Introduction to Linear Relationships

If a parent has a child on her 30th birthday, there will always be a mathematical relationship between their ages. The parent's age will always the 30 more than the child's age. When the child turns 1 the parent will turn 31, when the child turns 2 the parent will turn 32, when the child turns 3 the parent will turn 33 ….

A pattern is established which often means that the situation can be expressed in a mathematical way.

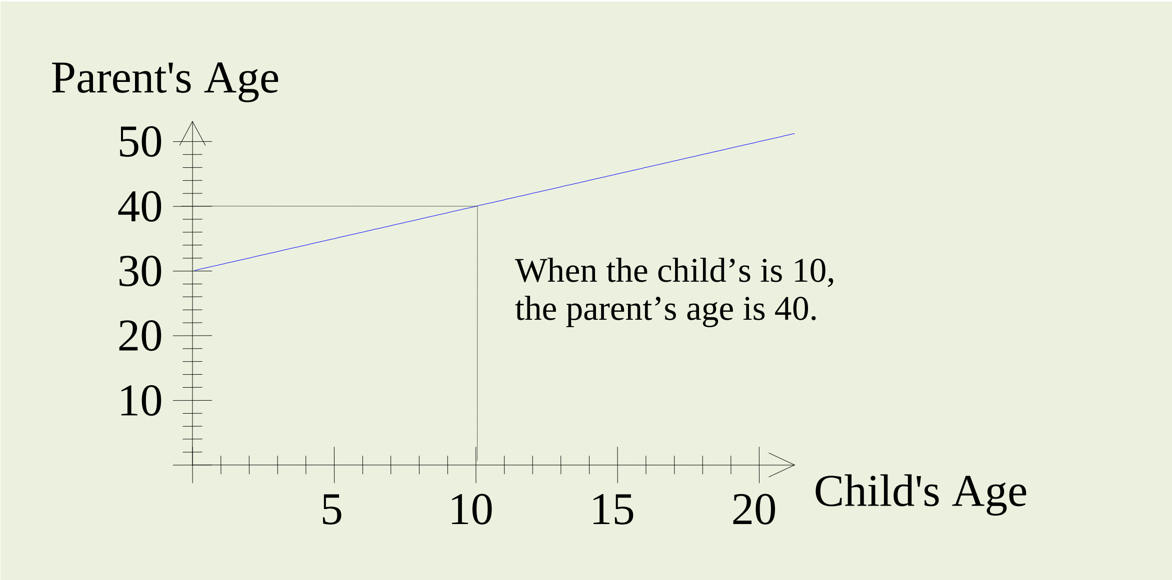
Written in a more general way:

Parent's Age = Child's Age + 30

This can be simplified to an equation:

$P = C + 30$
where P is the parent's age
and C is the child's age.

This relationship is called a Linear Relationship because the graph of the data will be a straight line.



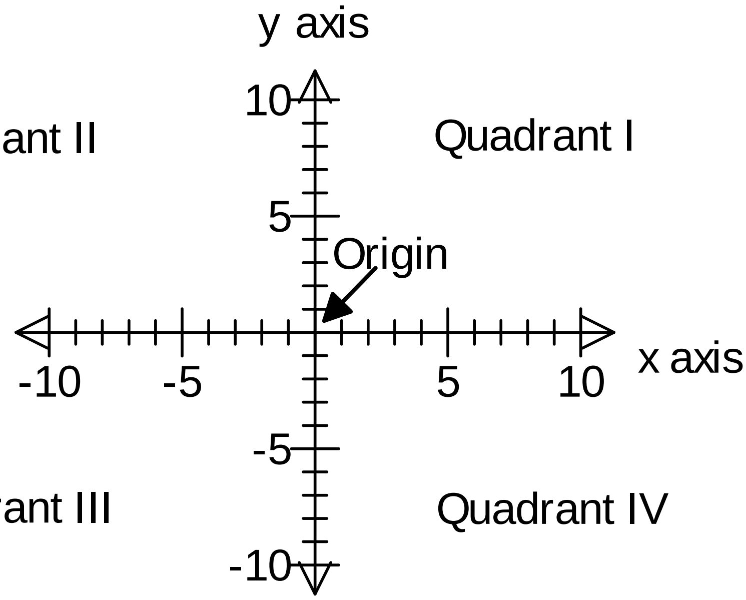
The written description, equation and graph are all describing this situation but in slightly different ways.

Revision of graphs: Axes, Quadrants and Points

The drawing below shows a **rectangular coordinate system** or **Cartesian Plane**. The plane (2 dimensional) is formed by two number lines crossing at right angles. The horizontal axis is called the ***x*-axis** unless given another pronumeral. The vertical axis is called the ***y* axis** unless given another pronumeral. The point where the axes intersect is called the **origin**.

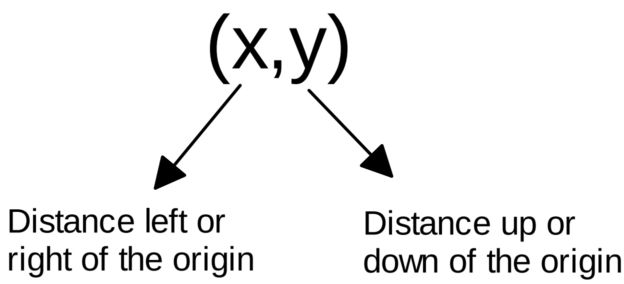
Each axis includes both positive and negative values, although for some graphs the use of negative values is unnecessary. The rectangular coordinate system contains four quadrants, these are numbered I to IV, starting in the top right quadrant, moving anti-clockwise.

Each axis must have a constant scale; however, the scale used on the *x*-axis can be different to the scale used on the *y*-axis.

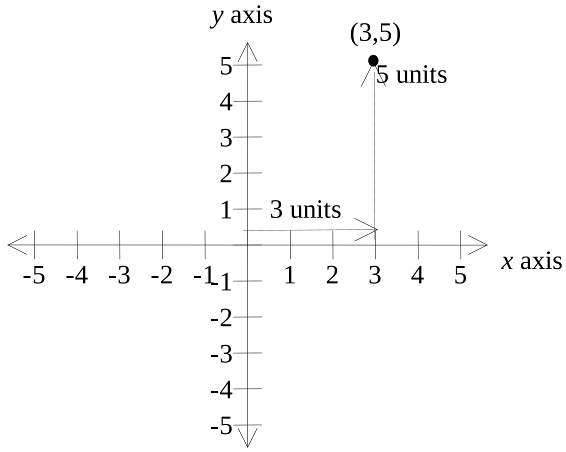


Points are located using an ordered pair, written in the form (*x*,*y*).

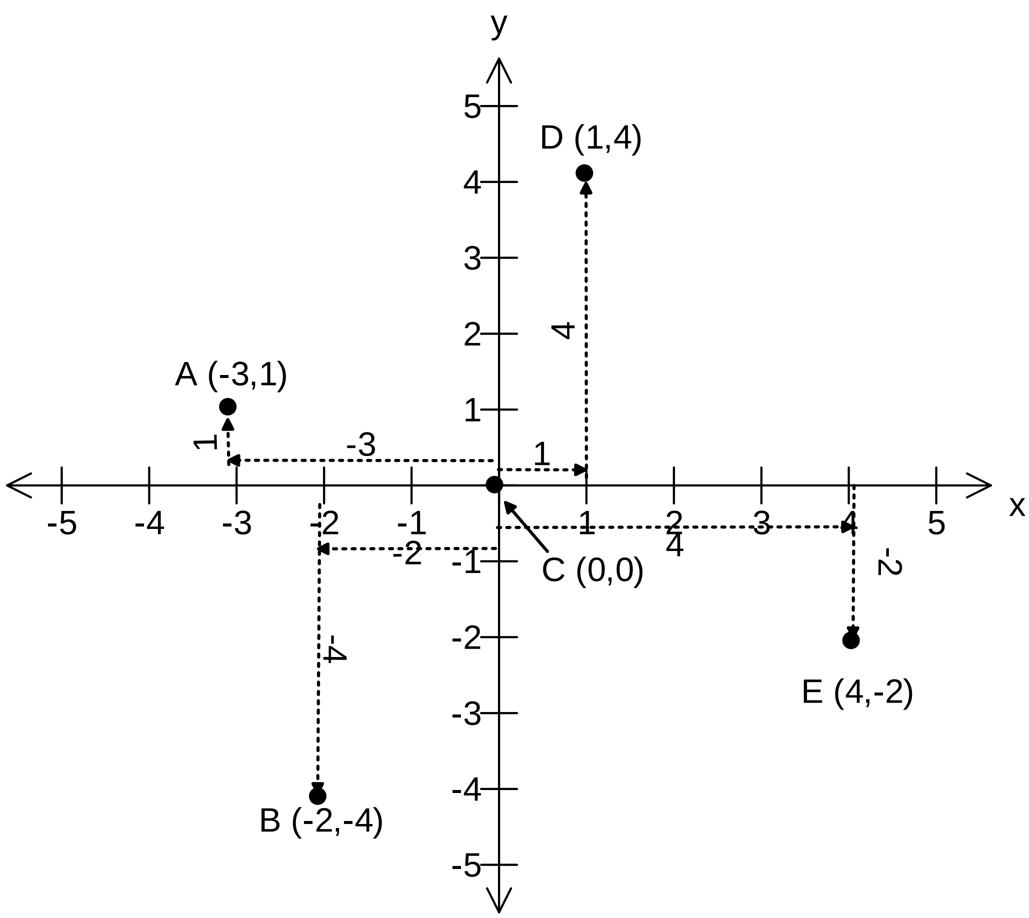
The first value *x*, refers to the distance along the *x*-axis, and the second value *y*, refers to the distance along the *y* axis. The direction moved along each axis depends upon whether the number is positive or negative.



The diagram below shows how to locate the point (3 , 5)



The diagram below shows how to locate the points: A(-3,1), B(-2,-4), C(0,0) ‘the origin’, D(1,4) and E(4,-2).



[Video ‘Revision of Co-ordinates](#)

Module contents

Introduction

- **Examples of linear relationships**
- **Graphing lines**
- **Finding equations of lines**
- **Lines of best fit – pen & paper methods**
- **Lines of best fit – regression (excel)**
- **Applications – break even analysis**

Answers to activity questions

Outcomes

- To describe relationships using graphs and equations.
- To construct graphs and determine equations.
- To find equations for lines of best fit.
- Use Excel to determine equations of regression lines.

Check your skills

This module covers the following concepts, if you can successfully answer these questions, you do not need to do this module. Check your answers from the answer section at the end of the module.

1. Draw the graph and write an equation to represent this story. "A tank holds 100 litres of water. Unfortunately the tap leaks at 10 litres per day. Assuming the tank is full now, what will the volume left at any time?"

2. Plot the graph $y = 4x - 3: \;\; -5 \leq x \leq 5$

3. Find the equation of the line that passes through (100,1000) and (500, 200)

4. Find the equation of the line of best fit by a pen and paper method for the data below:

d (days of growth)	0	3	5	8	12	15
h (height of plant in cm)	4	7.2	9.1	12.5	17	18.8

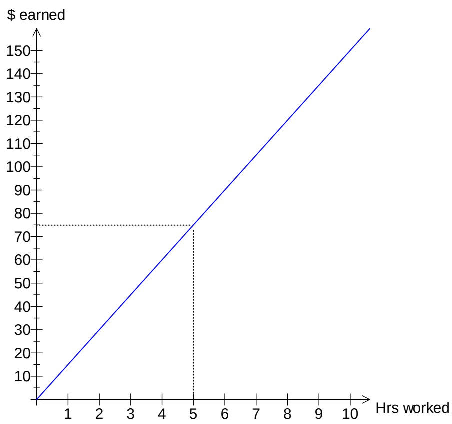
5. Use regression (calculator or formulas) to find the regression equation for the data from question 4.

6. If a farmer sells punnets of strawberries for $2. His variable cost is 50c per punnet and the fixed costs are $10 000, how many punnets will he need to sell to break-even?

Topic 1: Examples of Linear Relationships

Relationships between quantities give us another way of looking at the world. The relationship can be in the form of a picture (graph), an equation or a written statement (story). The graph and equation gives us the flexibility to predict values not given in the original information.

Example:

<table>
<tr><th>Story</th><th>Equation</th><th>Graph</th></tr>
<tr><td>Kylie earns $15 per hour working at a café.</td><td>$A = 15h$ - where A is the amount earned and h is the number of hours worked.</td><td>
</td></tr>
</table>

Kylie earns $15 per hour,

In 0 hours, Kylie will earn $15x0=$0, this is plotted on the graph as (0,0)
In 1 hour, Kylie will earn $15x1=$15, this is plotted on the graph as (1,15)
In 2 hours, Kylie will earn $15x2=$30, this is plotted on the graph as (2,30)
In 3 hours, Kylie will earn $15x3=$45, this is plotted on the graph as (3,45)
In 4 hours, Kylie will earn $15x4=$60, this is plotted on the graph as (4,60)

In h hours, Kylie will earn $15x$h$ = 15h

Using this idea, the equation becomes $A = 15h$

Having derived the equation and graph,
it is possible to determine new information about this situation.

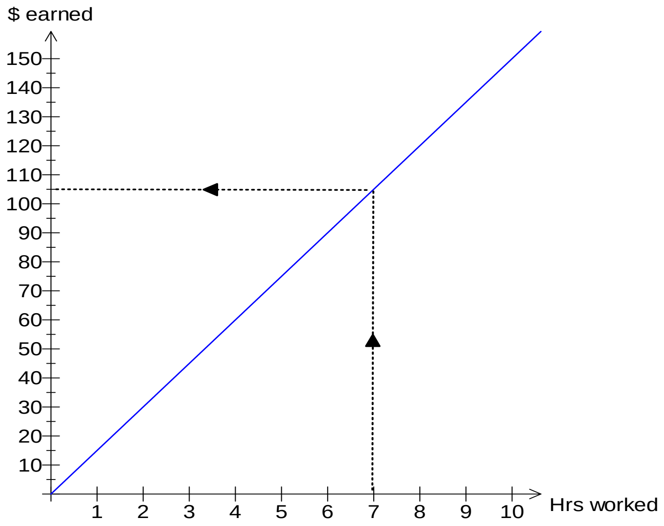
How much will Kylie earn if she works for 7 hours?

- Using the equation: working for 7 hours means $h=7$

$$A=15h$$
$$A=15\times 7$$
$$A=\$105$$

After 7 hours work, Kylies earns $105

- Using the graph: Locate 7 hours on the horizontal axis, make a dashed line moving vertically until the line of the equation is reached, then move horizontally until the value is obtained from the vertical axis.



After 7 hours, Kylie earns $105

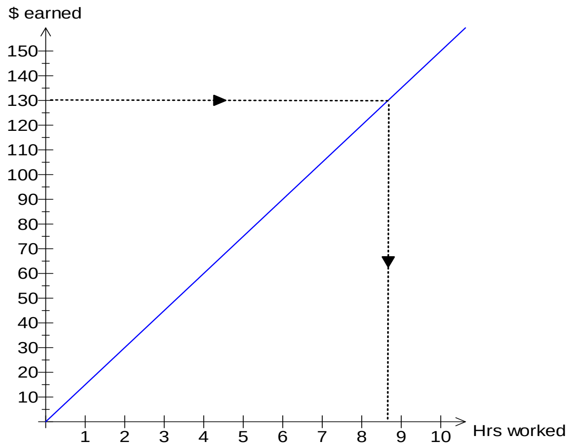
How long will it take Kylie to earn $130?

- Using the equation: The amount earned (A) = 130

$$A=15h$$
$$130=15h \ \left(\text{divide both sides by } 15\right)$$
$$\frac{130}{15}=h$$
$$h=8.6\overline{6}\ hours\left(\text{or 8hrs 40 mins}\right)$$

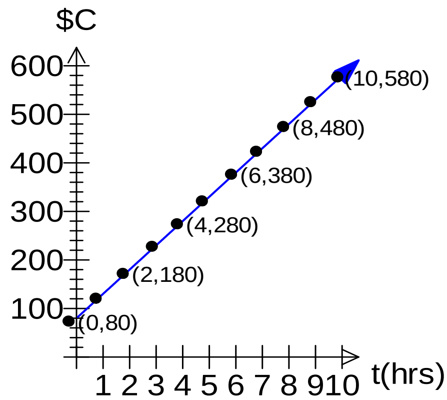
Kylie has to work for 8 hrs and 40 mins to earn $130.

