}x^{4\times\frac{1}{4}}}{\sqrt[4]{81}y^{8\times\frac{1}{4}}}$ $=\frac{2x^1}{3y^2}$ $=\frac{2x}{3y^2}$

If the exponent is a non-unit fraction, the power of a power rule can assist.

$$a^{\frac{2}{3}}=\left(a^2\right)^{\frac{1}{3}}=\sqrt[3]{a^2}$$

Generalising this:

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \text{or} \quad \left(\sqrt[n]{a}\right)^m$$

Examples:

Evaluate $8^{\frac{2}{3}}$

This question can be done in three ways.

In the first method, 8 is written in power form as 2^3 and the power of a power rule is used.

$$8^{\frac{2}{3}}$$
$$= \left(2^3\right)^{\frac{2}{3}}$$
$$= 2^{\cancel{3}^1 \times \frac{2}{\cancel{3}_1}}$$
$$= 2^2$$
$$= 4$$

The second method uses a calculator with a fraction exponent.

$$8^{\frac{2}{3}}$$
$$= \left(8^2\right)^{\frac{1}{3}}$$
$$= \sqrt[3]{8^2}$$

($\sqrt[3]{\blacksquare}$) $\boxed{8}$ $\boxed{x^2}$ $\boxed{=}$ 4

The third method involves changing the fraction exponent to a decimal.

$$8^{\frac{2}{3}}$$
$$= 8^{0.666666666666}$$
$$= 4$$

$\boxed{8}$ $\boxed{x^{\blacksquare}}$ $\boxed{\frac{\blacksquare}{\square}}$ $\boxed{2}$ REPLAY $\boxed{3}$ $\boxed{=}$ 4

$\boxed{8}$ $\boxed{x^{\blacksquare}}$ $\boxed{\cdot}$ $\boxed{6}$ $\boxed{6}$ $\boxed{6}$ $\boxed{6}$ $\boxed{6}$ $\boxed{6}$ $\boxed{=}$ 4

It is important to fill the display of your calculator with the recurring digit. Be careful with calculating with recurring decimals.

Evaluate $243^{\frac{3}{5}}$

$$243^{\frac{3}{5}}$$
$$= \left(3^5\right)^{\frac{3}{5}}$$
$$= 3^{\cancel{5}^1 \times \frac{3}{\cancel{5}_1}}$$
$$= 3^3$$
$$= 27$$

$$243^{\frac{3}{5}}$$
$$= \left(243^3\right)^{\frac{1}{5}}$$
$$= \sqrt[5]{243^3}$$

$\boxed{5}$ ($\sqrt[\blacksquare]{\square}$) $\boxed{2}$ $\boxed{4}$ $\boxed{3}$ $\boxed{x^3}$ $\boxed{=}$ 27

$$243^{\frac{3}{5}}$$
$$= 243^{0.6}$$
$$= 27$$

$\boxed{2}$ $\boxed{4}$ $\boxed{3}$ $\boxed{x^{\blacksquare}}$ $\boxed{0}$ $\boxed{\cdot}$ $\boxed{6}$ $\boxed{=}$ 27

or

$\boxed{2}$ $\boxed{4}$ $\boxed{3}$ w $\boxed{0}$ $\boxed{\cdot}$ $\boxed{6}$ $\boxed{=}$ 27

Evaluate $50^{\frac{2}{5}}$

$$50^{\frac{2}{5}}$$

As 50 cannot be written in power form, only a calculator solution is possible.

$\boxed{5}$ $\boxed{0}$ $\boxed{x^{\blacksquare}}$ $\boxed{\frac{\blacksquare}{\square}}$ $\boxed{2}$ REPLAY $\boxed{5}$ $\boxed{=}$

$$50^{\frac{2}{5}}$$
$$= \sqrt[5]{50^2}$$

$\boxed{5}$ ($\sqrt[\blacksquare]{\square}$) $\boxed{5}$ $\boxed{0}$ $\boxed{x^2}$ $\boxed{=}$

4.781762499

$$50^{\frac{2}{5}}$$
$$= 50^{0.4}$$

$\boxed{5}$ $\boxed{0}$ $\boxed{x^{\blacksquare}}$ $\boxed{0}$ $\boxed{\cdot}$ $\boxed{4}$ $\boxed{=}$

4.781762499

4.781762499

Evaluate $128^{-\frac{4}{7}}$

$$
\begin{aligned}
&128^{-\frac{4}{7}} \\
&= \frac{1}{128^{\frac{4}{7}}} \\
&= \frac{1}{\left(2^7\right)^{\frac{4}{7}}} \\
&= \frac{1}{2^{\cancel{7}^1 \times \frac{4}{\cancel{7}^1}}} \\
&= \frac{1}{2^4} \\
&= \frac{1}{16} \text{ or } 0.0625
\end{aligned}
$$

$$
\begin{aligned}
&128^{-\frac{4}{7}} \\
&= \sqrt[7]{128^{-4}}
\end{aligned}
$$

[7] (■√□) [1] [2] [8] [x■] [(–)] [4] [=] 0.0625

$$
\begin{aligned}
&128^{-\frac{4}{7}} \\
&= 128^{-0.571428571} \\
&= 0.0625
\end{aligned}
$$

[1] [2] [8] [x■] [(–)] [0] [•] [5] [7] [1] [4] [2] [8] [5] [7] [1] [=] 0.0625

or

[1] [2] [8] [x■] [▭/▭] [(–)] [4] [7] [=] 0.0625

Simplify $\left(625a^4b^8\right)^{\frac{3}{4}}$

$$
\begin{aligned}
&\left(625a^4b^8\right)^{\frac{3}{4}} \\
&= 5^{\cancel{4}^1 \times \frac{3}{\cancel{4}^1}} a^{\cancel{4}^1 \times \frac{3}{\cancel{4}^1}} b^{\cancel{8}^2 \times \frac{3}{\cancel{4}^1}} \\
&= 5^3 a^3 b^6 \\
&= 125a^3b^6
\end{aligned}
$$

$$
\begin{aligned}
&\left(625a^4b^8\right)^{\frac{3}{4}} \\
&= \sqrt[4]{625^3}\, a^{\cancel{4}^1 \times \frac{3}{\cancel{4}^1}} b^{\cancel{8}^2 \times \frac{3}{\cancel{4}^1}} \\
&= 125a^3b^6
\end{aligned}
$$

[4] (■√□) [6] [2] [5] [x■] [3] [=] 125

$$
\begin{aligned}
&\left(625a^4b^8\right)^{\frac{3}{4}} \\
&= 625^{0.75} a^{\cancel{4}^1 \times \frac{3}{\cancel{4}^1}} b^{\cancel{8}^2 \times \frac{3}{\cancel{4}^1}} \\
&= 125a^3b^6
\end{aligned}
$$

[6] [2] [5] [x■] [0] [•] [7] [5] [=] 125

Simplify $\sqrt[3]{64a^3b^5}$

$$
\begin{aligned}
&\sqrt[3]{64a^3b^5} \\
&= \left(64a^3b^5\right)^{\frac{1}{3}} \\
&= 4^{\cancel{3}^1 \times \frac{1}{\cancel{3}^1}} a^{\cancel{3}^1 \times \frac{1}{\cancel{3}^1}} b^{5 \times \frac{1}{3}} \\
&= 4ab^{\frac{5}{3}}
\end{aligned}
$$

$$
\begin{aligned}
&\sqrt[3]{64a^3b^5} \\
&= \sqrt[3]{64}\left(a^3b^5\right)^{\frac{1}{3}} \\
&= 4a^{\cancel{3}^1 \times \frac{1}{\cancel{3}^1}} b^{5 \times \frac{1}{3}} \\
&= 4ab^{\frac{5}{3}}
\end{aligned}
$$

[3] (³√■) [6] [4] [=] 4

$$
\begin{aligned}
&\sqrt[3]{64a^3b^5} \\
&= \left(64a^3b^5\right)^{\frac{1}{3}} \\
&= 64^{0.33\bar{3}} a^{\cancel{3}^1 \times \frac{1}{\cancel{3}^1}} b^{5 \times \frac{1}{3}} \\
&= 4ab^{\frac{5}{3}}
\end{aligned}
$$

[6] [4] [x■] [0] [•] [3] [3] [3] [3] [3] [3] [3] [3] [3] [=] 4

Simplify $\left(\dfrac{625a^6}{b^8}\right)^{-\frac{3}{4}}$

$$\left(\frac{b^8}{625a^6}\right)^{\frac{3}{4}}$$
$$=\left(\frac{b^8}{5^4a^6}\right)^{\frac{3}{4}}$$
$$=\frac{b^{\cancel{8}^2\times\frac{3}{\cancel{4}^1}}}{5^{\cancel{4}^1\times\frac{3}{\cancel{4}^1}}\,a^{\cancel{6}^3\times\frac{3}{\cancel{4}^2}}}$$
$$=\frac{b^6}{5^3a^{\frac{9}{2}}}\ \text{or}\ \frac{b^6}{125a^{\frac{9}{2}}}$$

$$\left(\frac{b^8}{625a^6}\right)^{\frac{3}{4}}$$
$$=\left(\frac{b^8}{625^{\frac{3}{4}}a^6}\right)^{\frac{3}{4}}$$
$$=\frac{b^{\cancel{8}^2\times\frac{3}{\cancel{4}^1}}}{\sqrt[4]{625^3}\,a^{\cancel{6}^3\times\frac{3}{\cancel{4}^2}}}$$
$$=\frac{b^6}{125a^{\frac{9}{2}}}$$

$$\left(\frac{b^8}{625a^6}\right)^{\frac{3}{4}}$$
$$=\left(\frac{b^8}{625^{\frac{3}{4}}a^6}\right)^{\frac{3}{4}}$$
$$=\frac{b^{\cancel{8}^2\times\frac{3}{\cancel{4}^1}}}{625^{0.75}\,a^{\cancel{6}^3\times\frac{3}{\cancel{4}^2}}}$$
$$=\frac{b^6}{125a^{\frac{9}{2}}}$$

[6] [2] [5] [$x^{■}$] [$\frac{■}{□}$] [3] [4] [=] 125

[4] ($^{■}\sqrt{□}$) [6] [2] [5] [$x^{■}$] [3] [=] 125

[6] [2] [5] [$x^{■}$] [0] [•] [7] [5] [=] 125

Video ‘Exponent Rules – Integer & Fraction Exponents’

Activity

1. Evaluate

(a) 36^{-1} (b) 5^{-2} (c) 10^{-4}

(d) $\dfrac{1}{4^{-3}}$ (e) $\left(\dfrac{1}{4}\right)^{-4}$ (f) $\dfrac{5^{-2}}{2^{3}}$

(g) 7×2^{-4} (h) $\left(3\times2^{-2}\right)^{-2}$

2. Simplify

(a) a^{-4} (b) c^{-7} (c) $\dfrac{1}{x^{-2}}$

(d) $2x^{-3}$ (e) $(3b)^{-2}$ (f) ab^{-1}

(g) $a^{3}\times b^{-4}$ (h) $4p^{-1}$ (i) $(4p)^{-1}$

(j) $\left(3^{-1}ab^{-2}\right)^{3}$ (k) $4p^{2}q^{-3}\times3p^{-1}q^{2}$ (l) $\left(\dfrac{4}{b^{2}}\right)^{-3}$

(m) $\dfrac{\left(xy^{3}\right)^{-4}}{4x^{4}y^{-2}}$ (n) $\dfrac{2\left(x^{2}y^{-1}\right)^{-3}}{\left(3x^{-1}y^{3}\right)^{2}}$ (o) $\dfrac{2x^{2}}{3y^{-3}}\times\left(\dfrac{4xy^{2}}{9x^{-4}}\right)^{-1}$

3. Evaluate without using a calculator

(a) $49^{\frac{1}{2}}$ (b) $8^{\frac{2}{3}}$ (c) $32^{\frac{3}{5}}$

(d) $625^{\frac{1}{4}}$ (e) $1028^{\frac{7}{10}}$ (f) $256^{-\frac{1}{4}}$

(g) $16^{-\frac{1}{2}}$ (h) $1028^{-\frac{7}{10}}$ (i) $\dfrac{3}{27^{-\frac{2}{3}}}$

(j) $7\times128^{\frac{3}{7}}$ (k) $\dfrac{16^{\frac{3}{4}}}{27^{\frac{1}{3}}}$ (l) $36^{\frac{1}{2}}\times32^{\frac{3}{5}}$

4. Use a calculator to answer the following

(a) $10^{\frac{3}{4}}$ (b) $450^{\frac{1}{7}}$ (c) $1.864297^{\frac{1}{10}}$

(d) $2.1209527^{-\frac{1}{240}}$ (e) $4.95^{2.5}$ (f) $1.01^{-0.75}$

5. Simplify

(a) $\left(\dfrac{8}{a^{6}}\right)^{\frac{1}{3}}$ (b) $\dfrac{4\left(a^{6}b^{3}\right)^{\frac{1}{3}}}{\left(16a^{8}b^{4}\right)^{\frac{1}{4}}}$ (c) $\left(\dfrac{a^{2}b^{6}}{16}\right)^{-\frac{1}{2}}$

(d) $\sqrt[3]{64a^{2}b^{4}}$ (e) $\sqrt[4]{32\left(a^{3}b^{2}\right)^{2}}$ (f) $\dfrac{\sqrt{x^{2}y^{6}}}{\sqrt[3]{x^{6}y^{3}}}$

Topic 3: Equations with exponents

Before starting to solve equations containing exponents, it is important to think about opposite operations just like in the Equations Topic.

You will recall the opposites below:

Operation	Opposite Operation
Addition +	Subtraction -
Subtraction -	Addition +
Multiplication x	Division ÷
Division ÷	Multiplication x
Squaring n^2	Square root $\sqrt{n}$
Square root $\sqrt{n}$	Squaring n^2

Revision example:

$2x^2+3=11$ $2x^2+3-3=11-3 \rightarrow \text{take 3 from both sides}$ $2x^2=8$ $\frac{2x^2}{2}=\frac{8}{2} \rightarrow \text{divide both sides by 2}$ $x^2=4$ $\sqrt{x^2}=\pm\sqrt{4} \rightarrow \text{take square root of both sides}$ $x=\pm 2$	$2x^2+3=11$ $2x^2=11-3 \rightarrow$ +3 becomes -3 on the RHS $2x^2=8$ $x^2=\frac{8}{2} \rightarrow \times 2$ becomes $\div 2$ on the RHS $x^2=4$ $x=\pm\sqrt{4} \rightarrow {}^2$ becomes $\sqrt{\ }$ on the RHS $x=\pm 2$

There are two solutions here because both 2 and -2 satisfy the equation. This always occurs with even roots.

This list can be expanded to cubes, cube roots, to the 4, forth root, etc.

Operation	**Opposite Operation**
Cubing $(\ )^3$	Cube root ($\sqrt[3]{\ }$) or $\frac{1}{3}$
To the 4th power	Forth root ($\sqrt[4]{\ }$) or $\frac{1}{4}$
To the 5th power	Fifth root ($\sqrt[5]{\ }$) or $\frac{1}{5}$
To the 6th power	Sixth root ($\sqrt[6]{\ }$) or $\frac{1}{6}$
To the 7th power	Seventh root ($\sqrt[7]{\ }$) or $\frac{1}{7}$
etc.	etc.
To the power $\frac{2}{3}$	To the power $\frac{3}{2}$
To the power $\frac{3}{5}$	To the power $\frac{5}{3}$

Examples: Solve for x

Both sides method	**Short -cut Method**
$x^4 = 256$ $\sqrt[4]{x^4} = \sqrt[4]{256} \rightarrow$ take the fourth root of both sides $x = \pm 4$ There are two solutions here because both 4 and -4 satisfy the equation. This always occurs with even roots	$x^4 = 256$ $x = \sqrt[4]{256} \rightarrow$ ^{4} becomes $\sqrt[4]{\ }$ on the RHS $x = \pm 4$
$\sqrt[6]{x} = 3$ $\left(\sqrt[6]{x}\right)^6 = 3^6$ raise both sides to the power of 6 $x = 729$	$\sqrt[6]{x} = 3$ $x = 3^6 \rightarrow \sqrt[6]{\ }$ becomes ^{6} on the RHS $x = 729$
$3\sqrt[5]{x} - 1 = 5$ $3\sqrt[5]{x} - 1 + 1 = 5 + 1 \rightarrow$ add 1 to both sides $3\sqrt[5]{x} = 6$ $\frac{3\sqrt[5]{x}}{3} = \frac{6}{3} \rightarrow$ divide both sides by 3 $\sqrt[5]{x} = 2$ $\left(\sqrt[5]{x}\right)^5 = 2^5 \rightarrow$ raise both side to the power of 5 $x = 32$	$3\sqrt[5]{x} - 1 = 5$ $3\sqrt[5]{x} = 5 + 1 \rightarrow$ - 1 becomes + 1 on the RHS $3\sqrt[5]{x} = 6$ $\sqrt[5]{x} = \frac{6}{3} \rightarrow \times$ by 3 becomes $\div$ by 3 on the RHS $\sqrt[5]{x} = 2$ $x = 2^5 \rightarrow \sqrt[5]{\ }$ becomes $(\ )^5$ on the RHS $x = 32$

$4x^{-2} = 40$ $\frac{\cancel{4}^1 x^{-2}}{\cancel{4}^1} = \frac{\cancel{40}^{10}}{\cancel{4}^1} \rightarrow$ divide both sides by 4 $x^{-2} = 10 \rightarrow$ raise both sides to the $-\frac{1}{2}$ $\left(x^{-2}\right)^{-\frac{1}{2}} = 10^{-\frac{1}{2}}$ $x = \frac{1}{10^{\frac{1}{2}}}$ $x = \frac{1}{\sqrt{10}}$ $x = \pm 0.3162$ (to 4 d.p.)	$4x^{-2} = 40$ $x^{-2} = \frac{\cancel{40}^{10}}{\cancel{4}^1} \rightarrow \times 4$ becomes $\div 4$ on the RHS $x^{-2} = 10$ $x = 10^{-\frac{1}{2}} \rightarrow {}^{-2}$ becomes ${}^{-\frac{1}{2}}$ on the RHS $x = \frac{1}{\sqrt{10}}$ $x = \pm 0.3162$ (to 4 d.p.)
$(2x-1)^2 = 49$ $\sqrt{(2x-1)^2} = \sqrt{49} \rightarrow$ take square root of both sides $2x-1 = \pm 7$ $2x-1+1 = 7+1$ or $-7+1 \rightarrow$ add 1 to both sides $2x = 8$ or -6 $\frac{2x}{2} = \frac{8}{2}$ or $\frac{-6}{2} \rightarrow$ divide both sides by 2 $x = 4$ or -3	$(2x-1)^2 = 49$ $2x-1 = \sqrt{49} \rightarrow {}^2$ becomes $\sqrt{\ }$ on the RHS $2x-1 = \pm 7$ $2x = 7+1$ or $-7+1 \rightarrow -1$ becomes $+1$ on the RHS $2x = 8$ or -6 $x = \frac{8}{2}$ or $\frac{-6}{2} \rightarrow \times 2$ becomes $\div 2$ on the RHS $x = 4$ or -3
In the question below, the opposite of raising a number to a power is raising it to the reciprocal power. Remember a number multiplied by its reciprocal gives 1.	In the question below, the fraction exponent becomes the reciprocal on the other side of the equation.
$x^{\frac{2}{3}} = 50$ $\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = 50^{\frac{3}{2}}$ $x = \sqrt{50^3}$ $x = 353.55$	$x^{\frac{2}{3}} = 50$ $x = 50^{\frac{3}{2}}$ $x = \sqrt{50^3}$ $x = 353.55$

It is also important to be able to rearrange formulae that contain powers.