- Using the graph: Locate $130 on the vertical axis, make a dashed line moving horizontally (to the right) until the line of the equation is reached and then move vertically (down) until the value is obtained from the horizontal axis.



From the graph, it is possible to say that Kylie should work somewhere between $8^1/_2$ and 9 hours to earn $130. The accuracy of the answer will depend upon how well the graph is drawn.

Example:

Story	Equation	Graph
A plumber charges a call out fee of $80 plus $50 per hour for work done.	$C = 50t + 80$ where C is the total cost and t is the time worked in hours.	

The call out fee of $80 is a set value. The cost of work done is found by multiplying the hourly rate ($50) by the number of hours worked. So the total cost of the plumber's charges is given by the equation: $C = 50t + 80$.

The graph is obtained by considering different times and calculating the charge.

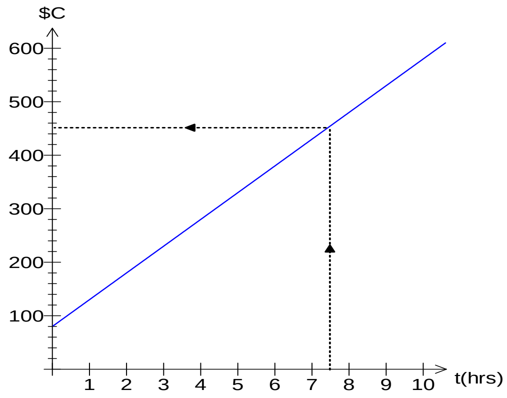
How much will the plumber charge for 7 hours 30 minutes work?

- Using the equation: the time must be expressed in hours, so t = 7.5hrs

$$C = 50t + 80$$
$$C = 50 \times 7.5 + 80$$
$$C = \$455$$

After 7 hours 30 mins, the plumber will charge $455.

- Using the graph: Locate 7.5hrs on the horizontal axis. Make a dashed line moving vertically (up) until the line of the equation is reached, then move horizontally (left) until the value is obtained from the vertical axis.



The charge is approximately $450, accuracy is limited by the size of the graph.

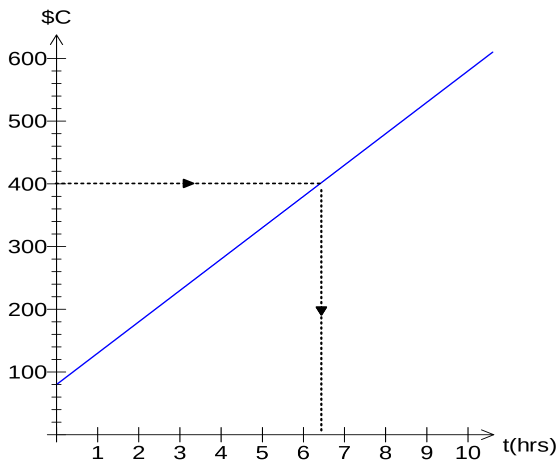
How many hours work can be carried out for $400?

- Using the equation: the charge is $400, so C=400

$$C = 50t + 80$$
$$400 = 50t + 80 \quad \text{subtract 80 from both sides}$$
$$320 = 50t \quad \text{divide both sides by 50}$$
$$6.4 = t$$

The plumber will work for 6.4 hours or 6 hrs 24min for $400

- Using the graph: Locate $400 on the vertical axis, make a dashed line moving horizontally (to the right) until the line of the equation is reached and then move vertically (down) until the value is obtained from the horizontal axis.



From the graph, it is possible to say that the plumber should work about 6.5 hours to earn $400. The accuracy of the answer is limited by the size and accuracy of the graph.

'Examples of Linear Relationships'

Activity

In questions 1 to 7, write an equation that describes each situation.

1. The amount earned *(A)* after *w* weeks if Garry earn $850 per week.

2. The revenue *(R)* from selling *q* calculators at $25 each.

3. The cost (C) of producing n badges if blank badges cost $1 each and the machine to make badges costs $150.

4. Sally uses her $650 tax refund cheque to start a holiday savings account and adds $100 per week.

5. A retiree has savings of $150 000. She uses $ 15 000 per year.

6. The daily cost of renting a car is made up of $50 hire fee plus 25 cents per kilometre.

7. A car costing $15 000 depreciates by $1200 per year.

8. Saline solution is being infused to a patient using IV drip. The amount of solution remaining (in mL) after *h* hours is given by the equation:

$$A = 1000 - 80h$$

(a) What amount is remaining after 3 hours?
(b) After how many hours will there be 500mL remaining?
(c) Explain the significance of the 80 in the equation.

9. The weight in kilograms of a truck loaded with *n* bales of wool is given by the equation:

$$W = 150n + 4500$$

(a) What is the weight of one bale of wool?
(b) If the maximum load is 20 bales, what will the total weight of the truck be?
(c) Complete the table below:

n	0	5	10	15	20
W					

(d) If a bridge has a load rating of only 5½ tonnes (5500kg), how many bales can the truck carry without damage to the bridge?

10. Julie rents a flat. She pays a bond of $1200 plus $250 per week.

(a) Complete this table

Weeks rented	0	1	2	3	4	5
Total Amount Paid						

(b) Which equation best describes this situation.

(i) $T = 1200w + 250$
(ii) $T = 250w + 1200$
(ii) $T = 1200 - 250w$
(iv) $T = \dfrac{1200 - w}{250}$

(c) Draw a graph based on the table from part (a).

Topic 2: Graphing lines

When graphing lines, the equation may be expressed in different forms:

Form	Example	
$y = mx + c$	$y = 2x + 5$	This is not applied to any story; it is an example of a mathematical linear equation.
	$C = 30q + 100$	This is in the same form as above but applied to a situation. This is a cost equation based on the quantity being made.
$Ax + By + c = 0$	$3x - 2y + 6 = 0$	This is called the general form of a linear equation, it is basically the same as the other form but just rearranged.

There are two methods for graphing lines:

1. Plotting – this involves constructing a table and substituting in values for *x*. The values of *x* used are based on the use of the equation. If the equation is just a mathematical equation, *x* values are whole numbers such as 0, 3, 6, etc. can be used. If the equation is applied to a context, the *x* values could be values such as 0, 100, 200, 300 etc.
2. Intercept method- this involves finding the points where the graph cuts the *x*-axis (called the *x*-intercept) and *y*-axis (called the *y*-intercept). This method will not work if the line passes through the origin as the *x*-intercept and the *y*-intercept are the same. Choosing another value for *x* will solve this problem.

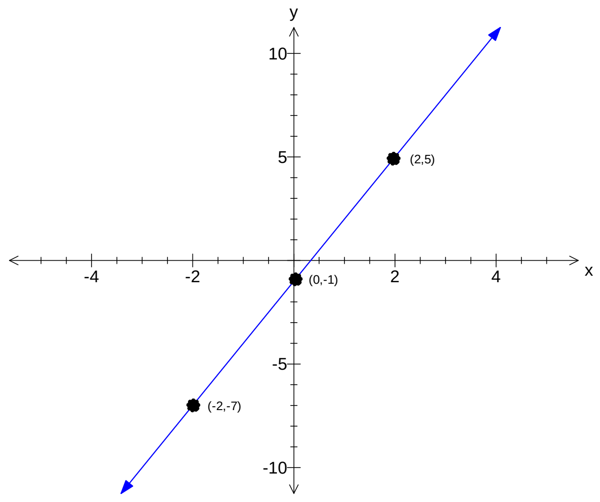
Plotting Graphs

Example: Plot the graph of $y = 3x - 1$

For this question the values selected for *x* are -2, 0, 2 .Three values for *x* give a checking mechanism because if the points do not line up then a calculation error has occurred.

x	-2	0	2
y (substitute in values of x)	$y = 3x - 1$ $y = 3\times{}^{-}2 - 1$ $y = -7$	$y = 3x - 1$ $y = 3\times 0 - 1$ $y = -1$	$y = 3x - 1$ $y = 3\times 2 - 1$ $y = 5$
Point *(x,y)*	(-2,-7)	(0,-1)	(2,5)

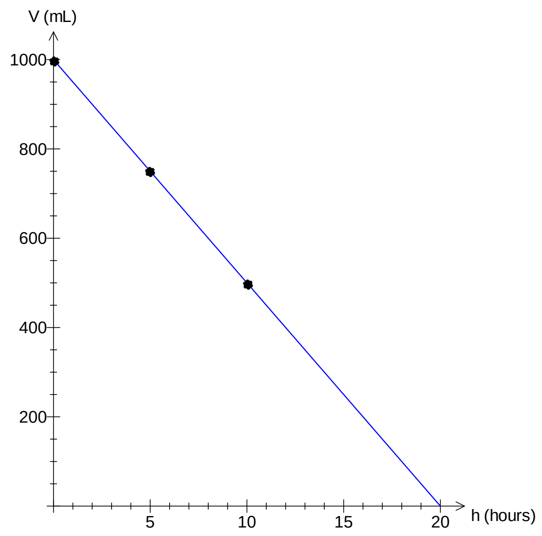
Gives the graph:



Example: Plot the graph of the equation $V = 1000 - 50h$ *or* $V = -50h + 1000$, where V is the volume of saline solution left in a bag after h hours (Nursing).

For this question the values selected for h are 0, 5, 10 as negative values of h are not possible. Also negative values of V are not possible. This means values for h (time) must be chosen carefully.

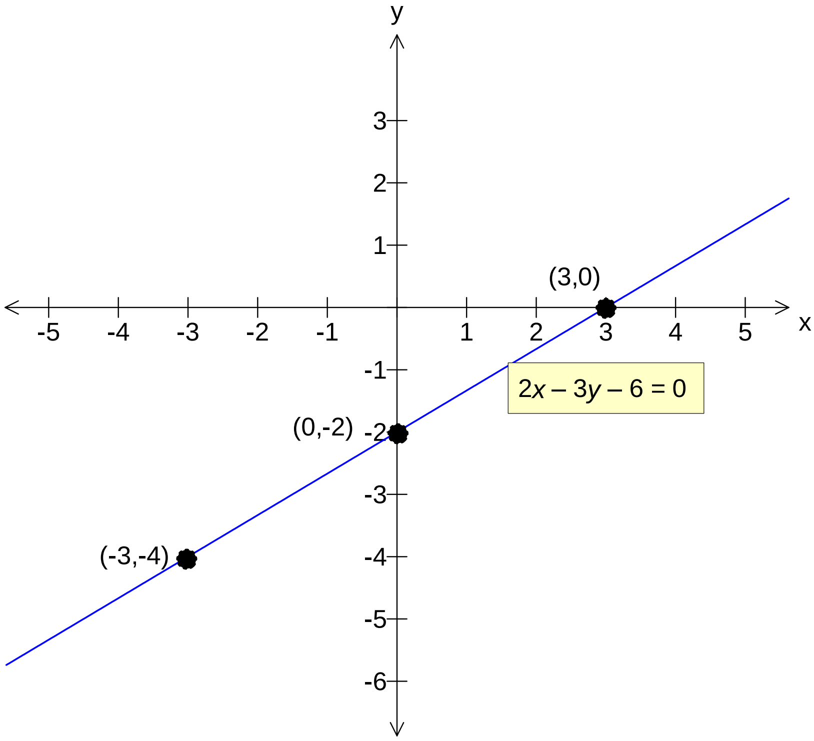
h	0	5	10
V	$V = 1000 - 50h$ $V = 1000 - 50 \times 0$ $V = 1000$	$V = 1000 - 50h$ $V = 1000 - 50 \times 5$ $V = 750$	$V = 1000 - 50h$ $V = 1000 - 50 \times 10$ $V = 500$
Point *(h,V)*	(0,1000)	(5,750)	(10,500)



Example: Plot the graph of the equation $2x - 3y - 6 = 0$

In this question the values selected for *x* are -3, 0, 3. Remember, values for *x* are selected to find values of *y*. The actual values selected can be anything relevant to the situation. Because this is not an applied question, so the *x* values can be anything.

x	-3	0	3
y	$2x-3y-6=0$ $2(-3)-3y-6=0$ $-6-3y-6=0$ $-3y-12=0$ $-3y=12$ $y=\frac{12}{-3}$ $y=-4$	$2x-3y-6=0$ $2(0)-3y-6=0$ $-3y-6=0$ $-3y=6$ $y=\frac{6}{-3}$ $y=-2$	$2x-3y-6=0$ $2(3)-3y-6=0$ $-3y=0$ $y=0$
Point *(x,y)*	(-3,-4)	(0,-2)	(3,0)



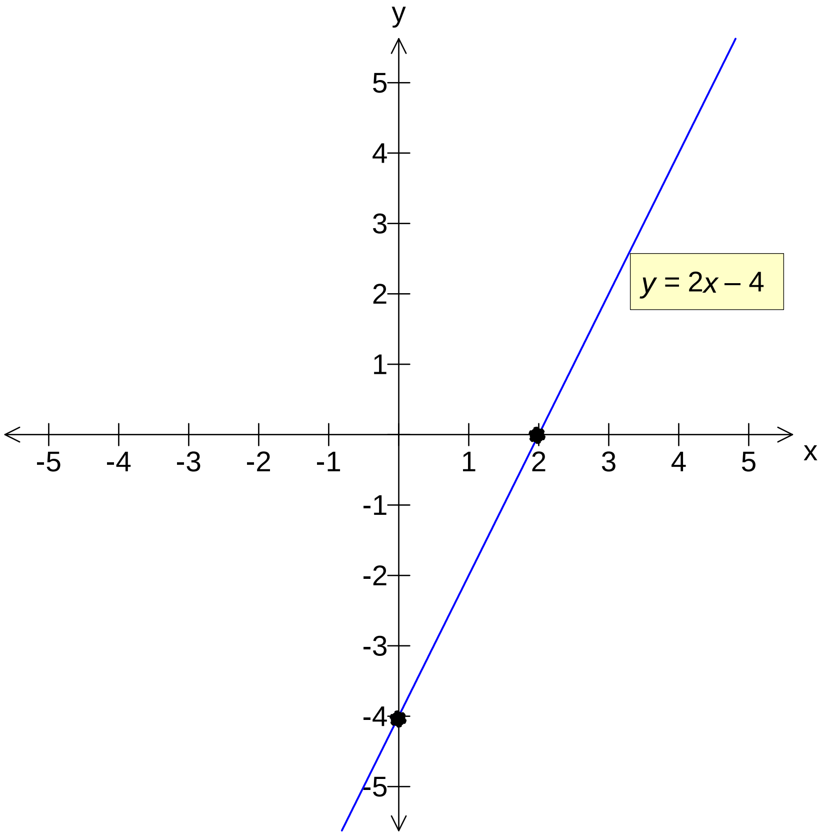
Intercept Method

This method involves finding the point where the graph cuts the *x* axis, called the *x* – intercept, and the point where the graph cuts the *y* axis, called the *y* intercept.

> Key Idea:
> At the *x*-intercept on the graph, the value of *y* will be 0. Likewise, at the *y*-intercept on the graph, the value of *x* will be 0.