Rearrange the formulae below to make the pronumeral in brackets the subject.	
$A = \pi r^2 \quad [r]$ $\frac{A}{\pi} = \frac{\cancel{\pi}^1 r^2}{\cancel{\pi}^1} \rightarrow$ divide both sides by π $\frac{A}{\pi} = r^2 \rightarrow$ take the square root of both sides $\sqrt{\frac{A}{\pi}} = \sqrt{r^2}$ $r = \sqrt{\frac{A}{\pi}}$	$A = \pi r^2 \quad [r]$ $\frac{A}{\pi} = r^2 \rightarrow \times\pi$ becomes $\div\pi$ on LHS $\sqrt{\frac{A}{\pi}} = r \rightarrow {}^2$ becomes $\sqrt{\ }$ on the LHS $r = \sqrt{\frac{A}{\pi}}$

$$V = \frac{4}{3}\pi r^3 \; [r] \text{ The first step is to remove the fraction}$$
$$3V = \frac{4}{\cancel{3}^1}\pi r^3 \times \cancel{3}^1 \rightarrow \text{Multiple both sides by 3}$$
$$3V = 4\pi r^3$$
$$\frac{3V}{4\pi} = \frac{\cancel{4\pi} r^3}{\cancel{4\pi}} \rightarrow \text{ Divide both sides by } 4\pi$$
$$\frac{3V}{4\pi} = r^3$$
$$\sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{r^3} \rightarrow \text{ take cube root of both sides}$$
$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

$$V = \frac{4}{3}\pi r^3 \; [r] \text{ The first step is to remove the fraction}$$
$$3V = 4\pi r^3 \rightarrow \div 3 \text{ becomes } \times 3 \text{ on LHS}$$
$$\frac{3V}{4\pi} = r^3 \rightarrow \; \times 4\pi \text{ becomes } \div 4\pi \text{ on LHS}$$
$$\sqrt[3]{\frac{3V}{4\pi}} = r \rightarrow \; ^3 \text{ becomes } \sqrt[3]{\;} \text{ on the LHS}$$
$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

$$f = \frac{1}{2\pi\sqrt{LC}} \quad [C]$$
$$f \times \sqrt{LC} = \frac{1}{2\pi\cancel{\sqrt{LC}}} \times \cancel{\sqrt{LC}} \rightarrow \text{multiply both sides by } \sqrt{LC} \text{ to get C into the numerator}$$
$$\frac{\cancel{f} \times \sqrt{LC}}{\cancel{f}} = \frac{1}{2\pi f} \rightarrow \text{ divide both sides by } f$$
$$\left(\sqrt{LC}\right)^2 = \left(\frac{1}{2\pi f}\right)^2 \rightarrow \text{ square both sides}$$
$$LC = \frac{1}{4\pi^2 f^2}$$
$$\frac{\cancel{L}C}{\cancel{L}} = \frac{1}{4\pi^2 f^2 L} \rightarrow \text{ divide both sides by } L$$
$$C = \frac{1}{4\pi^2 f^2 L}$$

Or

$$f = \frac{1}{2\pi\sqrt{LC}} \quad [C]$$
$$f \times \sqrt{LC} = \frac{1}{2\pi} \rightarrow \div\sqrt{LC} \text{ becomes } \times\sqrt{LC} \text{ on the LHS}$$
$$\sqrt{LC} = \frac{1}{2\pi f} \rightarrow \; \times f \text{ becomes } \div f \text{ on the RHS}$$
$$LC = \left(\frac{1}{2\pi f}\right)^2 \rightarrow \; \sqrt{\;} \text{ becomes } ^2 \text{ on the RHS}$$
$$LC = \frac{1}{4\pi^2 f^2}$$
$$C = \frac{1}{4\pi^2 f^2 L} \rightarrow \; \times L \text{ becomes } \div L \text{ on the RHS}$$

Video ‘Equations containing exponents’

Activity

1. Solve for x:

(a) $x^3 = 27$

(b) $x^5 + 1 = 244$

(c) $3\sqrt[4]{x} = 6$

(d) $\sqrt[3]{x+1} = 5$

(e) $\sqrt[12]{x} = 2$

(f) $4\sqrt{x} + 31 = 63$

(g) $5x^2 + 4 = 9$

(h) $7(x^4 + 1) = 119$

(i) $x^{-4} = 20$ (use calc.)

(j) $\dfrac{x^{-1}}{4} = 11$

(k) $4x^{-5} = 5$

(l) $3x^{\frac{1}{2}} + 7 = 19$

(m) $3x^4 + 256 = 1024$

2. Rearrange these formulae for the pronumeral in brackets

(a) $A = 4\pi r^2 \quad [r]$

(b) $V = \pi r^2 h \quad [r]$

(c) $c^2 = a^2 + b^2 \quad [b]$

(d) $F = \dfrac{1}{2}mv^2 \quad [v]$

(e) $Q = SLd^2 \quad [d]$

(f) $v^2 = u^2 + 2as \quad [u]$

(g) $Z = \sqrt{R^2 + \omega^2 L^2} \quad [L]$

(h) $R = \dfrac{2GM}{c^2} \quad [c]$

(i) $P = \dfrac{\pi^2 EI}{L^2} \quad [L]$

The next questions are challenging, take care!

(j) $T = 2\pi\sqrt{\dfrac{r^3}{GM}} \quad [r]$

(k) $F = m\left(g - \dfrac{v^2}{R}\right) \quad [v]$

(l) $R = \dfrac{\pi r^4 (p_2 - p_1)}{8nL} \quad [r]$

(m) $A = P(1+i)^n \quad [i]$

Topic 4: Scientific notation

Scientific Notation is a way of representing numbers that are very large or very small usually to avoid writing down numbers that contain many zeros.

When numbers are written in scientific notation they usually written in this form:

This number must be between 1 and 9.999999...	$n \times 10^e$	The exponent is an integer.

The number $4\,000\,000$ can be written as $4 \times 1\,000\,000$ and then 4×10^6.

The number $0.000\,8$ can be written as $\frac{8}{10\,000}$ and then $\frac{8}{10^4} = 8 \times 10^{-4}$.

The number $532\,000$ can be written as $5.32 \times 100\,000 = 5.32 \times 10^5$.

Mostly though, students look at the number of places that the decimal point has moved.

Writing numbers in Scientific Notation

Write $395\,000$ in scientific notation.

Step 1	From your number, obtain the number between 1 and 9.9999999 $395\,000 = 3.95 \times 10^?$
Step 2	Calculate how many places the decimal point moved. $395\,000 = 3.95 \times 10^?$ The decimal point moved 5 places, so the ? becomes 5. $395\,000 = 3.95 \times 10^5$
Step 3	The exponent can be positive, negative or 0. In this step, the sign of the exponent is checked. If the original number is smaller than 1, then the exponent is negative. Otherwise it is positive or possibly zero. Answer is $395\,000 = 3.95 \times 10^5$

Write 0.000452 in scientific notation.

Step 1	From your number, obtain the number between 1 and 9.9999999 $0.000452 = 4.52 \times 10^{?}$
Step 2	Calculate how many places the decimal point moved. $0.000452 = 4.52 \times 10^{?}$ The decimal point moved 4 places, so the ? becomes 4. $0.000452 = 4.52 \times 10^{4}$
Step 3	The original number is smaller than 1, so the exponent is negative. Answer is $0.000452 = 4.52 \times 10^{-4}$

Write 5.6 in scientific notation.

Step 1	From your number, obtain the number between 1 and 9.9999999 $5.6 = 5.6 \times 10^{?}$
Step 2	Calculate how many places the decimal point moved. $5.6 = 5.6 \times 10^{?}$ The decimal point has not moved, so the ? becomes 0. $5.6 = 5.6 \times 10^{0}$
Step 3	The exponent is zero (from the previous step). Answer is $5.6 = 5.6 \times 10^{0}$

Write 13600000 in scientific notation.

Step 1	From your number, obtain the number between 1 and 9.9999999 $13600000 = 1.36 \times 10^{?}$
Step 2	Calculate how many places the decimal point moved. $13600000 = 1.36 \times 10^{?}$ The decimal point moved 7 places, so the ? becomes 7. $13600000 = 1.36 \times 10^{7}$
Step 3	The original number is greater than 1 so the exponent is positive or zero. Answer is $13600000 = 1.36 \times 10^{7}$

Changing numbers in scientific notation to standard form

Write 4.2×10^5 in standard form.

Step 1	An exponent of 5 means that the number is greater than 10. Write the number with trailing zeros. $$4.2 \times 10^5 = 4200000000$$
Step 2	Move the decimal point 5 places in a direction that produces a larger number (to the right). $$4.2 \times 10^5 = 420000.0000$$ The number in standard form is: $$4.2 \times 10^5 = 420000$$

Write 8.2×10^{-3} in standard form.

Step 1	An exponent of -3 means that the number is smaller than 1. Write the number with preceding zeros. $$8.2 \times 10^{-3} = 00008.2$$
Step 2	Move the decimal point 3 places in a direction that produces a smaller number (to the left). $$8.2 \times 10^{-3} = 00.0082$$ The number in standard form is: $$8.2 \times 10^{-3} = 0.0082$$

Write 6.354×10^1 in standard form.

Step 1	An exponent of 1 means that the number is greater than 1. Write the number with trailing zeros. $$6.354 \times 10^1 = 6.35400$$
Step 2	Move the decimal point 3 places in a direction that produces a smaller number (to the left). $$6.354 \times 10^1 = 63.5400$$ The number in standard form is: $$6.354 \times 10^1 = 63.54$$

Write 1.92×10^{-9} in standard form.

Step 1	An exponent of -9 means that the number is smaller than 1. Write the number with preceding zeros. $$1.92\times10^{-9}=000000001.92$$
Step 2	Move the decimal point 3 places in a direction that produces a smaller number (to the left). $$1.92\times10^{-9}=00000000192$$ The number in standard form is: $$1.92\times10^{-9}=0.00000000192$$

[Video ‘Scientific Notation – changing notation’]

Activity

1. Write these numbers in scientific notation

(a)	4 000 000	(b)	500 000	(c)	9 300 000 000 000
(d)	345 900	(e)	29 500 000	(f)	354.85
(g)	5.56	(h)	0.000 04	(i)	0.000 000 000 955
(j)	745	(k)	9	(l)	0.25

2. Write these numbers in standard notation

(a)	6×10^{5}	(b)	7.5×10^{10}	(c)	4.2×10^{1}
(d)	1.325×10^{6}	(e)	3.15×10^{0}	(f)	6.1×10^{-5}
(g)	9.1×10^{-2}	(h)	9.5×10^{0}	(i)	5.945×10^{-6}
(j)	6.1×10^{5}	(k)	3.125×10^{1}	(l)	$1.1933\overline{3}\times10^{3}$

Topic 5: Operations with numbers in scientific notation

Multiplying

When multiplying numbers in scientific notation, the number part is multiplied and the exponent part is multiplied using exponent rules.

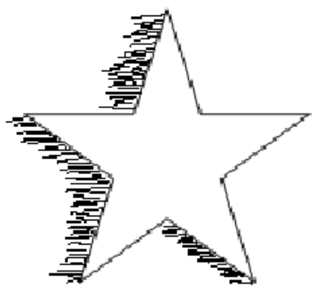
Example

$$\begin{aligned}
&\left(3.1\times10^{6}\right)\times\left(1.4\times10^{-2}\right)\\
&=\left(3.1\times1.4\right)\times\left(10^{6}\times10^{-2}\right) \ \rightarrow \text{rearrange}\\
&=4.34\times10^{6+{}^{-}2}\\
&=4.34\times10^{4}
\end{aligned}$$

It is important to look at the solution to see if the answer is still in scientific notation.

In the example below, an adjustment is required to put the answer into scientific notation.

$$\begin{aligned}
&\left(5.2\times10^{-5}\right)\times\left(3.5\times10^{-9}\right)\\
&=\left(5.2\times3.5\right)\times\left(10^{-5}\times10^{-9}\right) \ \rightarrow \text{rearrange}\\
&=18.2\times10^{-5+{}^{-}9}\\
&=18.2\times10^{-14} \quad \rightarrow \text{The number 18.2 is not in the range 1 to 9.9999}\ldots,\\
&=1.82\times10^{1}\times10^{-14} \quad \rightarrow \ 18.2 \ =1.82\times10^{1}\\
&=1.82\times10^{1+{}^{-}14}\\
&=1.82\times10^{-13}
\end{aligned}$$



The speed of light is approximately 3×10^{8} m/sec. How far will it travel in 5 hours?

$$\begin{aligned}
&5 \text{ hours}\\
&= 5\times60 \text{ minutes}\\
&= 5\times60\times60 \text{ seconds}\\
&=18000 \text{ seconds}\\
&=1.8\times10^{4}\text{seconds}
\end{aligned}$$

$$\begin{aligned}
\text{Distance covered } &=\text{speed}\times\text{time}\\
&=\left(3\times10^{8}\right)\times\left(1.8\times10^{4}\right)\\
&=5.4\times10^{12} \ \text{ metres}
\end{aligned}$$

Dividing

Dividing numbers in scientific notation is similar to multiplication. The number part is divided as usual and the exponent part is divided using exponent rules.

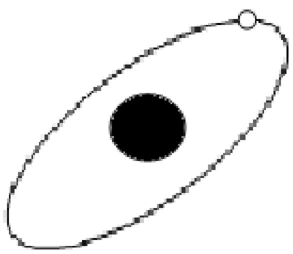
Example:

$$\begin{aligned} &\frac{6.25\times10^{7}}{5.1\times10^{-2}} \\ &=(6.25\div5.1)\times(10^{7}\div10^{-2}) \\ &=1.2255\times10^{7-^{-}2} \\ &=1.2255\times10^{9} \end{aligned}$$

As in multiplying, it is important to look at the solution to see if the answer is still in scientific notation.

In the example below, an adjustment is required to put the answer into scientific notation.

$$\begin{aligned} &\frac{3.12\times10^{-7}}{7.36\times10^{4}} \\ &=(3.12\div7.36)\times(10^{-7}\div10^{4}) \\ &=0.4239\times10^{-7-4} \\ &=0.4239\times10^{-11} \quad \rightarrow \text{The number } 0.4239 \text{ is not in the range 1 to } 9.9999\ldots, \\ &=4.239\times10^{-1}\times10^{-11} \quad \rightarrow 0.4239{=}4.239\times10^{-1} \\ &=4.239\times10^{-12} \end{aligned}$$



The mass of a hydrogen atom is 1.6×10^{-24} grams. How many atoms are there in $1\,\text{mg}$ (10^{-3} gram) of hydrogen?

$$\begin{aligned} \text{Number of atoms} &= \frac{1\times10^{-3}}{1.6\times10^{-24}} \\ &=(1\div1.6)\times(10^{-3}\div10^{-24}) \\ &=0.625\times10^{-3-^{-}24} \\ &=0.625\times10^{21} \\ &=6.25\times10^{-1}\times10^{21} \\ &=6.25\times10^{20} \end{aligned}$$

Adding and Subtracting

When adding or subtracting any numbers, it is important that numbers of the same place value are added or subtracted. When adding or subtracting whole numbers or decimals these means lining up the numbers in place value rows.

With numbers written in scientific notation, lining up numbers in place values is achieved by having exponent parts the same.

Add (3×10^{5}) and (4×10^{7})

One exponent must be changed to be the same as the other exponent. Choosing the higher exponent makes the finding then solution slightly easier.

$$
\begin{aligned}
&(3\times10^{5}) + (4\times10^{7})\\
&=3\times10^{-2}\times10^{7}+4\times10^{7}\\
&=0.03\times10^{7}+4\times10^{7} \quad \rightarrow \text{Both numbers are in the same place value}\\
&=4.03\times10^{7}
\end{aligned}
$$

Take 4.3×10^{4} from 1.95×10^{3}

$$
\begin{aligned}
&1.95\times10^{3} \;-\; 4.3\times10^{4}\\
&=1.95\times10^{-1}\times10^{4}-4.3\times10^{4}\\
&=0.195\times10^{3}-4.3\times10^{4}\\
&=\left(0.195-4.3\right)\times10^{4}\\
&= {}^{-}4.105\times10^{4}
\end{aligned}
$$

$$
\begin{array}{r}
\,{}^{2}\;{}^{\cancel{10}\,9}\;{}^{10}\\
4.\cancel{3}\,\cancel{0}\,\cancel{0}\\
-0.1\;9\;5\\
\hline
4.1\;0\;5
\end{array}
$$

The largest number is negative

$\rightarrow$ answer is negative

$\rightarrow$ ${}^{-}4.105$

Add 3.652×10^{-4} and 4.2×10^{-7}

Remember ${}^{-}4$ is larger than ${}^{-}7$.

$$
\begin{aligned}
&3.652\times10^{-4} \;+\; 4.2\times10^{-7}\\
&=3.652\times10^{-4} \;+\; 4.2\times10^{-3}\times10^{-4}\\
&=3.652\times10^{-4} \;+\; 0.0042\times10^{-4}\\
&=(3.652+0.0042)\times10^{-4}\\
&=3.6562\times10^{-4}
\end{aligned}
$$

Calculator use

To use scientific notation on your calculator, some calculators have a key marked [Exp], others K.

2×10^{11} is entered by the sequence: 2 [$\times10^x$] 11 or 2 [Exp] 11

4.25×10^{-6} is entered by the sequence [4] [•] [2] [5] [$\times10^x$] z [6] or [4] [•] [2] [5] [Exp] [(−)] [6]

-8.75×10^{15} is entered by the sequence [(−)] [8] [•] [7] [5] [$\times10^x$] [1] [5]

-1.6×10^{-19} is entered by the sequence [(−)] [1] [•] [6] [$\times10^x$] [(−)] [1] [9]

If you enter a number expressed in scientific notation using [Exp] or the [$\times10^x$] keys, the calculator treats this as a single number (which it is) ☺.

However, if you enter your number using the ($10^{\blacksquare}$) key, for example [2] [×] ($10^{\blacksquare}$) [1] [1], the calculator treats this as two numbers being multiplied and this can lead to incorrect answers when performing calculations ☹.