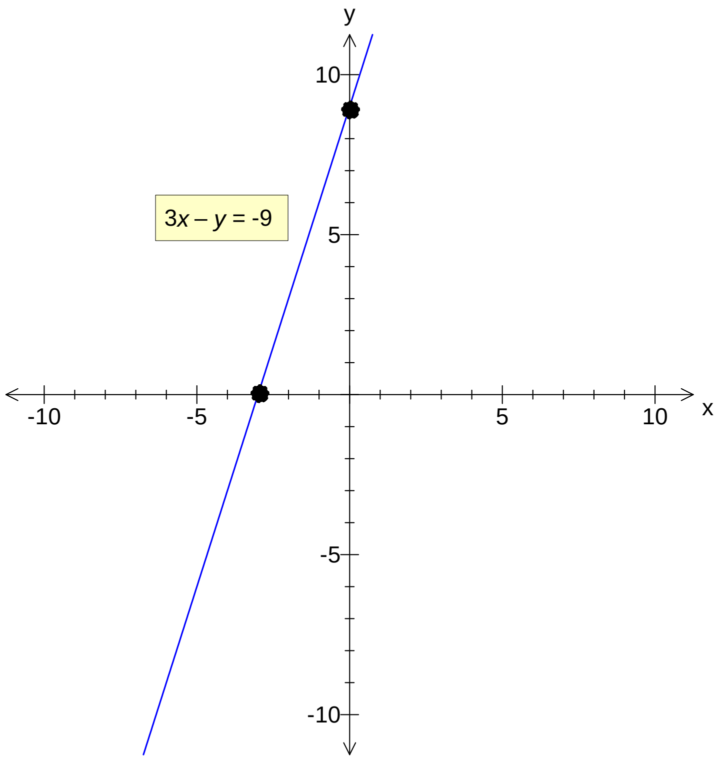
Example: Plot the graph of $y = 2x - 4$

When *x=0*	$y = 2x - 4$ $y = 2(0) - 4$ $y = -4$	(0,-4)
When *y=0*	$y = 2x - 4$ $0 = 2x - 4$ add 4 to both sides $4 = 2x$ divide both sides by 2 $2 = x$ or $x = 2$	(2,0)



Example: Plot the graph of the equation $3x - y = -9$

When $x=0$	$3x - y = -9$ $3(0) - y = -9$ $-y = -9$ $y = 9$	(0,9)
When $y=0$	$3x - y = -9$ $3x - 0 = -9$ $3x = -9$ $x = -3$	(-3,0)



The example below shows the problem associated with this method.

Example

If goods are sold for $5 each, the revenue equation is written as $R = 5q$, where R is the revenue from selling q items.

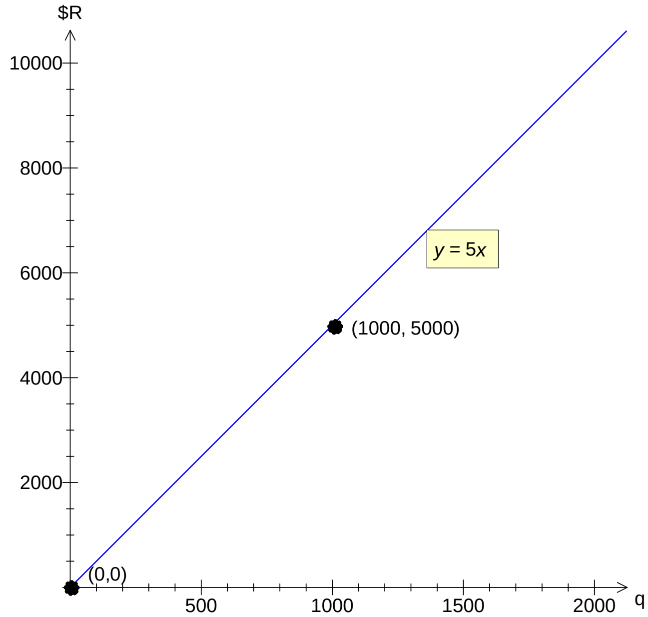
The maximum number sold is about 2000.

The number sold is positive so the graph only exists in quadrant 1.

When $q=0$	$R=5q$ $R=5(0)$ $R=0$	(0,0)
When $R=0$	$R=5q$ $0=5q$ $0=q$	(0,0)

The points obtained are the same so the line cannot be drawn until a second point is found. A value of q is substituted into the equation to obtain a second point.

When q = 1000	$R=5\times1000$ $R=5000$	(1000,5000)



Two Special Lines

The equation for Horizontal Lines has the form $y=c$.

Example

The line with equation $y=4$ is a horizontal line. The equation is simply stating that for every point on the line will have a y coordinate of 4. Points include:

(-5,4) (-1,4) (0,4) (2,4) (4,4) (5,4) (10,4) etc…

The line with equation $y={}^{-}1$ is a horizontal line. Every point on the line will have a y coordinate of -1. Points include:

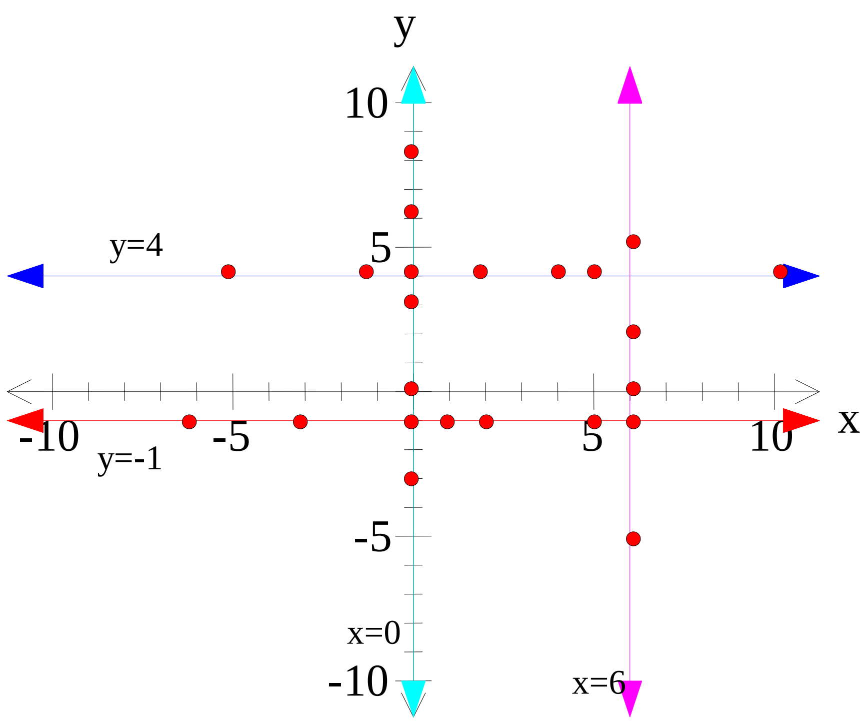
(-6,-1) (-3,-1) (0,-1) (1,-1) (2,-1) (5,-1) etc.…

The equation for Vertical Lines has the form $x=a$.

Example

The line with equation $x = 6$ is a vertical line. The equation is simply stating that for every point on the line will have a x coordinate of 6. Points include: (say)
(6,-5) (6,-1) (6,0) (6,2) (6,4) (6,5) (6,10) etc…

The line with equation $x = 0$ is a vertical line, in fact, this line is the y axis. Every point on the line will have a x coordinate of 0. Points include: (say)
(0,-3) (0,-1) (0,0) (0,4) (0,6) (0,8) etc….




[Video ‘Graphing Lines’]

Activity

1. Draw the following graphs. Use different methods

(a)	$y = 2x - 1$	(b)	$y = 5 - x$	(c)	$y = 4x$
(d)	$y = 4$	(e)	$x = -6$	(f)	$R = 25q$
(g)	$C = 5q + 1000$	(h)	$H = 0.2t - 0.5$	(i)	$A = 50H + 100$

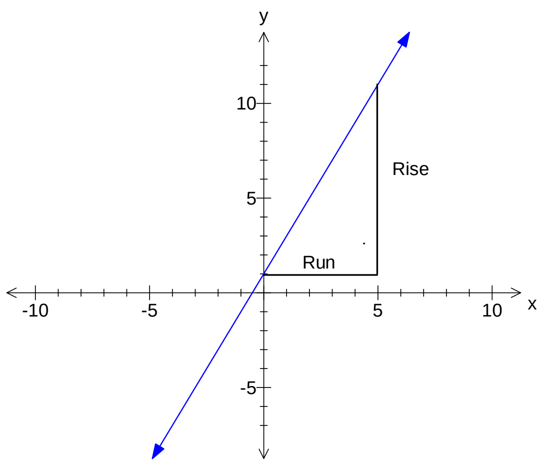
Topic 3: Finding Equations of Lines

In the module so far, a graph has been drawn using the equation. In this section information from a graph is used to determine the equation that best describes the situation.

The equation of a straight line has the general form $y = mx + c$, where m represents the slope (or gradient) of the line, and c represents the y intercept (see topic 2).

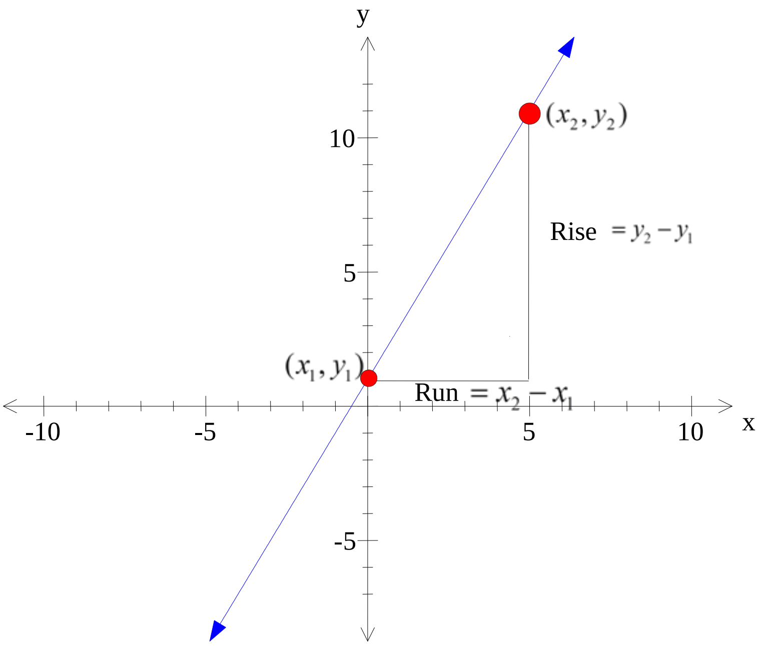
Calculating the Slope

The slope (or steepness) of the line is found using the formula: $slope = \frac{rise}{run}$.



Extending the formula: $slope = \frac{rise}{run} = \frac{\text{vertical distance}}{\text{horizontal distance}}$

If two points (x_1, y_1) and (x_2, y_2) are placed on the line, then the formula changes to

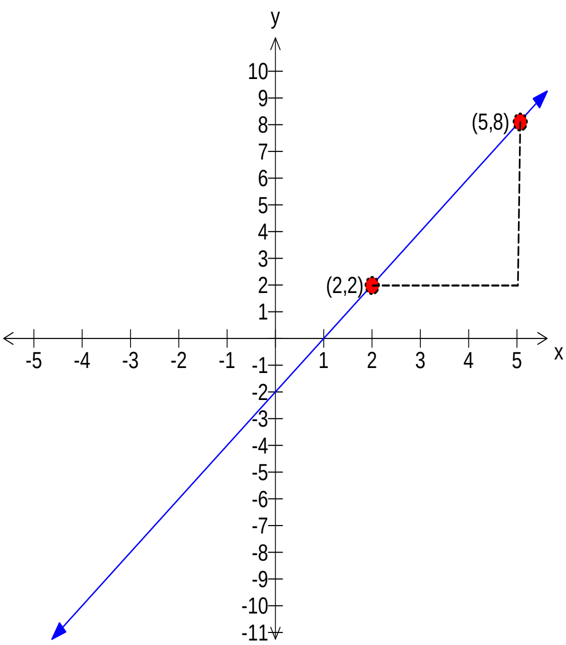


$$slope = \frac{rise}{run} = \frac{\text{vertical distance}}{\text{horizontal distance}}$$

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

Examples:

Find the slope of the line passing through the points (2,2) and (5,8).



$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

$$slope = \frac{8-2}{5-2}$$

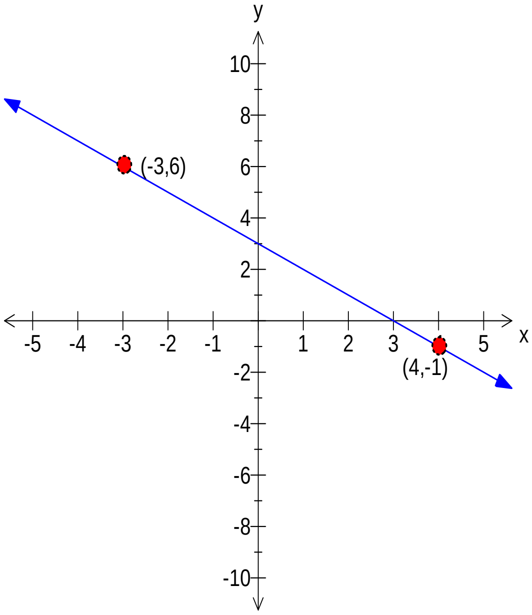
$$slope = \frac{6}{3}$$

$$slope = 2$$

This means that for every 1 increase in x, there will be a 2 increase in y.

A rising line has a positive slope.

Find the slope of the line passing through the points (-3,6) and (4,-1).



$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

$$slope = \frac{6 - {}^{-}1}{-3 - 4}$$

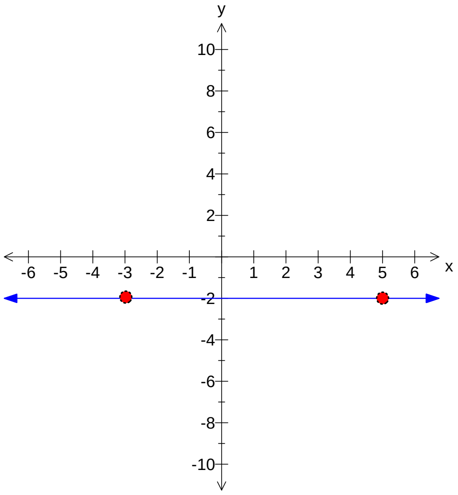
$$slope = \frac{7}{-7}$$

$$slope = -1$$

This means that for every 1 increase in x, there will be a 1 decrease in y.

A falling line has a negative slope.

Find the slope of the line passing through the points (-3,-2) and (5,-2).



$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

$$slope = \frac{{}^{-}2 - {}^{-}2}{5 - {}^{-}3}$$

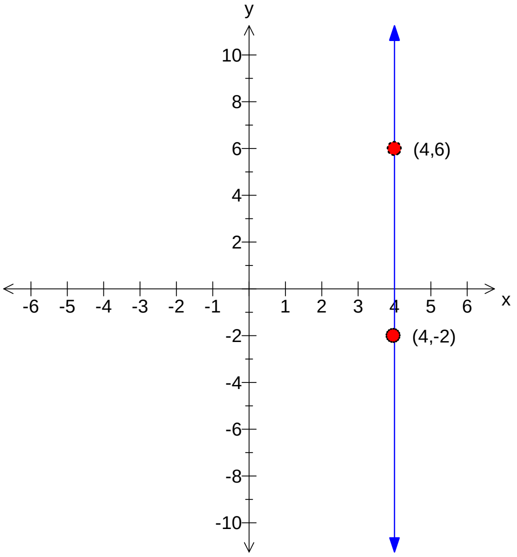
$$slope = \frac{0}{8}$$

$$slope = 0$$

This means that the line has a zero slope, it is a horizontal line, it does not rise or fall.

A horizontal line has a slope of zero.

Find the slope of the line passing through the points (4,6) and (4,-2).



$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

$$slope = \frac{6 - {}^{-}2}{4 - 4}$$

$$slope = \frac{8}{0}$$

$$slope = \text{undefined}$$

This means that the line has a undefined slope (division by 0), it is a vertical line, it does not run.

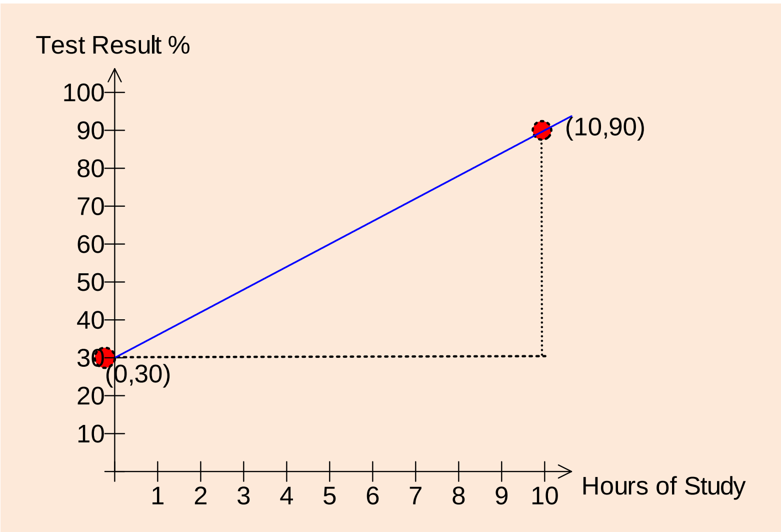
A vertical line has an undefined slope.