Using your calculator to change between scientific notation and standard notation can be quite difficult. If you require this you will need to obtain the instructions for your calculator and find out how to achieve this.

Performing an operation such as $2\times10^{11}\times4.25\times10^{-6}$ is entered as:

[2] [$\times10^x$] [1] [1] [X] [4] [•] [2] [5] [$\times10^x$] [(−)] [6] [=] 850000

The format of your answer will vary from calculator to calculator. Because the answer to this question is able to be displayed on a calculator in standard notation, the answer displayed could be 850 000, so you have to change this to 8.5×10^{5} if you specifically need the answer in scientific notation. Other calculators may automatically give the answer in scientific notation. If numbers are outside the range of what can be displayed in standard form on the calculator, your calculator should display the answer in scientific notation. Even when using a calculator, it is still important to be able to change from standard form to scientific notation and vice-versa.

Video 'Operations using Scientific Notation'

Activity

1. Perform the operations indicated. (Leave your answers in scientific notation)

(a) $6.02\times10^{23}\times1.6\times10^{-19}$

(b) $\left(3.432\times10^{3}\right)\div\left(3\times10^{-12}\right)$

(c) $3.4\times10^{6}+2.125\times10^{8}$

(d) $2.5\times10^{-6}-8.12\times10^{-5}$

2. Perform the operations indicated by changing to scientific notation.

(a) $2\,000\,000\,000\times0.000\,000\,03$

(b) $0.000\,000\,45\div0.000\,000\,000\,31$

(c) $340\,000\,000\,000\times0.000\,000\,125$

(d) $0.65\times12\,000\,000\div0.000\,000\,085$

Answers to activity questions

Check your skills

1.	(a) $x^6 \times 4x^3 \times x$ $= 4x^{6+3+1}$ $= 4x^{10}$	(b) $a^6 \div a^0$ $= a^6 \div 1$ $= a^6$	(c) $\left(x^3\right)^2 \times 4x^4 \div \left(2x^2\right)^4$ $= \dfrac{x^6 \times 4x^4}{2^4 x^8}$ $= \dfrac{\cancel{4}^1 x^{10}}{\cancel{16}^4 x^8}$ $= \dfrac{x^{10-8}}{4}$ $= \dfrac{x^2}{4}$
2.	(a) $\left(x^{-2}\right)^2 \times \left(x^{\frac{1}{2}}\right)^4 \times x^0$ $= x^{-2\times2} \times x^{\frac{1}{2}\times4} \times 1$ $= x^{-4} \times x^2$ $= x^{-2}$ or $\dfrac{1}{x^2}$	(b) $\sqrt[3]{512}$ $= \left(2^9\right)^{\frac{1}{3}}$ $= 2^{9\times\frac{1}{3}}$ $= 2^3$ $= 8$	(c) $\left(27x^9y^6\right)^{\frac{2}{3}}$ $= \left(3^3x^9y^6\right)^{\frac{2}{3}}$ $= 3^{3\times\frac{2}{3}}x^{9\times\frac{2}{3}}y^{6\times\frac{2}{3}}$ $= 9x^6y^4$
3.	(a) $x^4 = 81$ $x = \pm\sqrt[4]{81}$ $x = \pm3$	(b) $3\sqrt[4]{x} + 1 = 7$ $3\sqrt[4]{x} = 7 - 1$ $3\sqrt[4]{x} = 6$ $\sqrt[4]{x} = \dfrac{6}{3}$ $\sqrt[4]{x} = 2$ $x = 2^4$ $x = 16$	(c) $V = \pi r^3$ $\dfrac{V}{\pi} = r^3$ $\sqrt[3]{\dfrac{V}{\pi}} = r$

4.
(a) 634 000 000 in Scientific Notation is 6.34×10^8
(b) 9.12×10^{-5} in Standard Notation is 0.000 091 2
(c) 0.000 000 55 in Scientific Notation is 5.5×10^{-7}
(d) 7.25×10^8 in Standard Notation is 725 000 000

5.	(a) $3.2\times10^{8} \times 5.3\times10^{-5}$ $=3.2\times5.3\times10^{8+{}^{-}5}$ $=16.96\times10^{3}$ $=1.696\times10^{4}$	(b) $\dfrac{\left(2.8\times10^{8}\right)}{\left(7.2\times10^{-12}\right)}$ $=0.388\overline{8}\times10^{8-{}^{-}12}$ $=0.388\overline{8}\times10^{20}$ $=3.88\overline{8}\times10^{19}$	(c) $7.25\times10^{8}+2.61\times10^{4}$ $=7.25\times10^{8}+2.61\times10^{-4}\times10^{8}$ $=7.25\times10^{8}+0.000261\times10^{8}$ $=7.250261\times10^{8}$

Exponent rules – Whole number exponents

1. Evaluate the following powers.

(a) 3^{5}
$=3\times3\times3\times3\times3$
$=243$

(b) 5^{3}
$=5\times5\times5$
$=125$

(c) 7^{3}
$=7\times7\times7$
$=343$

(d) 2^{9}
$=2\times2\times2\times2\times2\times2\times2\times2\times$
$=512$

(e) 14^{3}
$=14\times14\times14$
$=2744$

(f) 5^{5}
$=5\times5\times5\times5\times5$
$=3125$

(g) 3^{0}
$=1$

(h) $\left(2^{7}\right)^{0}$
$=1$

(i) $\left(3^{2}\right)^{3}$
$=3^{2\times3}$
$=3^{6}=729$

(j) $2^{4}\times2^{6}$
$=2^{4+6}$
$=2^{10}$
$=1024$

(k) $4^{7}\div4^{5}$
$=\dfrac{4^{7}}{4^{5}}$
$=4^{7-5}$
$=4^{2}=16$

(l) -4^{3}
$=-(4\times4\times4)$
$=-64$

(m) $(-5)^{3}$
$={}^{-}5\times{}^{-}5\times{}^{-}5$
$={}^{-}125$

(n) 0.85^{6}
0.85W6é0.377149515

(o) $4.5^{1.25}$
4.5W1.25é
6.554138918

(p) $(1.006)^{240}$
1.006W240é
4.202574033

(q) $(1.05)^{20}$
1.05W20é
2.653297705

2. Simplify the following (leave your answer in power form).

(a) $5^{2}\times5^{5}$
$=5^{2+5}$
$=5^{7}$

(b) $m^{4}\times m^{5}$
$=m^{4+5}$
$=m^{9}$

(c) $a^{4}\times a^{3}\times a$
$=a^{4+3+1}$
$=a^{8}$

(d) $10^{4}\times10\times10^{0}$
$=10^{4+1+0}$
$=10^{5}$

(e) $\dfrac{r^{4}}{2}\times\dfrac{r}{4}$
$=\dfrac{r^{4+1}}{8}$
$=\dfrac{r^{5}}{8}$

(f) $5c^{2}\times3c^{4}$
$=15c^{2+4}$
$=15c^{6}$

(g) $a^2b^4 \times a^3bc$
$= a^{2+3}b^{4+1}c$
$= a^5b^5c$

(h) $4x^3y^4z^2 \times 5x^2y$
$= 20x^{3+2}y^{4+1}z^2$
$= 20x^5y^5z^2$

(i) $5^2ef^3g^5 \times 5^3e^2f^2g^0$
$= 5^{2+3}e^{1+2}f^{3+2}g^{5+0}$
$= 5^5e^3f^5g^5$

(j) $4^9 \div 4^5$
$= 4^{9-5}$
$= 4^4$

(k) $x^8 \div x^5$
$= x^{8-5}$
$= x^3$

(l) $h^5 \div h$
$= h^{5-1}$
$= h^4$

(m) $3^x \div 3^4$
$= 3^{x-4}$

(n) $\dfrac{16a^5}{4a^2}$
$= 4a^{5-2}$
$= 4a^3$

(o) $\dfrac{5a^2b^4}{25a^2b}$
$= \dfrac{a^{2-2}b^{4-1}}{5}$
$= \dfrac{a^0b^3}{5}$
$= \dfrac{b^3}{5}$

(p) $a^{\frac{1}{2}} \div a^{\frac{1}{4}}$
$= a^{\frac{1}{2}-\frac{1}{4}}$
$= a^{\frac{2}{4}-\frac{1}{4}}$
$= a^{\frac{1}{4}}$

(q) $\left(2^4\right)^3$
$= 2^{4\times3}$
$= 2^{12}$

(r) $\left(5^4\right)^{\frac{1}{2}}$
$= 5^{4\times\frac{1}{2}}$
$= 5^2$

(s) $\left(b^4\right)^5$
$= b^{4\times5}$
$= b^{20}$

(t) $(3t)^4$
$= 3^4t^4$

(u) $\left(2a^4b^3\right)^3$
$= 2^3a^{4\times3}b^{3\times3}$
$= 2^3a^{12}b^9$

(v) $\left(\dfrac{4}{y}\right)^3$
$= \dfrac{4^3}{y^3}$

(w) $\dfrac{\left(2y^4\right)^2}{\left(2x^2\right)^4}$
$= \dfrac{2^2y^{4\times2}}{2^4x^{2\times4}}$
$= \dfrac{y^8}{2^2x^8}$

(x) $\left(2^5\right)^b$
$= 2^{5\times b}$
$= 2^{5b}$

3. Simplify the following.

(a)
$$\left(a^2\right)^4 \times a^3$$
$$= a^{2\times4} \times a^3$$
$$= a^{8+3}$$
$$= a^{11}$$

(b)
$$\frac{\left(3y^4\right)^2}{2y^5}$$
$$= \frac{3^2 y^{4\times2}}{2y^5}$$
$$= \frac{9y^8}{2y^5}$$
$$= \frac{9y^{8-5}}{2}$$
$$= \frac{9y^3}{2}$$

(c)
$$\frac{2x^5 \times x^4}{\left(x^2\right)^3}$$
$$= \frac{2x^{5+4}}{x^{2\times3}}$$
$$= \frac{2x^9}{x^6}$$
$$= 2x^{9-6}$$
$$= 2x^3$$

(d)
$$3b^0 + \left(3b\right)^0$$
$$= 3\times1+1$$
$$= 4$$

(e)
$$\frac{\left(3n\right)^2 \times 3n^2}{9n^4}$$
$$= \frac{3^2 n^2 \times 3n^2}{9n^4}$$
$$= \frac{\cancel{9}^1 \times 3 \times n^{2+2}}{\cancel{9}^1 n^4}$$
$$= \frac{3\cancel{n^4}^1}{\cancel{n^4}^1}$$
$$= 3$$

(f)
$$5\left(mn\right)^2 \times \left(mn^2\right)^3$$
$$= 5m^2n^2 \times m^3 n^{2\times3}$$
$$= 5m^{2+3} n^{2+6}$$
$$= 5m^5 n^8$$

(g)
$$m^2 n^5 p^7 \times m^3 p^4 \times n^2 p$$
$$= m^{2+3} n^{5+2} p^{7+4+1}$$
$$= m^5 n^7 p^{12}$$

(h)
$$-4(a^2 b^5)^0$$
$$= -4\times1$$
$$= -4$$

(i)
$$2a^2b^3 \times 3ab^4$$
$$= 6a^{2+1} b^{3+4}$$
$$= 6a^3 b^7$$

(j)
$$\frac{\cancel{2}^1 x^5 y^4}{\cancel{10}^5 x^2 y^3}$$
$$= \frac{x^{5-2} y^{4-3}}{5}$$
$$= \frac{x^3 y^1}{5} \quad or \quad \frac{x^3 y}{5}$$

(k)
$$\left(\frac{2x^3}{y^4}\right)^4 \times \left(\frac{y}{x}\right)^2$$
$$= \frac{2^4 x^{3\times4}}{y^{4\times4}} \times \frac{y^2}{x^2}$$
$$= \frac{16x^{12} y^2}{x^2 y^{16}}$$
$$= 16x^{12-2} y^{2-16}$$
$$= 16x^{10} y^{-14} \quad or \quad \frac{16x^{10}}{y^{14}}$$

(l)
$$4a^0 - \left(\frac{a}{4}\right)^0$$
$$= 4\times1-1$$
$$= 3$$

(m)
$$\frac{25p^6}{36q^8}$$
$$=\frac{\left(5p^3\right)^2}{\left(6q^4\right)^2}$$
$$=\left(\frac{5p^3}{6q^4}\right)^2$$

(n)
$$27a^6b^9c^3$$
$$=3^3a^6b^9c^3$$
$$=\left(3a^2b^3c\right)^3$$

(o)
$$\frac{\left(3xy^2\right)^3}{2x\left(2y\right)^2}\div\frac{6xy^0}{4\left(xy^2\right)^3}$$
$$=\frac{\left(3xy^2\right)^3}{2x\left(2y\right)^2}\times\frac{4\left(xy^2\right)^3}{6xy^0}$$
$$=\frac{3^3x^3y^{2\times3}}{2x\times2^2y^2}\times\frac{4x^3y^{2\times3}}{6x}$$
$$=\frac{\cancel{27}^9\times\cancel{4}^1\times x^{3+3}y^{6+6}}{\cancel{8}^2\times\cancel{6}^2\times x^2y^2}$$
$$=\frac{9x^6y^{12}}{4x^2y^2}$$
$$=\frac{9x^{6-2}y^{12-2}}{4}$$
$$=\frac{9x^4y^{10}}{4}$$

Exponent rules – Integer and fraction exponents

1. Evaluate

(a)
$$36^{-1}$$
$$=\frac{1}{36^1}$$
$$=\frac{1}{36}$$

(b)
$$5^{-2}$$
$$=\frac{1}{5^2}$$
$$=\frac{1}{25}$$

(c)
$$10^{-4}$$
$$=\frac{1}{10^4}$$
$$=\frac{1}{10000}$$

(d)
$$\frac{1}{4^{-3}}$$
$$=4^3$$
$$=64$$

(e)
$$\left(\frac{1}{4}\right)^{-4}$$
$$=\left(4\right)^4$$
$$=256$$

(f)
$$\frac{5^{-2}}{2^3}$$
$$=\frac{1}{2^3\times5^2}$$
$$=\frac{1}{8\times25}$$
$$=\frac{1}{200}$$

(g)
$$7\times2^{-4}$$
$$=7\times\frac{1}{2^4}$$
$$=\frac{7}{16}$$

(h)
$$\left(3\times2^{-2}\right)^{-2}$$
$$=\frac{1}{\left(3\times2^{-2}\right)^2}$$
$$=\frac{1}{3^2\times2^{-4}}$$
$$=\frac{2^4}{3^2}$$
$$=\frac{16}{9}\ or\ 1\frac{7}{9}$$

2. Simplify

(a) $a^{-4} = \frac{1}{a^4}$

(b) $c^{-7} = \frac{1}{c^7}$

(c) $\frac{1}{x^{-2}} = x^2$

(d) $2x^{-3} = \frac{2}{x^3}$

(e)
$$(3b)^{-2} = \frac{1}{(3b)^2} = \frac{1}{3^2b^2} = \frac{1}{9b^2}$$

(f) $ab^{-1} = \frac{a}{b}$

(g) $a^3 \times b^{-4} = \frac{a^3}{b^4}$

(h) $4p^{-1} = \frac{4}{p}$

(i) $(4p)^{-1} = \frac{1}{4p}$

(j)
$$\left(3^{-1}ab^{-2}\right)^3 = \left(\frac{a}{3b^2}\right)^3 = \frac{a^3}{3^3b^{2\times3}} = \frac{a^3}{27b^6}$$

(k)
$$4p^2q^{-3} \times 3p^{-1}q^2 = \frac{4p^2}{q^3} \times \frac{3q^2}{p} = \frac{12p^2q^2}{pq^3} = 12p^{2-1}q^{2-3} = 12p^1q^{-1} = \frac{12p}{q}$$

(l)
$$\left(\frac{4}{b^2}\right)^{-3} = \left(\frac{b^2}{4}\right)^3 = \frac{b^{2\times3}}{4^3} = \frac{b^6}{64}$$

(m)
$$\frac{\left(xy^3\right)^{-4}}{4x^4y^{-2}} = \frac{x^{-4}y^{3\times-4}}{4x^4y^{-2}} = \frac{x^{-4-4}y^{-12-{}^{-}2}}{4} = \frac{x^{-8}y^{-10}}{4} = \frac{1}{4x^8y^{10}}$$

(n)
$$\frac{2\left(x^2y^{-1}\right)^{-3}}{\left(3x^{-1}y^3\right)^2} = \frac{2x^{2\times-3}y^{-1\times-3}}{3^2x^{-1\times2}y^{3\times2}} = \frac{2x^{-6}y^3}{9x^{-2}y^6} = \frac{2x^{-6-{}^{-}2}y^{3-6}}{9} = \frac{2x^{-4}y^{-3}}{9} = \frac{2}{9x^4y^3}$$

(o)
$$\frac{2x^2}{3y^{-3}} \times \left(\frac{4xy^2}{9x^{-4}}\right)^{-1} = \frac{2x^2}{3y^{-3}} \times \left(\frac{9x^{-4}}{4xy^2}\right)^1 = \frac{\cancel{2}^1 \times \cancel{9}^3 \times x^{2+{}^{-}4}}{\cancel{3}^1 \times \cancel{4}^2 \times xy^{-3+2}} = \frac{3x^{-2}}{2xy^{-1}} = \frac{3x^{-2-1}}{2y^{-1}} = \frac{3x^{-3}}{2y^{-1}} = \frac{3y}{2x^3}$$

3. Evaluate without using a calculator

(a)
$$49^{\frac{1}{2}} = \left(7^{\cancel{2}^1}\right)^{\frac{1}{\cancel{2}_1}} = 7^1 \quad or \quad 7$$

(b)
$$8^{\frac{2}{3}} = \left(2^{\cancel{3}^1}\right)^{\frac{2}{\cancel{3}_1}} = 2^2 = 4$$

(c)
$$32^{\frac{3}{5}} = \left(2^{\cancel{5}^1}\right)^{\frac{3}{\cancel{5}_1}} = 2^3 = 8$$

(d)
$$625^{\frac{1}{4}}$$
$$=\left(5^{\cancel{4}^{1}}\right)^{\frac{1}{\cancel{4}^{1}}}$$
$$=5^1=5$$

(e)
$$1028^{\frac{7}{10}}$$
$$=\left(2^{\cancel{10}^{1}}\right)^{\frac{7}{\cancel{10}^{1}}}$$
$$=2^7$$
$$=128$$

(f)
$$256^{-\frac{1}{4}}$$
$$=\frac{1}{256^{\frac{1}{4}}}$$
$$=\frac{1}{\left(2^{\cancel{8}^{2}}\right)^{\frac{1}{\cancel{4}^{1}}}}$$
$$=\frac{1}{2^2}$$
$$=\frac{1}{4}$$

(g)
$$16^{-\frac{1}{2}}$$
$$=\frac{1}{\left(2^{\cancel{4}^{2}}\right)^{\frac{1}{\cancel{2}^{1}}}}$$
$$=\frac{1}{2^2}$$
$$=\frac{1}{4}$$