What is the Significance of the Slope

If a graph has a slope of 200, explain the significance of this?

Because slope is the amount of rise for a run of 1, this graph increases *y* by 200 for every 1 increase in *x*.

The graph below shows the Test Results of a class of students based on the Hours of Study prior to the test. The points (0,30) and (10,90) lie on the extremes of the line.



$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

$$slope = \frac{90 - 30}{10 - 0}$$

$$slope = \frac{60}{10}$$

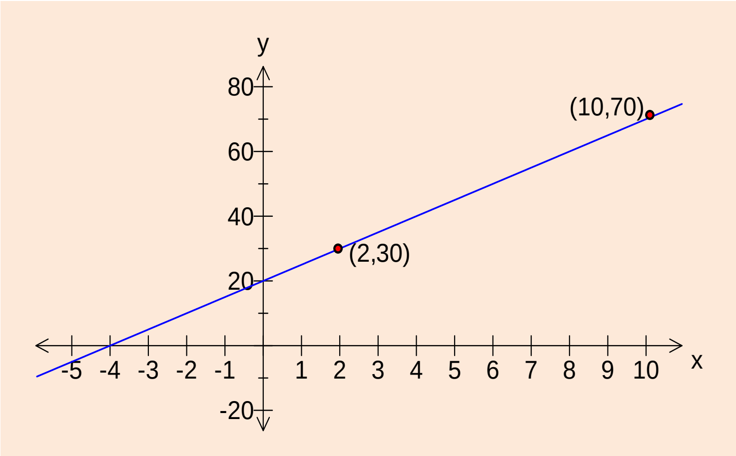
$$slope = 6$$

The graph above shows the effect of the length of time spent studying on the test result. The slope of this line is 6. This means that: for every hour spent studying there will be an increase of 6% in the test result.

Finding Equations of Lines

Example

Find the Equation of the Line in the graph with the y intercept given.



$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$
$$slope = \frac{70 - 30}{10 - 2}$$
$$slope = \frac{40}{8}$$
$$slope = 5$$

From the graph, the y-intercept is 20.

Using the general form of the equation $y = mx + c$, the equation is $y = 5x + 20$.

Example

Find the equation of the line that passes through (2,20) and (5,8). If the y-intercept is unknown, the process has an extra step.

$$(2,20) \rightarrow (x_2, y_2)$$
$$(5,8) \rightarrow (x_1, y_1)$$

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$
$$slope = \frac{20 - 8}{2 - 5}$$
$$slope = \frac{12}{^{-}3}$$
$$slope = {}^{-}4$$

So far the equation is $y = {}^{-}4x + c$

One of the two points can be used to find the y-intercept. Substituting in a point's x and y values will enable c to be calculated. The point (2,20) will be used.

Using (2,20)

$$y = mx + c$$
$$20 = {}^{-}4 \times 2 + c$$
$$20 = {}^{-}8 + c$$
$$20 + 8 = c$$
$$28 = c$$

The equation is $y = {}^{-}4x + 28$ or $y = 28 - 4x$

Example

A business manufactures an orange flavoured chocolate bar. If they sell 1000 bars they make a weekly profit of $385, if they sell 2000 bars they make a weekly profit of $1285.

Determine the equation of the profit based on the number sold.

Let the x variable be n the number sold. Let the y variable be P the profit made. The two points are (1000,385) and (2000,1285)	$slope = \frac{y_2 - y_1}{x_2 - x_1}$ $slope = \frac{1285 - 385}{2000 - 1000}$ $slope = \frac{900}{1000}$ $slope = 0.9$

So far the equation is $P = 0.9n + c$

One of the two points can be used to find the P (y)-intercept. Substituting in a point's n and P values will enable c to be calculated. The point (1000, 385) will be used.

Using (1000,385)	$P = 0.9n + c$ $385 = 0.9 \times 1000 + c$ $385 = 900 + c$ $385 - 900 = c$ $^{-}515 = c$

The equation is $P = 0.9n - 515$

Once an equation is found, questions about the context of the equation can be answered and a graph drawn.

Example

On the first visit, a plumber charged \$450 for 5 hours work. On the second visit, the same plumber charged \$210 for 2 hours work.

(a) Determine the equation that represents his fee structure.
(b) Draw a graph of the equation.
(c) Comment on the significance of the slope and y intercept.
(d) What would the charge be for 6.5 hours work?
(e) How long did the plumber work for if the fee charged was \$890?

(a) The two points are (5,450) and (2,210)

Let the x variable be h the number of hours worked. Let the y variable be F the fee charged.	$slope = \frac{y_2 - y_1}{x_2 - x_1}$ $slope = \frac{450 - 210}{5 - 2}$ $slope = \frac{240}{3}$ $slope = 80$

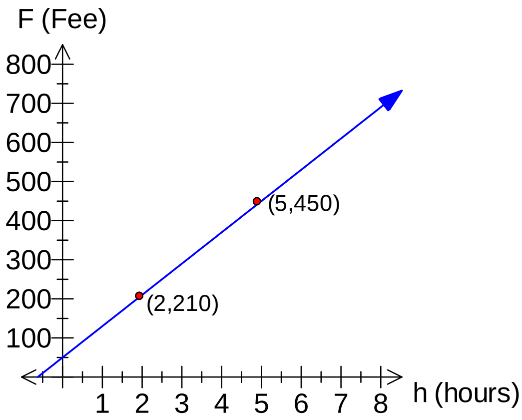
So far the equation is $F = 80h + c$

One of the two points can be used to find the *P* (*y*)-intercept. Substituting in the point *h* and *F* values enable *c* to be calculated.

Using (1000,385)	$\begin{aligned} F &= mh + c \\ 450 &= 80 \times 5 + c \\ 450 &= 400 + c \\ 450 - 400 &= c \\ 50 &= c \end{aligned}$

The equation is $F = 80h + 50$

(b) Using the two points, the graph can be plotted.



(c) The slope ($80) represents the change in cost for each hour, this is the hourly rate - $80/hr. The y intercept ($50) represents a set amount; this is the call-out fee.

(d) Using the equation, the cost for 6.5 hours work is:

$$\begin{aligned} F &= 80h + 50 \\ F &= 80 \times 6.5 + 50 \\ F &= 570 \end{aligned}$$

The fee for 6.5 hours work is $570

(e) Using the equation, for $890 the plumber will work for:

$$\begin{aligned} F &= 80h + 50 \\ 890 &= 80h + 50 \\ 890 - 50 &= 80h \\ 840 &= 80h \\ \frac{840}{80} &= h \\ 10.5 &= h \end{aligned}$$

The plumber will work for 10.5 hours.

Video ‘Finding Equations of Lines’

Activity

1. Find the equation of the line given the information below

(a)	Slope=2 and y intercept = -10	(b)	Slope = -1 and passes through (5,1)	(c)	(-5,2) with y intercept 4.
(d)	(3,6) and (2,3)	(e)	(-3,-4) and (1,4)	(f)	(-1,-2) and (2,4)
(g)	(-4,8) and (1,-2)	(h)	(3,8) and (3,-1)	(i)	(0,6) and (9,6)

2. For each of the answers to question 1 above, complete the table below.

	Slope	Nature of Slope	Y-intercept
(a)	2	Positive	-10
(b)	-1		
(c)			
(d)			
(e)			
(f)			
(g)			
(h)			
(i)			

3. At a car-wash fund raiser, the cost equation was $C = 2.5q + 50$.

    (a) What is the cost if 50 cars were washed?
    (b) If the costs were $175, how many cars were washed?
    (c) Explain the practical significance of the slope.
    (d) Explain the practical significance of the y-intercept.

4. On the first visit, an electrician charged $180 for 2 hours work. On a second visit, the electrician charged $405 for 5 hours work.

    (a) From the information given, develop the equation $F = 75h + 30$, where F is the Fee charged for h hours work.
    (b) Explain the practical significance of the slope.
    (c) Explain the practical significance of the y-intercept.
    (d) How much would the electrician charge for three and a quarter hours work?
    (e) If the electrician charged $300, how many hours did they work for?
    (f) The electrician decides to increase the hourly rate by 4% to reflect inflation. What will be the new fee equation?

5. The number of photo-plankton found in a sample of sea water varies with the level of contaminates in the water. When the level of contaminates is 100ppm (parts per million) the number of photo- plankton is 95. When the level of contaminates increases to 500ppm, the number of photo-plankton is reduced to nil.

    (a) Which is the independent and dependent variable? (You may have to read the start of the next topic)

(b) From the information given, develop the equation. Use l for the level of contaminates and n for the number of photo-plankton.
(c) Explain why the slope is negative?
(d) Explain the practical significance of the y-intercept.
(e) If it is desirable to have a contaminate level less that 250ppm, what photo-plankton count would you expect to find?

6. In straight line depreciation, the value of an item depreciates by a set amount each year until its value is nil. The value (V) of a car after y years is given by the equation $V = 35000 - 2800y$

(a) Which is the independent and dependent variable? (You may have to read the start of the next topic)
(b) What was the original cost of the car?
(c) Explain the practical significance of the slope.
(d) Draw the graph of this equation. Use $0 \le y \le 10$
(e) Using the graph or equation, what is the value of the car after 4 years?
(f) When will the value of the car be effectively nil?

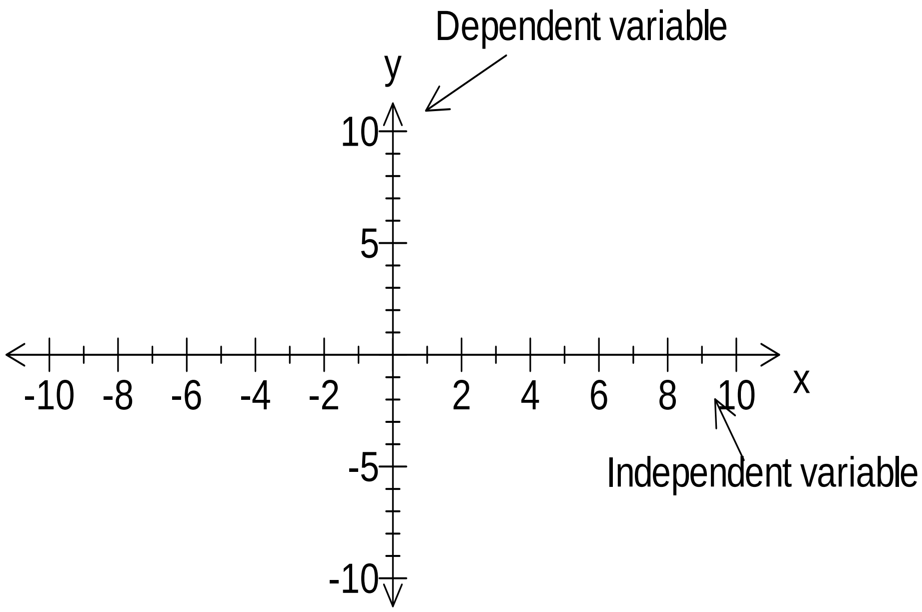
Topic 4: Lines of Best Fit – pen & paper methods

Let's use an example to illustrate the two methods to be discussed.

A shopkeeper records the daily temperature (°C) and the number of cans of soft drink sold over a 10 day period.

Daily Temp °C	18	22	19	25	29	12	8	15	20	24
Number sold	55	68	60	73	87	40	32	51	60	71

Looking at this data there appears to be a relationship between the temperature and the number of cans of soft drink sold. A better way to see if there is a relationship is to draw a graph; this type of graph is called a scatterplot. One variable is plotted on each axis but the big question is which variable should go on which axis?



In this example it appears that the temperature is affecting soft drink sales, or put another way, the number of cans of soft drink sold <u>depends</u> upon the temperature. The number of cans of soft drink is the dependent variable (y) and temperature is the independent variable (x).

This example is fairly straight forward, if in doubt, ask yourself the question and then ask it again with the variables swapped over.

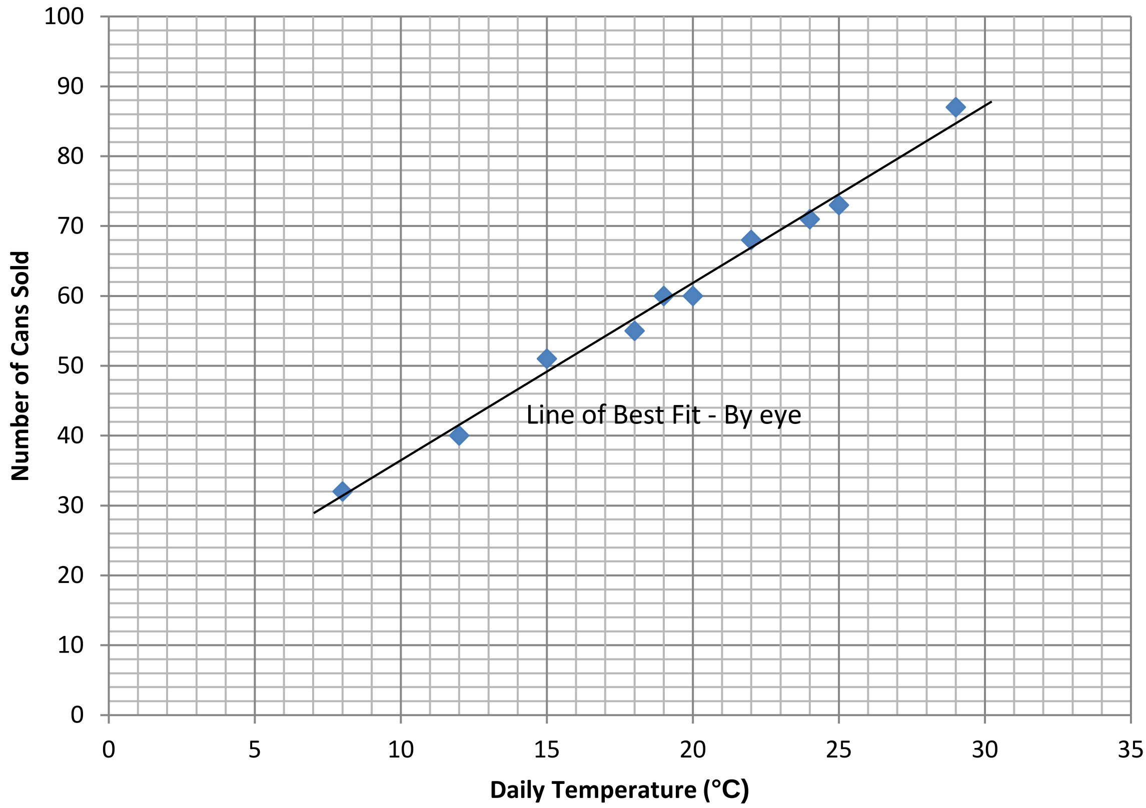
Does the number of cans of soft drink sold <u>depend</u> upon the temperature?
or

Does the temperature depend upon the number of cans of soft drink sold?

The first statement sounds much more appropriate confirming the number of cans of soft drink is the dependent variable (y) and temperature is the independent variable (x).

Another way that often helps is to consider a science experiment where the scientist controls one variable to measure another. For example, if an agricultural scientist is performing an experiment to find out the optimum amount of fertilizer to apply to maximise plant height, the amount of fertilizer applied is the independent variable and the plant height is the dependent variable.