(h)
$$1028^{-\frac{7}{10}}$$
$$=\frac{1}{\left(2^{\cancel{10}^{1}}\right)^{\frac{7}{\cancel{10}^{1}}}}$$
$$=\frac{1}{2^7}$$
$$=\frac{1}{128}$$

(i)
$$\frac{3}{27^{-\frac{2}{3}}}$$
$$=3\times 27^{\frac{2}{3}}$$
$$=3\times\left(3^{\cancel{3}^{1}}\right)^{\frac{2}{\cancel{3}^{1}}}$$
$$=3^1\times 3^2$$
$$=3^3 \quad or \quad 27$$

(j)
$$7\times 128^{\frac{3}{7}}$$
$$=7\times\left(2^{\cancel{7}^{1}}\right)^{\frac{3}{\cancel{7}^{1}}}$$
$$=7\times 2^3$$
$$=56$$

(k)
$$\frac{16^{\frac{3}{4}}}{27^{\frac{1}{3}}}$$
$$=\frac{\left(2^{\cancel{4}^{1}}\right)^{\frac{3}{\cancel{4}^{1}}}}{\left(3^{\cancel{3}^{1}}\right)^{\frac{1}{\cancel{3}^{1}}}}$$
$$=\frac{2^3}{3}$$
$$=\frac{8}{3} \quad or \quad 2\frac{2}{3}$$

(l)
$$36^{\frac{1}{2}}\times 32^{\frac{3}{5}}$$
$$=\left(6^{\cancel{2}^{1}}\right)^{\frac{1}{\cancel{2}^{1}}}\times\left(2^{\cancel{5}^{1}}\right)^{\frac{3}{\cancel{5}^{1}}}$$
$$=6\times 2^3$$
$$=48$$

4. Use a calculator to answer the following

(a)
$$10^{\frac{3}{4}}$$
$$=5.623413252$$

(b)
$$450^{\frac{1}{7}}$$
$$=2.393483066$$

(c)
$$1.864297^{\frac{1}{10}}$$
$$=1.06426924$$

(d)
$$2.1209527^{-\frac{1}{240}}$$
$$=0.996872129$$

(e)
$$4.95^{2.5}$$
$$=54.51462103$$

(f)
$$1.01^{-0.75}$$
$$=0.992565029$$

5. Simplify

(a)
$$\left(\frac{8}{a^6}\right)^{\frac{1}{3}}$$
$$=\frac{\left(2^{\cancel{3}^1}\right)^{\frac{1}{\cancel{3}^1}}}{a^{\cancel{6}^2\times\frac{1}{\cancel{3}^1}}}$$
$$=\frac{2^1}{a^2}$$
$$=\frac{2}{a^2}$$

(b)
$$\frac{4\left(a^6b^3\right)^{\frac{1}{3}}}{\left(16a^8b^4\right)^{\frac{1}{4}}}$$
$$=\frac{4a^{6\times\frac{1}{3}}b^{3\times\frac{1}{3}}}{\left(2^{\cancel{4}^1}\right)^{\frac{1}{\cancel{4}^1}}a^{8\times\frac{1}{4}}b^{4\times\frac{1}{4}}}$$
$$=\frac{4a^2b^1}{2a^2b^1}$$
$$=2$$

(c)
$$\left(\frac{a^2b^6}{16}\right)^{-\frac{1}{2}}$$
$$=\left(\frac{4^2}{\left(ab^3\right)^2}\right)^{\frac{1}{2}}$$
$$=\frac{4^{\cancel{2}^1\times\frac{1}{\cancel{2}^1}}}{\left(ab^3\right)^{\cancel{2}^1\times\frac{1}{\cancel{2}^1}}}$$
$$=\frac{4}{ab^3}$$

(d)
$$\sqrt[3]{64a^2b^4}$$
$$=\left(2^6a^2b^4\right)^{\frac{1}{3}}$$
$$=2^{6\times\frac{1}{3}}a^{2\times\frac{1}{3}}b^{4\times\frac{1}{3}}$$
$$=2^2a^{\frac{2}{3}}b^{\frac{4}{3}}$$
$$=4a^{\frac{2}{3}}b^{\frac{4}{3}}$$

(e)
$$\sqrt[4]{32\left(a^3b^2\right)^2}$$
$$=\left[32\left(a^3b^2\right)^2\right]^{\frac{1}{4}}$$
$$=2^{5\times\frac{1}{4}}\left(a^3b^2\right)^{\cancel{2}^1\times\frac{1}{\cancel{4}^2}}$$
$$=2^{\frac{5}{4}}\left(a^3b^2\right)^{\frac{1}{2}}$$
$$=2^{\frac{5}{4}}a^{\frac{3}{2}}b^1 \quad or \quad 2.3784a^{1.5}b$$

(f)
$$\frac{\sqrt{x^2y^6}}{\sqrt[3]{x^6y^3}}$$
$$=\frac{\left(x^2y^6\right)^{\frac{1}{2}}}{\left(x^6y^3\right)^{\frac{1}{3}}}$$
$$=\frac{xy^3}{x^2y}$$
$$=\frac{y^2}{x}$$

Equation with exponents

1. Solve for x:

(a)
$$x^3=27$$
$$x=\sqrt[3]{27}$$
$$x=3$$

(b)
$$x^5+1=244$$
$$x^5+1-1=244-1$$
$$x^5=243$$
$$\sqrt[5]{\left(x^5\right)}=\sqrt[5]{243}$$
$$x=3$$

(c)
$$3\sqrt[4]{x}=6$$
$$\frac{3\sqrt[4]{x}}{3}=\frac{6}{3}$$
$$\sqrt[4]{x}=2$$
$$\left(\sqrt[4]{x}\right)^4=2^4$$
$$x=16$$

(d)
$$\sqrt[3]{x+1}=5$$
$$\left(\sqrt[3]{x+1}\right)^3=5^3$$
$$x+1=125$$
$$x+1-1=125-1$$
$$x=124$$

(e)
$$\sqrt[12]{x}=2$$
$$\left(\sqrt[12]{x}\right)^{12}=2^{12}$$
$$x=4096$$

(f)
$$4\sqrt{x}+31=63$$
$$4\sqrt{x}+31-31=63-31$$
$$4\sqrt{x}=32$$
$$\frac{4\sqrt{x}}{4}=\frac{32}{4}$$
$$\sqrt{x}=8$$
$$\left(\sqrt{x}\right)^2=8^2$$
$$x=64$$

(g)
$$\begin{aligned} 5x^2+4&=9\\ 5x^2+4-4&=9-4\\ 5x^2&=5\\ \frac{5x^2}{5}&=\frac{5}{5}\\ x^2&=1\\ x&=\pm 1 \end{aligned}$$

(h)
$$\begin{aligned} 7(x^4+1)&=119\\ \frac{7(x^4+1)}{7}&=\frac{119}{7}\\ x^4+1-1&=17-1\\ x^4&=16\\ x&=\sqrt[4]{16}\\ x&=\pm 2 \end{aligned}$$

(i)
$$\begin{aligned} x^{-4}&=20\\ \frac{1}{x^4}&=\frac{20}{1}\\ x^4&=\frac{1}{20}\\ \sqrt[4]{x^4}&=\sqrt[4]{\frac{1}{20}}\\ x&\approx 0.47287 \end{aligned}$$

(j)
$$\begin{aligned} \frac{x^{-1}}{4}&=11\\ \frac{x^{-1}}{4}\times 4&=11\times 4\\ x^{-1}&=44\\ x&=\frac{1}{44} \end{aligned}$$

(k)
$$\begin{aligned} 4x^{-5}&=5\\ \frac{4x^{-5}}{4}&=\frac{5}{4}\\ x^{-5}&=1.25\\ x^5&=\frac{1}{1.25}\\ x^5&=0.8\\ \sqrt[5]{x^5}&=\sqrt[5]{0.8}\\ x&\approx 0.9565 \end{aligned}$$

(l)
$$\begin{aligned} 3x^{\frac{1}{2}}+7&=19\\ 3x^{\frac{1}{2}}+7-7&=19-7\\ 3x^{\frac{1}{2}}&=12\\ \frac{3x^{\frac{1}{2}}}{3}&=\frac{12}{3}\\ x^{\frac{1}{2}}&=4\\ \left(x^{\frac{1}{2}}\right)^2&=4^2\\ x&=16 \end{aligned}$$

(m)
$$\begin{aligned} 3x^4+256&=1024\\ 3x^4+256-256&=1024-256\\ \frac{3x^4}{3}&=\frac{768}{3}\\ x^4&=256\\ \sqrt[4]{x^4}&=\sqrt[4]{256}\\ x&=4 \end{aligned}$$

2. Rearrange these formulae for the pronumeral in brackets

(a)
$$\begin{aligned} A&=4\pi r^2 \quad [r]\\ \frac{A}{4\pi}&=\frac{4\pi r^2}{4\pi}\\ \frac{A}{4\pi}&=r^2\\ \sqrt{\frac{A}{4\pi}}&=\sqrt{r^2}\\ \sqrt{\frac{A}{4\pi}}&=r \end{aligned}$$

(b)
$$\begin{aligned} V&=\pi r^2 h \quad [r]\\ \frac{V}{\pi h}&=\frac{\pi r^2 h}{\pi h}\\ \frac{V}{\pi h}&=r^2\\ \sqrt{\frac{V}{\pi h}}&=\sqrt{r^2}\\ \sqrt{\frac{V}{\pi h}}&=r \end{aligned}$$