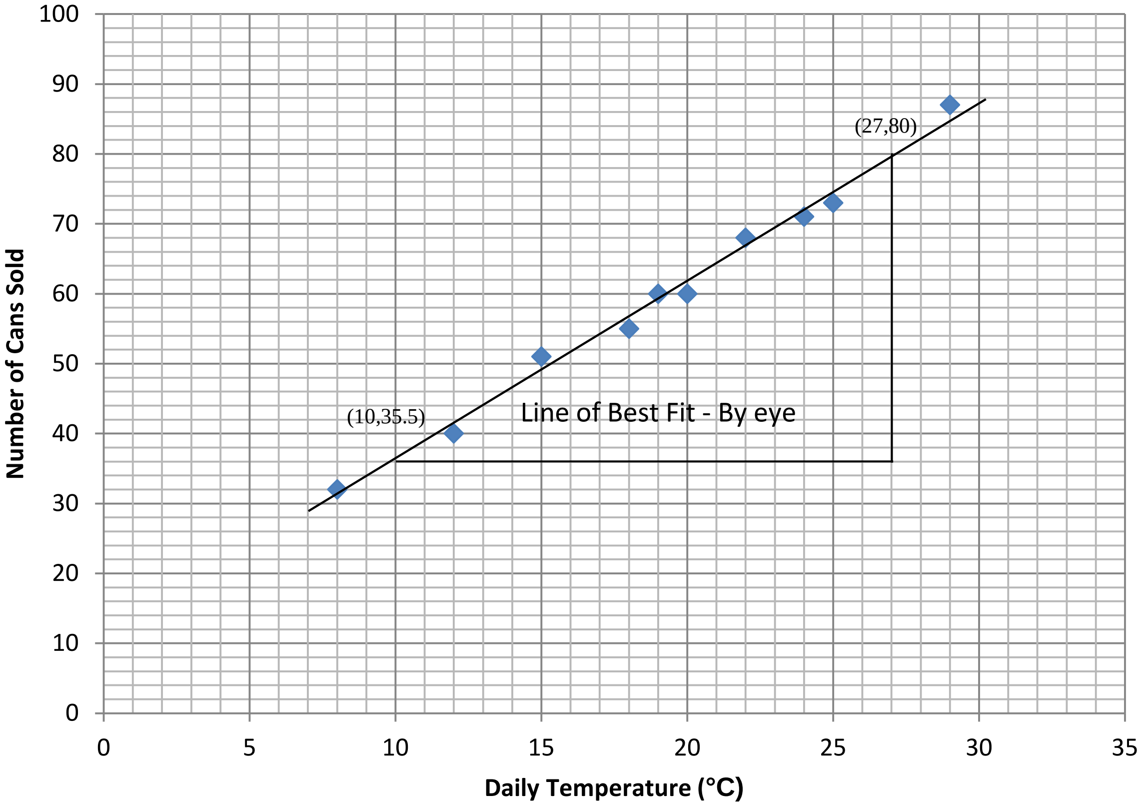
In the original example, the Daily Temperature will be on the x-axis and the Number of Cans of Soft Drink Sold will be on the y-axis. On the first day, the temperature was 18°C and the number of cans sold was 55, this gives a point (18,55). Plotting all 10 days gives the graph below.



From the scatterplot, there appears to be a linear relationship between the variables. A straight line will not run through every point but a line of best fit can be drawn. There are numerous methods to obtain a Line of Best Fit, three will be given in this module.

Finding the Line of Best Fit – By Eye

In this method you judge to most suitable line by eye. You should try to have an equal number of points above and below the line at both ends. Once the line is drawn, you can use the method outlined in the previous section ‘Finding the Equations of Lines’



Two points on the line are (10,35.5) and (27,80)	$slope = \frac{y_2 - y_1}{x_2 - x_1}$ $slope = \frac{80 - 35.5}{27 - 10}$ $slope = \frac{44.5}{17}$ $slope = 2.62$ (to 2 dp)

So far the equation is $N = 2.62T + c$. One of the two points can be used to find the *P* (*y*)-intercept. Substituting in the point *n* and *P* values enable *c* to be calculated.

Using (27,80)	$N = 2.62T + c$ $80 = 2.62 \times 27 + c$ $80 = 70.74 + c$ $80 - 70.74 = c$ $9.26 = c$

The equation, using the By Eye method, is $N = 2.62T + 9.26$

Finding the Line of Best Fit – Two Means Method

This method attempts to take some of the guess work out of locating the line of best fit.

Daily Temp °C	18	22	19	25	29	12	8	15	20	24
Number sold	55	68	60	73	87	40	32	51	60	71

The data must be put in order based on the independent variable, the daily temperature. Be careful to preserve points.

	Lower Half x Mean = 14.4					Upper Half x Mean = 24				
Daily Temp °C	8	12	15	18	19	20	22	24	25	29
Number sold	32	40	51	55	60	60	68	71	73	87
	Lower Half y Mean = 47.6					Upper Half y Mean = 71.8				

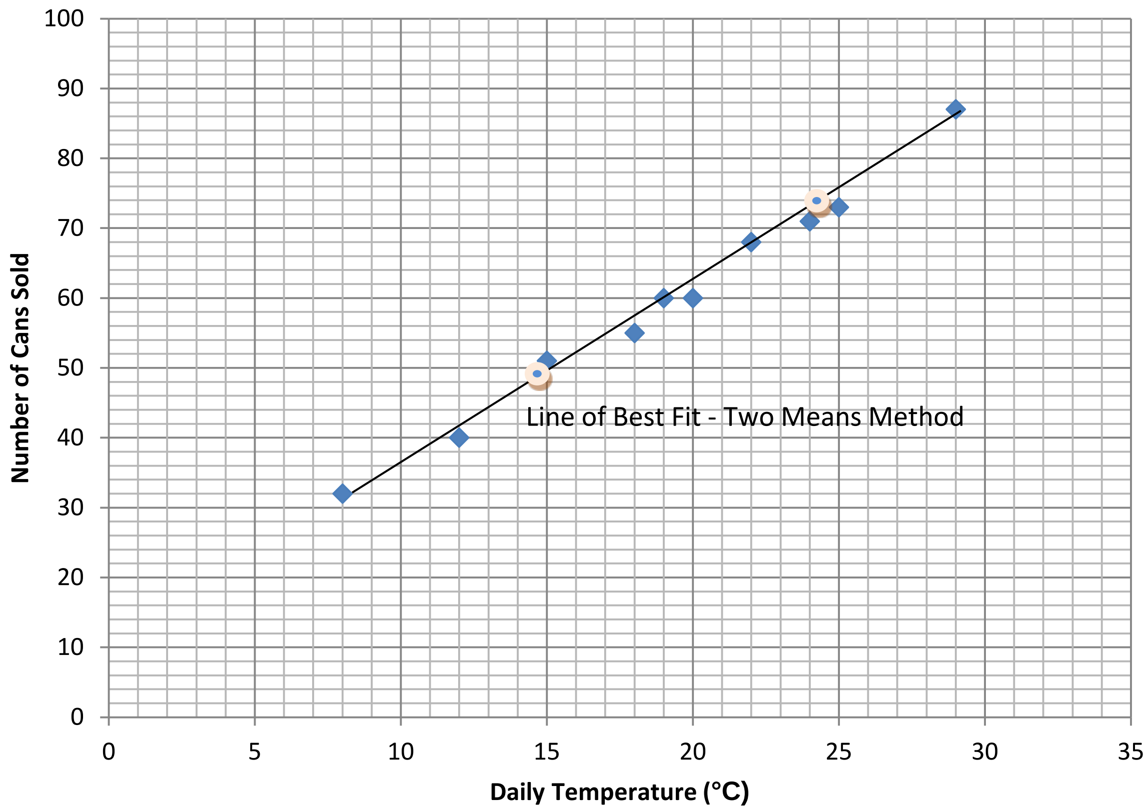
Two points on the line are (14.4, 47.6) and (24, 71.8)	$slope = \frac{y_2 - y_1}{x_2 - x_1}$ $slope = \frac{71.8 - 47.6}{24 - 14.4}$ $slope = \frac{24.2}{9.6}$ $slope = 2.52$ (to 2 dp)

So far the equation is $N = 2.52T + c$

One of the two points can be used to find the P (y)-intercept. Substituting in the point n and P values enable c to be calculated.

Using (24, 71.8)	$N = mT + c$ $71.8 = 2.52 \times 24 + c$ $71.8 = 60.48 + c$ $71.8 - 60.48 = c$ $11.32 = c$

The equation, using the Two Means Method, is $N = 2.52T + 11.32$



Note:
If there are an odd number of points, include the middle point in both the lower and upper halves.

Using the equation of the line of best fit it is possible to make predictions about one variable given a value for the other variable.

The original data ranged from a temperature of 8°C to 29°C with corresponding values of 32 to 87 cans sold. Predictions can be made within or outside this range.

Making predictions within this range is called Interpolation. Interpolation is considered reliable if the data points lie close to the line of best fit.
Making predictions outside this range is called Extrapolation. Between temperatures of 8°C to 29°C, there is a linear relationship. Extrapolating data would assume that the linear relationship would continue in either or both directions and there is no evidence to support this assumption.

Example:

Find the number of cans sold if the temperature is 27°C.

Using the equation $N = 2.52T + 11.32$ from the Two Means Method, The number sold should be:

$$N = 2.52T + 11.32$$
$$N = 2.52 \times 27 + 11.32$$
$$N = 79.36$$

The number sold is either 79 or 80. This answer is considered reliable because the answer was found by interpolation.

Find the number of cans sold if the temperature is 45°C.

Using the equation $N = 2.52T + 11.32$ from the Two Means Method, The number sold should be:

$$N = 2.52T + 11.32$$
$$N = 2.52 \times 45 + 11.32$$
$$N = 124.72$$

The number sold is either 124 or 125. This answer is not reliable because the answer was found by extrapolation; 45°C is well outside the range of the data.

Video ‘Lines of Best Fit’

Activity

Identify the dependent and independent variables in questions 1 - 5 for the scenario presented.

1. A scientist applies different amounts of fertilizer to different test plots and measures the yield of corn in each plot.

2. A researcher applies different amounts of insecticide to different test plots and measures the number of insects found in each plot.

3. The heights of seedlings are measured every day for a month.

4. A racehorse is timed over 1600m carrying various weights.

5. A subject has their reaction time to a stimulus measured for different levels of alcohol in the blood-stream.

6. The data below gives data for different Honda cars, all manual transmission.

Engine size (Litres)	Power (kW)	Torque(Nm)	Fuel Consumption (L/100km)
1.3	73	127	5.8
1.5	88	145	6.3
1.8	103	174	6.9
2	114	188	8.3
2.4	148	230	8.9
3.6	226	370	11.3

Using engine size as the independent variable, construct scatter-plots of

(a) Power vs Engine Size
(b) Torque vs Engine Size
(c) Fuel Consumption vs Engine Size

For each graph, determine the equation of the line of best fit by either method.

Comment on the significance of the slope.

Predict the Power, Torque and Fuel Consumption if Honda made an engine 3L in size.

Predict the Power, Torque and Fuel Consumption if Honda made an engine 0.5L in size.

Comment on the validity of your prediction?

7. The troposphere is a layer of air around the Earth. It extends from surface level to an altitude of 11km. The table below show the air temperature at various altitudes.

Construct a scatterplot for the data and then determine the equation of the line of best fit by either method.

Altitude (km)	Temperature (°C)
0	14
2	-3
4	-14
6	-24
8	-38
10	-52

Topic 5: Lines of Best Fit – Regression (Excel)

The method that takes total guesswork out of find the Line of Best Fit is called Regression. The approach used here is very practical and the theory behind this will not be discussed. In the most basic form, a table can be constructed and the formulas below used.

If $y = mx + c$

Then the slope can be calculated by:

$$m = \frac{\sum xy - n\overline{x}\,\overline{y}}{\sum x^2 - n\overline{x}^2}$$

The y-intercept can be calculated by:

$$c = \overline{y} - m\overline{x}$$

You can also find out the slope and y intercept by using a scientific or graphics calculator. You should read the calculator manual to find out if this is possible.

The most common method is to use a computer program called a spreadsheet. The most popular spreadsheet program used is Excel.

Temp(x)	18	22	19	25	29	12	8	15	20	24
Cans(y)	55	68	60	73	87	40	32	51	60	71

The data can be entered into Excel arranged horizontally (like the table above) or vertically (best for using the Data Analysis Add-In).

To find the slope of the regression line:

In a cell near the table, enter a formula using the menus:
> formulas (tab)
> (click) more functions
> (click) statistical
> look down list to find slope
> enter known y's (highlight the section of the table)
> enter known x's
>(click) ok

To find the y-intercept of the regression line:

In a different cell near the table, enter a formula using the menus:
> formulas (tab)
> (click) more functions
> (click) statistical
> look down list to find intercept
> enter known y's (highlight the section of the table)
> enter known x's
>(click) ok

Following this procedure, the regression line is $N = 2.56T + 10.49$

The video below will step you through this process.

[Video ‘Regression with Excel’]()

An alternative method is to perform Data Analysis. This is found in Excel under the Data tab. If data analysis cannot be found, you have to go into the Excel settings to get this add-in included.

[Video ‘Obtaining the Data Analysis Add-In in Excel’]()

Click on the Data tab, find Data Analysis. Click on Data Analysis and a new window will open giving a list of analysis tools. Click on regression and then click Ok. You will have to enter the Y Range and X Range individually by highlighting the cells or writing in the cell locations.

The only other part worth considering is the Output Location.

The video below demonstrate the process.

[Video ‘Using the Data Analysis Add-In for Regression’]()

There is considerable amount of information given from this process. A sample output is below

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.977265
R Square	0.955046
Adjusted R Square	0.940061
Standard Error	1.086642
Observations	5

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	75.25763	75.25763	63.73493	0.004101
Residual	3	3.542373	1.180791		
Total	4	78.8			

Y intercept

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	0.728814	1.010289	0.721391	0.522808	-2.48638
X Variable 1	1.262712	0.158167	7.983416	0.004101	0.759354

Slope

This means the regression equation is $y = 1.26x + 0.7288$

Activity

1. The data below gives data for different Honda cars, all manual transmission.

engine size (Litres)	Power (kW)	Torque(Nm)	Fuel Consumption (L/100km)
1.3	73	127	5.8
1.5	88	145	6.3
1.8	103	174	6.9
2	114	188	8.3
2.4	148	230	8.9
3.6	226	370	11.3

Using engine size as the independent variable, use Excel to obtain scatter-plots of
(a) Power vs Engine Size
(b) Torque vs Engine Size
(c) Fuel Consumption vs Engine Size

For each graph, determine the equation of the line of best fit by regression using Excel.

2. An archaeologist finds 11 skeletons from an ancient tribe. Their Humerus Bone Lengths and Skeleton Heights are measured.

Humerus Bone (cm)	Skeleton Height (cm)
38	188
34	173
40	194
38	187
28	157
30	164
31	165
37	185
20	130
26	149
29	160

(a) Using Excel, determine the Regression Line. (Use the humerus length as the independent variable because the equation is used to predict height)

(b) Predict the height of the skeleton if a 35cm long humerus is found.

(c) Obtain the scatterplot and comment on the closeness of the points to the regression line.

Topic 6: Applications – Break Even Analysis

Break even analysis is about looking at costs and revenue in a business setting and determining when the business will break even (revenue = costs), make a loss (revenue < costs) or make a profit (revenue > costs).

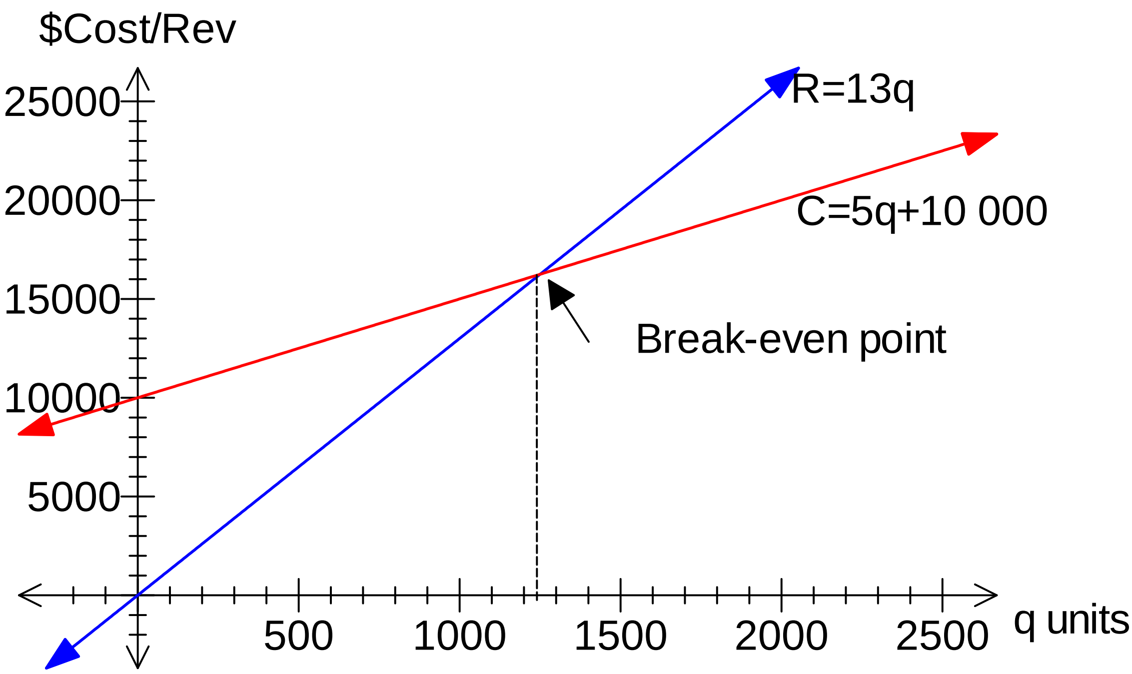
Revenue is the money coming into a business from sales. To simplify things, consider that the business sells a single item. The revenue depends upon the quantity sold and the price per item. If q is the quantity sold and items sell for $13, then the revenue equation is:

$$R = 13q$$

Costs usually consist of 2 parts, the variable cost and the fixed cost. The variable cost could be the cost of picking fruit and the fixed cost is the cost of establishing the orchard. Picking fruit is a variable cost because it depends on the amount picked.
If the variable cost is $5 per item and the establishment cost is $10 000, then the cost equation is:

$$C = 5q + 10\,000$$

Both revenue and cost equations can be represented graphically by linear graphs.



For break-even to occur, revenue is equal to costs. On the graph this point can be found where the two lines intersect. It can be seen that the break-even point occurs when approximately 1250 items are produced.

Algebraically, it is also possible to find the break-even point.

$$
\begin{aligned}
R &= C \\
13q &= 5q + 10\,000 \\
13q - 5q &= 5q - 5q + 10\,000 \\
8q &= 10\,000 \\
\frac{\cancel{8}q}{\cancel{8}} &= \frac{10\,000}{8} \\
q &= 1250
\end{aligned}
$$

The break-even point is 1250 items. It follows that the business makes a profit when the number of items exceeds 1250, and similarly the business makes a loss if the number of items is fewer than 1250.

In summary,

The revenue equation is written as: $R = Sq$ where S - selling price, q - quantity

The cost equation is written as: $C = Vq + F$ where V - variable cost, q - quantity, F - fixed cost

Break – even occurs when

$$
\begin{aligned}
R &= C \\
Sq &= Vq + F
\end{aligned}
$$

Activity

1. Find the break-even point both graphically and algebraically given the cost and revenue equations below.

$$
\begin{aligned}
C &= 5q + 150 \\
R &= 8q
\end{aligned}
$$

2. Jane has a small T-shirt printing business. She purchases T-shirts for $5 each and hires a suitable printing press for $200 per week. The printed T-shirts sell for $12.50.
   (a) Write down the cost and revenue equations.
   (b) How many shirts must be produced per week to break-even?
   (c) What profit or loss is made when 25 are sold in a week?

3. The cost of a rent a car is offered two different ways. Hirers can pay either $75 a day or $50 a day plus 20 cents a kilometre.
   (a) At how many kilometres is the cost the same?
   (b) What advice would you give to people renting a car about the method of rental payment based on the number of kilometres travelled?

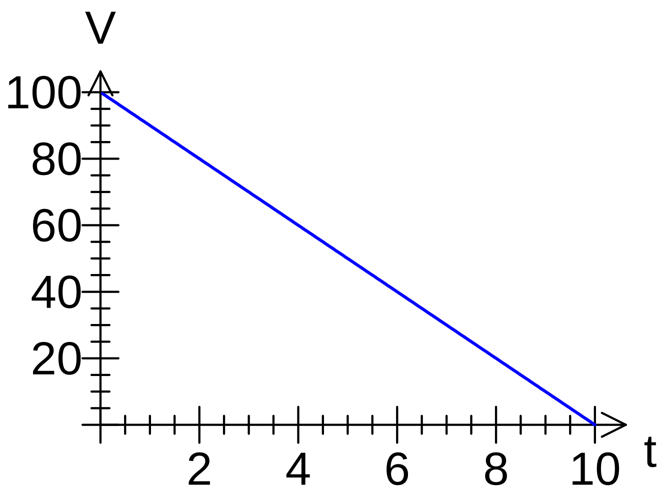
Answers to activity questions

Check your skills

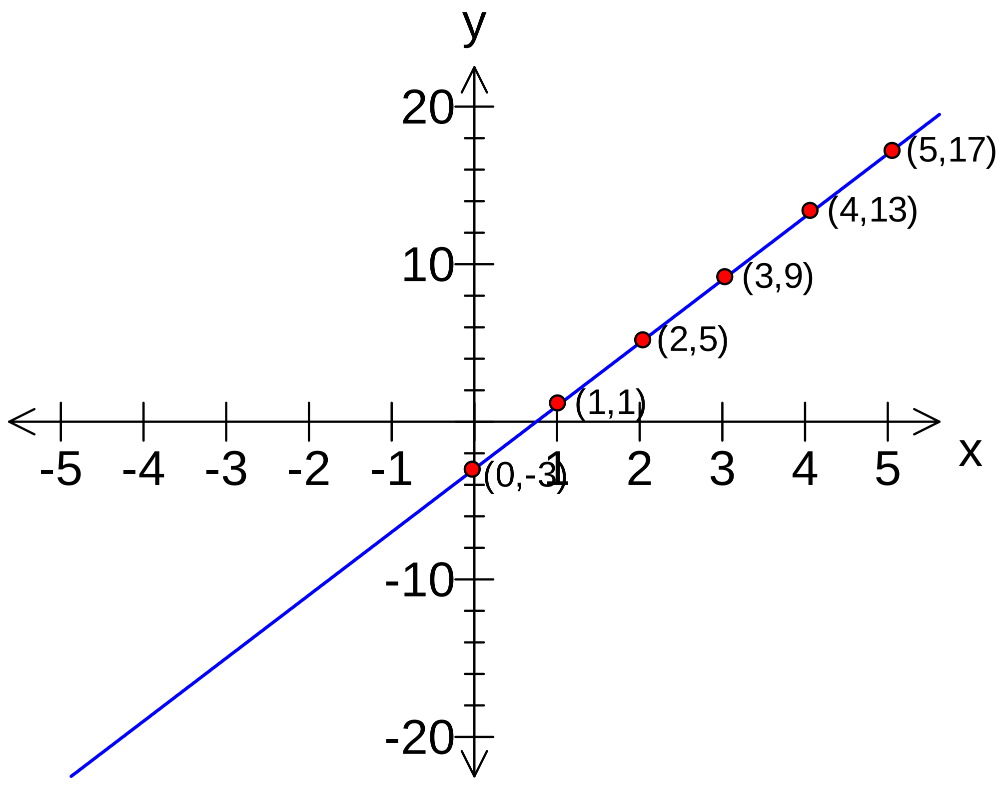
1. Equation Graph

$V = 100 - 10t$

Where V is the volume of water in the tank and t is the time (in days) since the tank was full.



2. The graph of $y = 4x - 3 : -5 \le x \le 5$



3.

$(100,1000) \rightarrow (x_2, y_2)$

$(500,200) \rightarrow (x_1, y_1)$

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

$$slope = \frac{1000 - 200}{100 - 500}$$

$$slope = \frac{800}{^{-}400}$$

$$slope = {^{-}2}$$

So far the equation is $y = {^{-}2}x + c$

One of the two points can be used to find the y-intercept. Substituting in a point's *x* and *y* values will enable *c* to be calculated. The point (100,1000) will be used.

Using (100,1000)

$$y = mx + c$$

$$1000 = {^{-}2} \times 100 + c$$

$$1000 = {^{-}200} + c$$

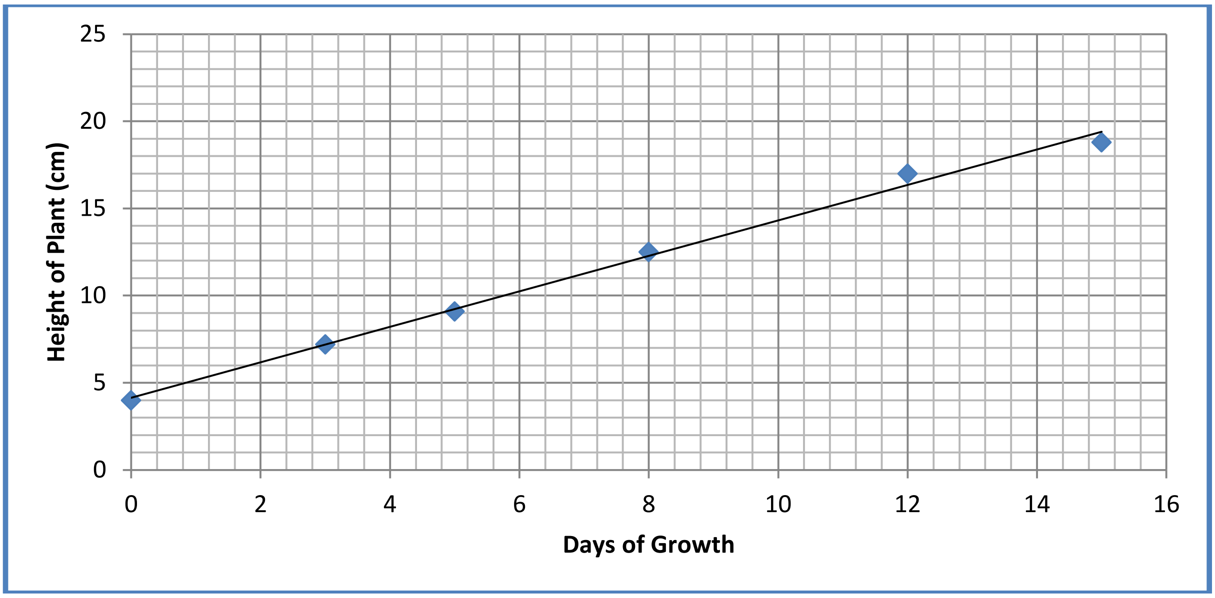
$$1000 + 200 = c$$

$$1200 = c$$

The equation of the line is $y = {^{-}2}x + 1200$

4.

d (days of growth)	0	3	5	8	12	15
h (height of plant in cm)	4	7.2	9.1	12.5	17	18.8



Using the two means method:

	Lower half x mean = 2.7			Upper half x mean = 11.7		
d (days of growth)	0	3	5	8	12	15
h (height in cm)	4	7.2	9.1	12.5	17	18.8
	Lower half y mean = 6.8			Upper half y mean = 16.1		

$(11.7,16.1) \rightarrow (d_2,h_2) \rightarrow (x_2,y_2)$

$(2.7,6.8) \rightarrow (d_1,h_1) \rightarrow (x_1,y_1)$

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

$$slope = \frac{16.1 - 6.8}{11.7 - 2.7}$$

$$slope = \frac{9.3}{9}$$

$$slope = 1.033$$

So far the equation is $h = 1.033d + c$

Using (2.7,6.8)

$$y = mx + c$$

$$6.8 = 1.033 \times 2.7 + c$$

$$6.8 = 2.8 + c$$

$$6.8 - 2.8 = c$$

$$4 = c$$

The equation of the line is $h = 1.033d + 4$

Lines of best fit drawn by eye should be similar to this.

5. The equation by regression (Excel output)

SUMMARY OUTPUT

Regression Statistics

	0.99736
Multiple R	1
R Square	0.99473

Adjusted R Square	0.993412
Standard Error	0.466282
Observations	6

ANOVA

	df	SS	MS	F	Significance F
Regression	1	164.1437	164.1437	755	1E-05
Residual	4	0.869675	0.217419		
Total	5	165.0133			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	4.147849	0.326408	12.70757	2E-04	3.2416	5.0541	3.2416	5.054
X Variable 1	1.016579	0.036998	27.47664	1E-05	0.9139	1.1193	0.9139	1.119

The equation by regression is $h = 1.02d + 4.15$

6. The revenue equation is $R = 2q$

The cost equation is $C = 0.5q + 10000$

For break-even

$$\begin{aligned} R &= C \\ 2q &= 0.5q + 10000 \\ 2q - 0.5q &= 10000 \\ 1.5q &= 10000 \\ q &= \frac{10000}{1.5} \\ q &= 6666.666667 \end{aligned}$$

Approximately 6667 punnets to be sold to break even.

Examples of Linear Relationships

1. The amount earned *(A)* after *w* weeks if Garry earn $850 per week. $A = 850w$

2. The revenue *(R)* from selling *q* calculators at $25 each. $R = 25q$

3. The cost (C) of producing n badges if blank badges cost $1 each and the machine to make badges costs $150.

$C = 1q + 150$ or $C = q + 150$

4. Sally uses her $650 tax refund cheque to start a holiday savings account and adds $100 per week.

$S = 100w + 650$
Where S is the amount saved after w weeks

5. A retiree has savings of $150 000. She uses $ 15000 per year.

$S = 150000 - 15000y$
Where S is amount of savings remaining after y years

6. The daily cost of renting a car is made up of $50 hire fee plus 25 cents per kilometre.

$R = 0.25k + 50$
Where R is the daily rental cost after travelling k kilometres

7. A car costing $15 000 depreciates by $1200 per year.

$V = 15000 - 1200y$ or $V = -1200y + 15000$
Where V is the value of the car after y years.

8. Saline solution is being infused to a patient using IV drip. The amount (in mL) of solution remaining after h hours is given by the equation:

$$A = 1000 - 80h$$

(a) What amount is remaining after 3 hours?

$$A = 1000 - 80h$$
$$A = 1000 - 80 \times 3$$
$$A = 1000 - 240$$
$$A = 760mL$$

(b) After how many hours will there be 500mL remaining?

$$A = 1000 - 80h$$
$$500 = 1000 - 80h$$
$$500 + 80h = 1000 \cancel{-80h} \cancel{+80h}$$
$$\cancel{500} \cancel{-500} + 80h = 1000 - 500$$
$$80h = 500$$
$$\frac{\cancel{80}h}{\cancel{80}} = \frac{500}{80}$$
$$h = 6.25hrs$$

After 6.25hrs or 6 hours 15 minutes

(c) Explain the significance of the 80 in the equation.

The volume in the flask <u>decreases</u> by 80 mL each hour.

9. The weight in kilograms of a truck loaded with n bales of wool is given by the equation:

$$W = 150n + 4500$$

(a) What is the weight of one bale of wool?

$W = 150n + 4500$
From the equation, 150 must be the weight of one bale. The 4500 represents the weight of the truck.

(b) If the maximum load is 20 bales, what will the total weight of the truck be?

$$W = 150n + 4500$$
$$W = 150 \times 20 + 4500$$
$$W = 3000 + 4500$$
$$W = 7500kg$$

(c) Complete the table below:

n	0	5	10	15	20
W	4500	5250	6000	6750	7500

(d) If a bridge has a load rating of only 5½ tonnes (5500kg), how many bales can the truck carry without damage to the bridge?

$$W = 150n + 4500$$
$$5500 = 150n + 4500$$
$$5500 - 4500 = 150n$$
$$1000 = 150n$$
$$\frac{1000}{150} = n$$
$$n = 6.6\overline{6}bales$$

The truck can only carry 6 bales. (Although $n = 6.6\overline{6}bales$ rounds to 7 bales, this would be too heavy.)

10. Julie rents a flat. She pays a bond of \$1200 plus \$250 per week.

(a) Complete this table

Weeks rented	0	1	2	3	4	5
Total Amount Paid	1200	1450	1700	1950	2200	2450

(b) Which equation best describes this situation.