(c)
$$\begin{aligned} c^2&=a^2+b^2 \quad [b]\\ c^2-a^2&=a^2+b^2-a^2\\ c^2-a^2&=b^2\\ \sqrt{c^2-a^2}&=\sqrt{b^2}\\ \sqrt{c^2-a^2}&=b \end{aligned}$$

(d)

$$F = \frac{1}{2}mv^2 \quad [v]$$
$$2\times F = 2\times\frac{1}{2}mv^2$$
$$2F = mv^2$$
$$\frac{2F}{m} = \frac{mv^2}{m}$$
$$\frac{2F}{m} = v^2$$
$$\sqrt{\frac{2F}{m}} = \sqrt{v^2}$$
$$\sqrt{\frac{2F}{m}} = v$$

(e)

$$Q = SLd^2 \quad [d]$$
$$\frac{Q}{SL} = \frac{SLd^2}{SL}$$
$$\frac{Q}{SL} = d^2$$
$$\sqrt{\frac{Q}{SL}} = \sqrt{d^2}$$
$$\sqrt{\frac{Q}{SL}} = d$$

(f)

$$v^2 = u^2 + 2as \quad [u]$$
$$v^2 - 2as = u^2 + 2as - 2as$$
$$v^2 - 2as = u^2$$
$$\sqrt{v^2 - 2as} = \sqrt{u^2}$$
$$\sqrt{v^2 - 2as} = u$$

(g)

$$Z = \sqrt{R^2 + \omega^2 L^2} \quad [L]$$
$$Z^2 = \left(\sqrt{R^2 + \omega^2 L^2}\right)^2$$
$$Z^2 = R^2 + \omega^2 L^2$$
$$Z^2 - R^2 = R^2 + \omega^2 L^2 - R^2$$
$$Z^2 - R^2 = \omega^2 L^2$$
$$\frac{Z^2 - R^2}{\omega^2} = \frac{\omega^2 L^2}{\omega^2}$$
$$\frac{Z^2 - R^2}{\omega^2} = L^2$$
$$\sqrt{\frac{Z^2 - R^2}{\omega^2}} = \sqrt{L^2}$$
$$\sqrt{\frac{Z^2 - R^2}{\omega^2}} = L$$

(h)

$$R = \frac{2GM}{c^2} \quad [c]$$
$$c^2 R = \frac{2GM}{c^2}\times c^2$$
$$c^2 R = 2GM$$
$$\frac{c^2 R}{R} = \frac{2GM}{R}$$
$$c^2 = \frac{2GM}{R}$$
$$\sqrt{c^2} = \sqrt{\frac{2GM}{R}}$$
$$c = \sqrt{\frac{2GM}{R}}$$

(i)

$$P = \frac{\pi^2 EI}{L^2} \quad [L]$$
$$L^2 \times P = \frac{\pi^2 EI}{L^2}\times L^2$$
$$L^2 P = \pi^2 EI$$
$$\frac{L^2 P}{P} = \frac{\pi^2 EI}{P}$$
$$L^2 = \frac{\pi^2 EI}{P}$$
$$\sqrt{L^2} = \sqrt{\frac{\pi^2 EI}{P}}$$
$$L = \sqrt{\frac{\pi^2 EI}{P}}$$

The next questions are challenging, take care!

(j)

$$T = 2\pi\sqrt{\frac{r^3}{GM}} \quad [r]$$
$$\frac{T}{2\pi} = \frac{2\pi\sqrt{\frac{r^3}{GM}}}{2\pi}$$
$$\frac{T}{2\pi} = \sqrt{\frac{r^3}{GM}}$$
$$\left(\frac{T}{2\pi}\right)^2 = \left(\sqrt{\frac{r^3}{GM}}\right)^2$$
$$\left(\frac{T}{2\pi}\right)^2 = \frac{r^3}{GM}$$
$$GM\times\left(\frac{T}{2\pi}\right)^2 = \frac{r^3}{GM}\times GM$$
$$GM\left(\frac{T}{2\pi}\right)^2 = r^3$$
$$\sqrt[3]{GM\left(\frac{T}{2\pi}\right)^2} = \sqrt[3]{r^3}$$
$$\sqrt[3]{GM\left(\frac{T}{2\pi}\right)^2} = r$$

(k)

$$F = m\left(g - \frac{v^2}{R}\right) \quad [v]$$
$$\frac{F}{m} = \frac{m\left(g - \frac{v^2}{R}\right)}{m}$$
$$\frac{F}{m} = g - \frac{v^2}{R}$$
$$\frac{F}{m} - g = g - \frac{v^2}{R} - g$$
$$\frac{F}{m} - g = -\frac{v^2}{R}$$
$$g - \frac{F}{m} = \frac{v^2}{R}$$
$$R\left(g - \frac{F}{m}\right) = \frac{v^2}{R}\times R$$
$$R\left(g - \frac{F}{m}\right) = v^2$$
$$\sqrt{R\left(g - \frac{F}{m}\right)} = \sqrt{v^2}$$
$$\sqrt{R\left(g - \frac{F}{m}\right)} = v$$

(l)

$$R = \frac{\pi r^4\left(p_2 - p_1\right)}{8nL} \quad [r]$$
$$8nL\times R = \frac{\pi r^4\left(p_2 - p_1\right)}{8nL}\times 8nL$$
$$8nLR = \pi r^4\left(p_2 - p_1\right)$$
$$\frac{8nLR}{\pi\left(p_2 - p_1\right)} = r^4$$
$$\sqrt[4]{\frac{8nLR}{\pi\left(p_2 - p_1\right)}} = \sqrt[4]{r^4}$$
$$\sqrt[4]{\frac{8nLR}{\pi\left(p_2 - p_1\right)}} = r$$

(m) $A = P(1+i)^n \quad [i]$

$$\frac{A}{P} = \frac{P(1+i)^n}{P}$$

$$\frac{A}{P} = (1+i)^n$$

$$\sqrt[n]{\frac{A}{P}} = \sqrt[n]{(1+i)^n}$$

$$\sqrt[n]{\frac{A}{P}} = 1+i$$

$$\sqrt[n]{\frac{A}{P}} - 1 = 1+i-1$$

$$\sqrt[n]{\frac{A}{P}} - 1 = i$$

Scientific notation

1. Write these numbers in scientific notation

(a) 4 000 000
$= 4\times10^6$

(b) 500 000
$= 5\times10^5$

(c) 9 300 000 000 000
$= 9.3\times10^{12}$

(d) 345 900
$= 3.459\times10^5$

(e) 29 500 000
$= 2.95\times10^7$

(f) 354.85
$= 3.5485\times10^2$

(g) 5.56
$= 5.56\times10^0$

(h) 0.000 04
$= 4\times10^{-5}$

(i) 0.000 000 000 955
$= 9.55\times10^{-10}$

(j) 745
$= 7.45\times10^2$

(k) 9
$= 9\times10^0$

(l) 0.25
$= 2.5\times10^{-1}$

2. Write these numbers in standard notation

(a) 6×10^5
$= 600000$

(b) 7.5×10^{10}
$= 75000000000$

(c) 4.2×10^1
$= 42$

(d) 1.325×10^6
1325000

(e) 3.15×10^0
$= 3.15$

(f) 6.1×10^{-5}
$= 0.000061$

(g) 9.1×10^{-2}
$= 0.091$

(h) 9.5×10^0
$= 9.5$

(i) 5.945×10^{-6}
$= 0.000005945$

(j) 6.1×10^5
$= 610000$

(k) 3.125×10^1
$= 31.25$

(l) $1.1933\overline{3}\times10^3$
$= 1193.3\overline{3}$

Operations with numbers in scientific notation

1. Perform the operations indicated. (Leave your answers in scientific notation)

(a) $6.02\times10^{23}\times1.6\times10^{-19}$
$= 6.02\times1.6\times10^{23+{}^-19}$
$= 9.696\times10^4$

(b) $\left(3.432\times10^3\right)\div\left(3\times10^{-12}\right)$
$= (3.432\div3)\times10^{3-{}^-12}$
$= 1.144\times10^{15}$

(c) $3.4\times10^{6}+2.125\times10^{8}$
$=3.4\times10^{-2}\times10^{8}+2.125\times10^{8}$
$=0.034\times10^{8}+2.125\times10^{8}$
$=2.159\times10^{8}$

(d) $2.5\times10^{-6}-8.12\times10^{-5}$
$=2.5\times10^{-1}\times10^{-5}-8.12\times10^{-5}$
$=0.25\times10^{-5}-8.12\times10^{-5}$
$={}^{-}7.87\times10^{-5}$

2. Perform the operations indicated by changing to scientific notation.

(a) $2\,000\,000\,000\times0.000\,000\,03$
$=2\times10^{9}\times3\times10^{-8}$
$=6\times10^{1}$
or 60

(b) $0.000\,000\,45\div0.000\,000\,000\,31$
$=\left(4.5\times10^{-7}\right)\div\left(3.1\times10^{-10}\right)$
$=\left(4.5\div3.1\right)\times10^{-7-{}^{-}10}$
$=1.452\times10^{3}$
or 1452

(c) $340\,000\,000\,000\times0.000\,000\,125$
$=3.4\times10^{11}\times1.25\times10^{-7}$
$=4.25\times10^{4}$
or $42\,500$

(d) $0.65\times12\,000\,000\div0.000\,000\,085$
$=6.5\times10^{-1}\ \times\ 1.2\times10^{7}\ \div\ \left(8.5\times10^{-8}\right)$
$=7.8\times10^{6}\ \div\ \left(8.5\times10^{-8}\right)$
$=0.9176\times10^{6-{}^{-}8}$
$=0.9176\times10^{14}$
$=9.176\times10^{13}$