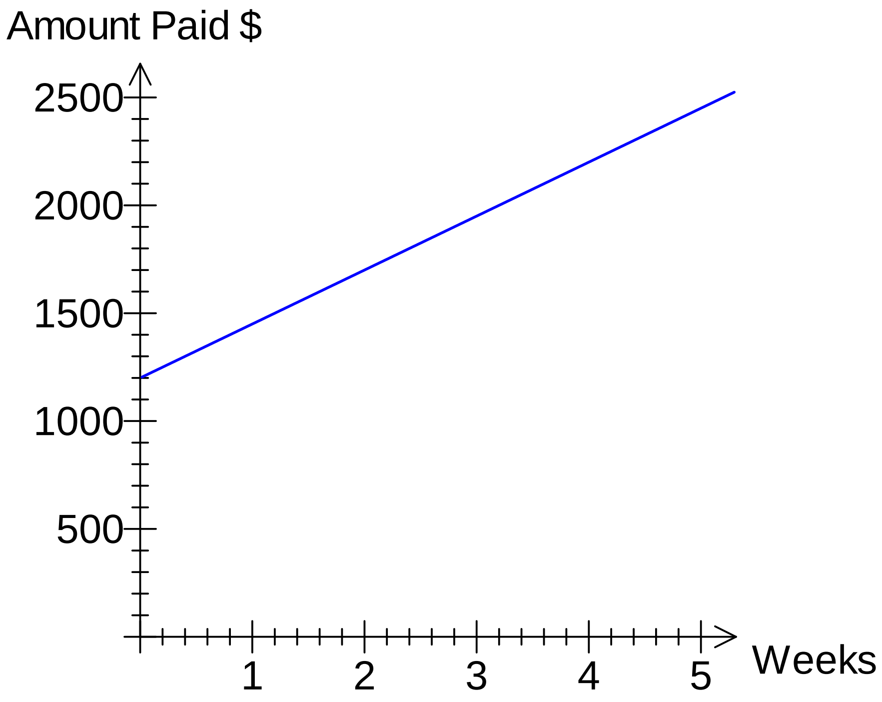
(i) $T = 1200w + 250$

(ii) $T = 250w + 1200$

(ii) $T = 1200 - 250w$

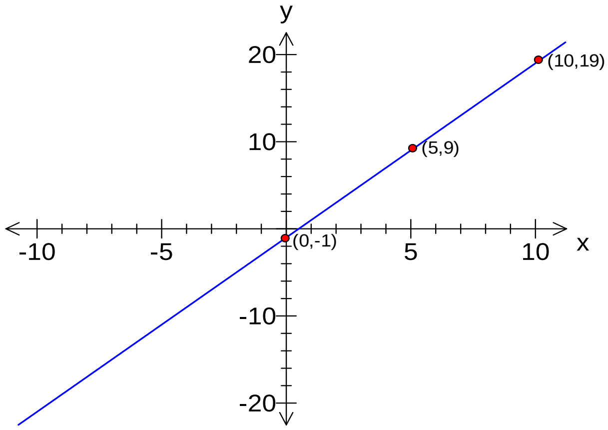
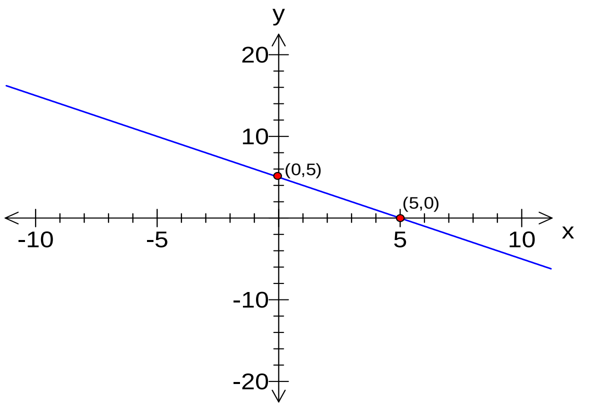
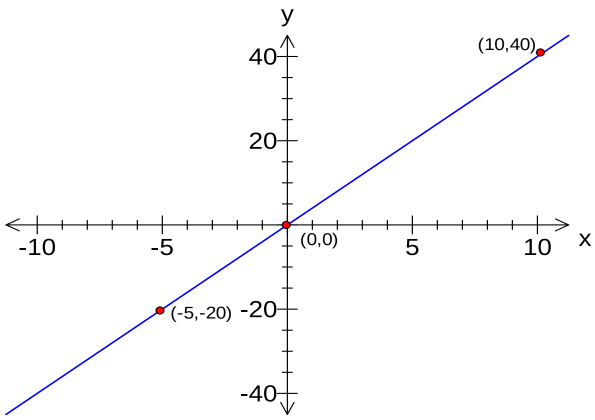
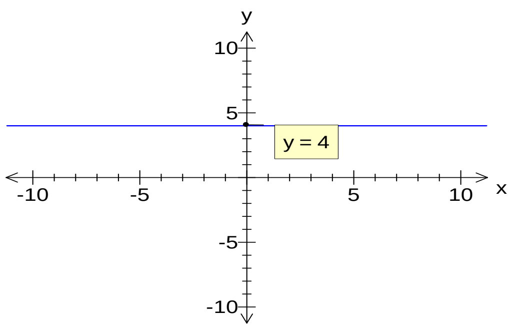
(iv) $T = \frac{1200 - w}{250}$

(c) Draw a graph based on the table from part (a).


Amount Paid $
2500
2000
1500
1000
500
1
2
3
4
5
Weeks

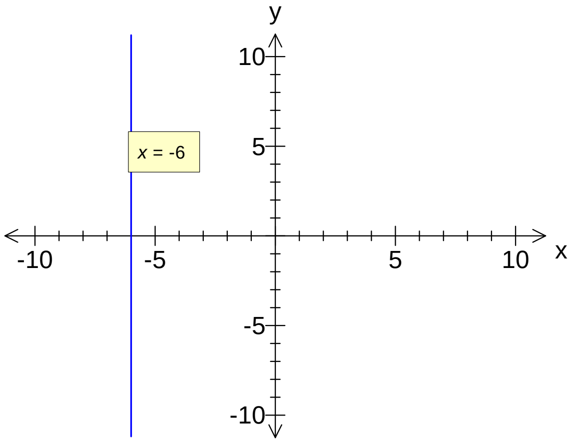
Graphing Lines

1. Draw the following graphs

(a) $y = 2x - 1$ When $x = 0,\ y = 2\times 0 - 1 = -1 \ \rightarrow\ (0,-1)$ When $x = 5,\ y = 2\times 5 - 1 = 9 \ \rightarrow\ (0,9)$ When $x = 10,\ y = 2\times 10 - 1 = 19 \ \rightarrow\ (0,19)$	
(b) $y = 5 - x$ y-intecept When $x = 0,\ y = 5 - 0 = 5 \ \rightarrow\ (0,5)$ x-intercept When y = 0, $0 = 5 - x$ $x = 5 \ \rightarrow\ (5,0)$	
(c) $y = 4x$ When $x = 0,\ y = 4\times 0 = 0 \ \rightarrow\ (0,0)$ When $x = -5,\ y = 4\times -5 = -20 \ \rightarrow\ (-5,-20)$ When $x = 10,\ y = 4\times 10 = 40 \ \rightarrow\ (10,40)$	
(d) $y = 4$ This equation is a special line. Because every y coordinate is 4, it is a horizontal line.	

(e) $x = {}^{-}6$

This equation is a special line. Because every *x* coordinate is -6, it is a vertical line.



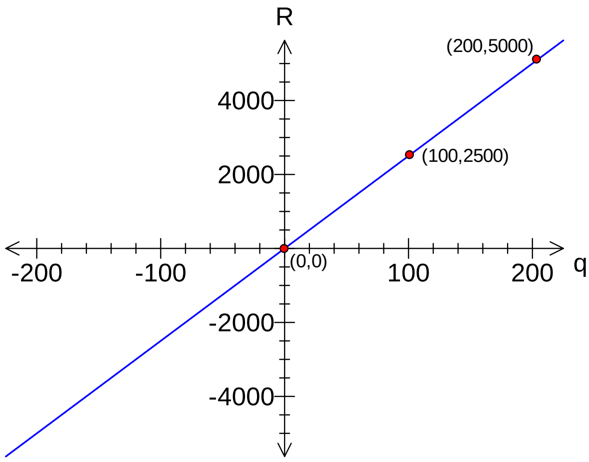
(f) $R = 25q$

This is a revenue equation so values of *q* need to reflect this.

When $q = 0,\ R = 25 \times 0 = 0 \ \rightarrow \ (0,0)$

When $q = 100,\ R = 25 \times 100 = 2500 \ \rightarrow \ (100, 2500)$

When $q = 200,\ y = 25 \times 200 = 5000 \ \rightarrow \ (200, 5000)$

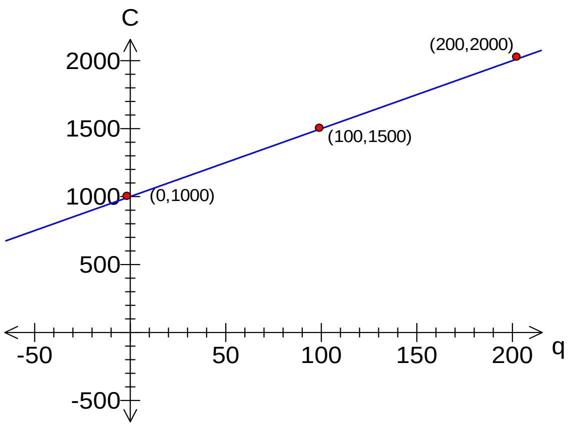


(g) $C = 5q + 1000$

When $q = 0,\ \mathrm{C} = 5 \times 0 + 1000 = 1000 \ \rightarrow (0,1000)$

When $q = 100,\ \mathrm{C} = 5 \times 100 + 1000 = 1500 \ \rightarrow (100,1500)$

When $q = 200,\ y = 5 \times 200 + 1000 = 2000 \ \rightarrow \ (200, 2000)$



(h) $H = 0.2t - 0.5$

y-intecept

When $x = 0,\ H = 0.2 \times 0 - 0.5 = -0.5 \rightarrow (0, -0.5)$

x-intercept

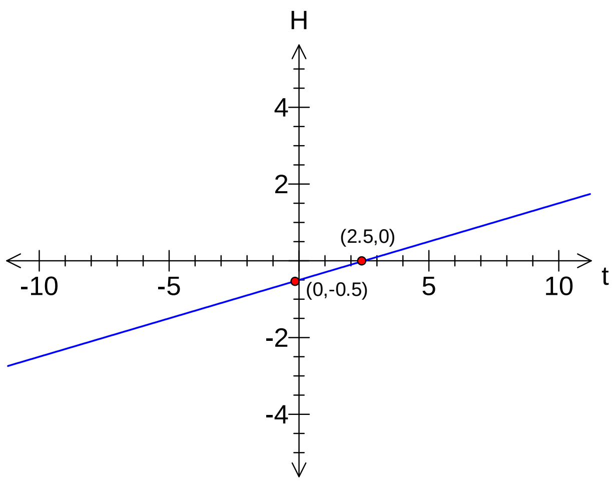
When y = 0,

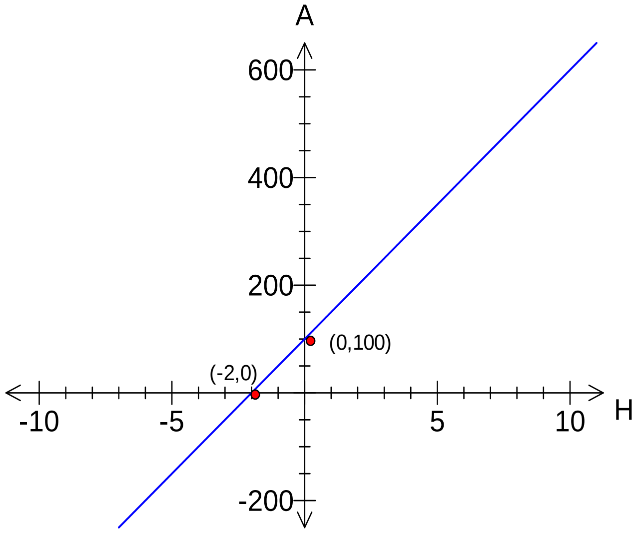
$0 = 0.2t - 0.5$

$0.5 = 0.2t$

$\frac{0.5}{0.2} = t$

$t = 2.5 \rightarrow \quad (2.5, 0)$



(i) $A = 50H + 100$ y-intecept When $x = 0,\ A = 50\times 0 + 100 = 100 \rightarrow (0,100)$ x-intercept When y $= 0,$ $0 = 50H + 100$ $-100 = 50H$ $\frac{-100}{50} = H$ $t = -2 \rightarrow \quad (-2,0)$	

Finding Equations of Lines

1. Find the equation of the line given the information below.

(a) Slope=2 and y intercept = -10. Using the general equation $y = mx + c$ the equation becomes $y = 2x - 10$

(b) Slope = -1 and passes through (5,1). Using the general equation $y = mx + c$ the equation so far is $y = -1x + c$

The point (5,1) can be used to find the y-intercept. Substituting in a point's *x* and *y* values will enable *c* to be calculated.

Using (5,1)

$$\begin{aligned} y &= mx + c \\ 1 &= {}^{-}1 \times 5 + c \\ 1 &= {}^{-}5 + c \\ 1 + 5 &= c \\ 6 &= c \end{aligned}$$

The equation is $y = -1x + 6$ or $y = -x + 6$

(c) (-5,2) with y intercept 4. Using the general equation $y = mx + c$ the equation so far is $y = mx + 4$
Using the y intercept (0,4) and the second point (-5,2) the slope and equation can be found.

$(0,4) \rightarrow (x_2, y_2)$
$(-5,2) \rightarrow (x_1, y_1)$

$$\begin{aligned} slope &= \frac{y_2 - y_1}{x_2 - x_1} \\ slope &= \frac{4 - 2}{0 - {}^{-}5} \\ slope &= \frac{2}{5} \\ slope &= 0.4 \end{aligned}$$

The equation is $y = 0.4x + 4$

(d) (3,6) and (2,3)

$(3,6) \rightarrow (x_2, y_2)$
$(2,3) \rightarrow (x_1, y_1)$

$$\begin{aligned} slope &= \frac{y_2 - y_1}{x_2 - x_1} \\ slope &= \frac{6 - 3}{3 - 2} \\ slope &= \frac{3}{1} \\ slope &= 3 \end{aligned}$$

The equation so far is $y = 3x + c$
The point (3,6) or (2,3) can be used to find the y-intercept. Substituting in a point's *x* and *y* values will enable *c* to be calculated.

Using (3,6)

$$\begin{aligned} y &= mx + c \\ 6 &= 3 \times 3 + c \\ 6 &= 9 + c \\ 6 - 9 &= c \\ -3 &= c \end{aligned}$$

The equation is $y = 3x - 3$

(e) (-3,-4) and (1,4)

(1,4) $\rightarrow$ (x_2, y_2)
(-3,-4) $\rightarrow$ (x_1, y_1)

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$
$$slope = \frac{4 - {}^-4}{1 - {}^-3}$$
$$slope = \frac{8}{4}$$
$$slope = 2$$

The equation so far is $y = 2x + c$
The point (1,4) or (-3,-4) can be used to find the y-intercept. Substituting in a point's *x* and *y* values will enable *c* to be calculated.

Using (1,4)

$$y = mx + c$$
$$4 = 2 \times 1 + c$$
$$4 = 2 + c$$
$$4 - 2 = c$$
$$2 = c$$

The equation is $y = 2x + 2$

(f) (-1,-2) and (2,4)

(2,4) $\rightarrow$ (x_2, y_2)
(-1,-2) $\rightarrow$ (x_1, y_1)

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$
$$slope = \frac{4 - {}^-2}{2 - {}^-1}$$
$$slope = \frac{6}{3}$$
$$slope = 2$$

The equation so far is $y = 2x + c$
The point (2,4) or (-1,-2) can be used to find the y-intercept. Substituting in a point's *x* and *y* values will enable *c* to be calculated.

Using (2,4)

$$y = mx + c$$
$$4 = 2 \times 2 + c$$
$$4 = 4 + c$$
$$4 - 4 = c$$
$$0 = c$$

The equation is $y = 2x + 0$ or $y = 2x$

(g) (-4,8) and (1,-2)

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

(-4,8) $\rightarrow$ (x_2, y_2)
(1,-2) $\rightarrow$ (x_1, y_1)

$$slope = \frac{8 - {}^{-}2}{{}^{-}4 - 1}$$

$$slope = \frac{10}{-5}$$

$$slope = {}^{-}2$$

The equation so far is $y = {}^{-}2x + c$
The point (-4,8) or (1,-2) can be used to find the y-intercept. Substituting in a point's *x* and *y* values will enable *c* to be calculated.

Using (1,-2)

$$y = mx + c$$
$$-2 = {}^{-}2 \times 1 + c$$
$$-2 = -2 + c$$
$$-2 + 2 = c$$
$$0 = c$$

The equation is $y = {}^{-}2x + 0$ or $y = {}^{-}2x$

(h) (3,8) and (3,-1)

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

(3,8) $\rightarrow$ (x_2, y_2)
(3,-1) $\rightarrow$ (x_1, y_1)

$$slope = \frac{8 - {}^{-}1}{3 - 3}$$

$$slope = \frac{9}{0}$$

$$slope = undefined$$

This is a vertical line, notice that the x coordinate of each point is 3, the equation of this line is $x = 3$.

(i) (0,6) and (9,6)

$$(0,6) \rightarrow (x_2, y_2)$$
$$(9,6) \rightarrow (x_1, y_1)$$

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$
$$slope = \frac{6-6}{0-9}$$
$$slope = \frac{0}{-9}$$
$$slope = 0$$

This is a horizontal line, notice the y coordinate of each point is 6, the equation of this line is $y = 6$

2. For each of the answers to question 1 above, complete the table below.

	Slope	Nature of Slope	Y-intercept
(a)	2	Positive	-10
(b)	-1	Negative	6
(c)	0.4	Positive	4
(d)	3	Positive	-3
(e)	2	Positive	2
(f)	2	Positive	0
(g)	-2	Negative	0
(h)	undefined	Vertical Line	None
(i)	0	Horizontal Line	6

3. At a car-wash fund raiser, the cost equation was $C = 2.5q + 50$.

(a) What is the cost if 50 cars were washed?

$$C = 2.5q + 50$$
$$C = 2.5 \times 50 + 50$$
$$C = 175$$

(b) If the costs were \$175, how many cars were washed?

$$C = 2.5q + 50$$
$$175 = 2.5q + 50$$
$$175 - 50 = 2.5q$$
$$125 = 2.5q$$
$$\frac{125}{2.5} = q$$
$$q = 50$$

(c) Explain the practical significance of the slope. The slope, 2.5, is the cost of washing each car.

(d) Explain the practical significance of the y-intercept The y-intercept, 50, is the fixed cost of running the car wash.

4. On the first visit, an electrician charged \$180 for 2 hours work. On a second visit, the electrician charged \$405 for 5 hours work.

(a) From the information given, develop the equation $F = 75h + 30$, where F is the Fee charged for h hours work.

Two points are (2,180) and (5,405)

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

$$slope = \frac{405 - 180}{5 - 2}$$

$$slope = \frac{225}{3}$$

$$slope = 75$$

The equation is $F = 75h + c$

Substituting (2,180)

$$180 = 75 \times 2 + c$$

$$c = 30$$

The equation is $F = 75h + 30$

(b) Explain the practical significance of the slope.

The slope, 75, is the hourly rate of the electrician.

(c) Explain the practical significance of the y-intercept.

The y-intercept, 30, is the call out fee or a fixed fee.

(d) How much would the electrician charge for three and a quarter hours work?

$$F = 75h + 30$$

$$F = 75 \times 3.25 + 30$$

$$F = 273.75$$

(e) If the electrician charged \$300, how many hours did they work for?

$$F = 75h + 30$$

$$300 = 75h + 30$$

$$300 - 30 = 75h$$

$$270 = 75h$$

$$\frac{270}{75} = h$$

$$h = 3.6hrs$$

The electrician worked for 3.6hrs or 3hrs 36mins.

(f) The electrician decides to increase the hourly rate by 4% to reflect inflation. What will be the new fee equation?

New hourly rate = 1.04 x 75 = 78

New equation is $F = 78h + 30$

5. The number of photo-plankton found in a sample of sea water varies with the level of contaminates in the water. When the level of contaminates is 100ppm (parts per million) the number of photo- plankton is 95. When the level of contaminates increases to 500ppm, the number of photo-plankton is reduced to nil.

(a) Which is the independent and dependent variable? (You may have to read the start of the next topic)

The number of photo-plankton depends on the level of contaminates. Therefore: number of photo-plankton – dependent, level of contaminates – independent.

(b) From the information given, develop the equation. Use l for the level of contaminates and N for the number of photo-plankton.

Two points are (100,95) and (500,0)

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$
$$slope = \frac{0-95}{500-100}$$
$$slope = \frac{-95}{400}$$
$$slope = -0.2375$$

The equation is $N = -0.2375l + c$

Substituting (500,0)

$$0 = -0.2375 \times 500 + c$$
$$c = 118.75$$

The equation is:

$$N = -0.2375l + 118.75$$
$$or$$
$$N = 118.75 - 0.2375l$$

(c) Explain why the slope is negative?

The slope is negative due to decreasing numbers of photo-plankton with increasing level of contaminates.

(d) Explain the practical significance of the y-intercept.

The y-intercept represents the number of photo-plankton when there is no contamination.

(e) If it is desirable to have a contaminate level less that 250ppm, what photo-plankton count would you expect to find?

$$N = 118.75 - 0.2375l$$
$$N = 118.75 - 0.2375 \times 250$$
$$N = 59.375$$

6. In straight line depreciation, the value of an item depreciates by a set amount each year until its value is nil. The value (V) of a car after y years is given by the equation $V = 35000 - 2800y$

(a) Which is the independent and dependent variable? (You may have to read the start of the next topic)

The value of the car depends on its age in years. Therefore the value is the dependent variable and the age in years is the independent variable.

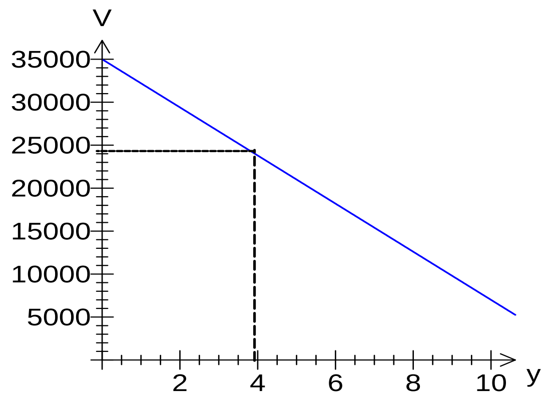
(b) What was the original cost of the car?

The original cost occurs when $y=0$, giving the value $V=35000$. This is the y-intercept.

(c) Explain the practical significance of the slope.

The slope -2800 is the decrease in value (depreciation) each year.

(d) Draw the graph of this equation. Use $0 \le y \le 10$



(e) Using the graph or equation, what is the value of the car after 4 years?

The graph suggests the value is $24000
Using the equation:

$$V = 35000 - 2800y$$
$$V = 35000 - 2800 \times 4$$
$$V = 35000 - 11200$$
$$V = 23800$$

(f) When will the value of the car be effectively nil?

Using the equation

$$V = 35000 - 2800y$$
$$0 = 35000 - 2800y$$
$$2800y = 35000$$
$$y = \frac{35000}{2800}$$
$$y = 12.5 years$$

Lines of Best Fit – Pen & Paper Methods

Identify the dependent and independent variables for the scenario given.

independent

1. A scientist applies different amounts of fertilizer to different test plots and measures the yield of corn in each plot.

dependent

independent

2. A researcher applies different amounts of insecticide to different test plots and measures the number of insects found in each plot.

dependent

3. The heights of seedlings are measured everyday for a month.

dependent

independent

4. A racehorse is timed over 1600m carrying various weights.

dependent

independent

dependent

5. A subject has their reaction time to a stimulus measured for different levels of alcohol in the blood-stream.

independent

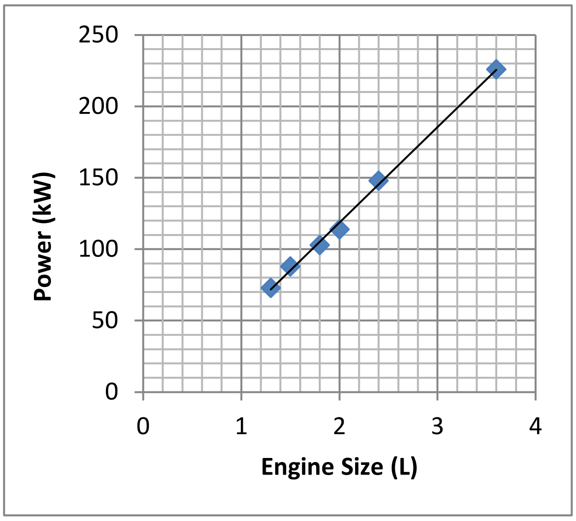
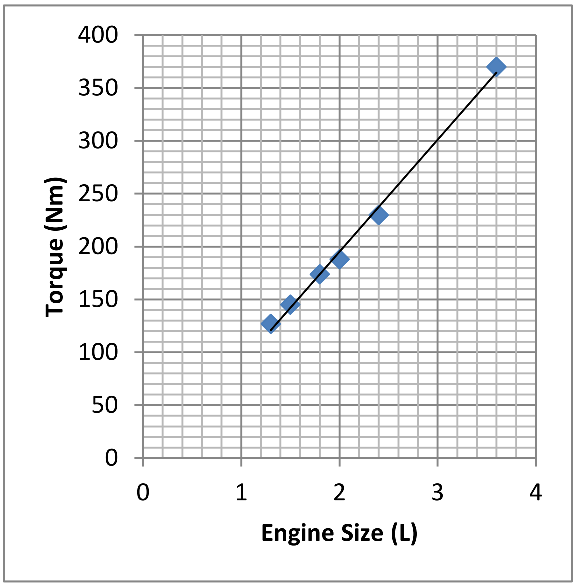
6. The data below gives data for different Honda cars, all manual transmission.

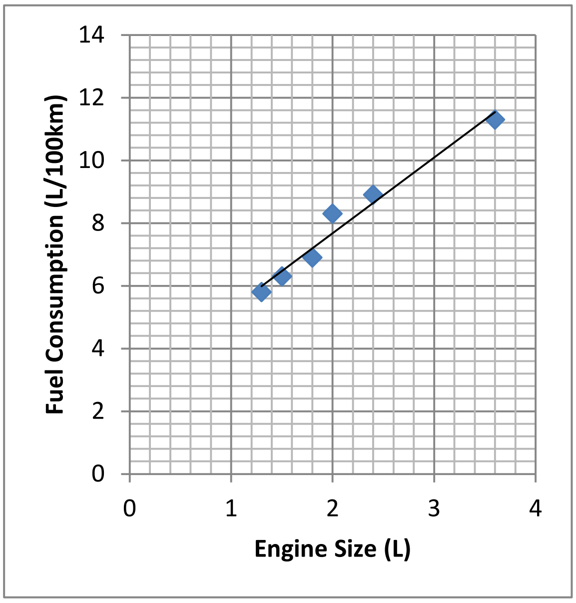
Engine size (Litres)	Power (kW)	Torque(Nm)	Fuel Consumption (L/100km)
1.3	73	127	5.8
1.5	88	145	6.3
1.8	103	174	6.9
2	114	188	8.3
2.4	148	230	8.9
3.6	226	370	11.3

Using engine size as the independent variable, construct scatter-plots of

(a) Power vs Engine Size
(b) Torque vs Engine Size
(c) Fuel Consumption vs Engine Size

For each graph, determine the equation of the line of best fit by either method.

	Graph (Line is a guide only)	Equation – Two Means Method
(a) Power vs Engine Size		Lower and upper mean points are: (1.533, 88) and (2.667, 162.667) Slope=72.9 Y-intercept=-23.8 $P = 72.9e - 23.8$
(b) Torque vs Engine Size		Lower and upper mean points are: (1.533, 148.667) and (2.667, 262.667) Slope=100.5 Y-intercept=-5.4 $T = 100.5e - 5.4$

(c) Fuel Consumption vs Engine Size		Lower and upper mean points are: (1.533, 6.333) and (2.667, 9.5) Slope=2.79 Y-intercept=2.05 $FC = 2.79e + 2.05$

Comment on the significance of the slope.

The slope indicates the Power, Torque or Power for each litre of engine size.

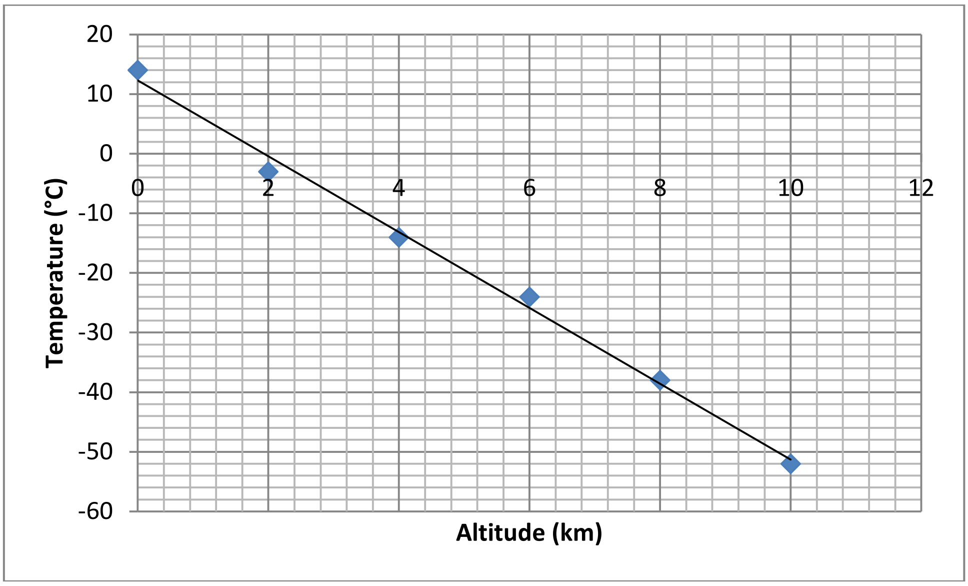
Predict the Power, Torque and Fuel Consumption if Honda made an engine 3L in size.
Predict the Power, Torque and Fuel Consumption if Honda made an engine 0.5L in size.
Comment on the validity of your prediction?

	For 3L engine	For a 0.5L engine
Using the Power equation, the predicted power is:	$P = 72.9e - 23.8$ $P = 72.9 \times 3 - 23.8$ $P = 195kW$	$P = 72.9e - 23.8$ $P = 72.9 \times 0.5 - 23.8$ $P = 12.65kW$
Using the Torque equation, the predicted power is:	$T = 100.5e - 5.4$ $T = 100.5 \times 3 - 5.4$ $T = 296Nm$	$T = 100.5e - 5.4$ $T = 100.5 \times 0.5 - 5.4$ $T = 44.85Nm$
Using the Fuel Consumption equation, the predicted fuel consumption equation is:	$FC = 2.79e + 2.05$ $FC = 2.79 \times 3 + 2.05$ $FC = 10.4L/100km$	$FC = 2.79e + 2.05$ $FC = 2.79 \times 0.5 + 2.05$ $FC = 3.45L/100km$
Comment on predictions.	This is Interpolation – these answers are probably reliable.	This is Extrapolation – these answers should be considered unreliable.

7. The troposphere is a layer of air around the Earth. It extends from surface level to an altitude of 11km. The table below show the air temperature at various altitudes.

Construct a scatterplot for the data and then determine the equation of the line of best fit by either method.

Altitude (km)	Temperature (°C)
0	14
2	-3
4	-14
6	-24
8	-38
10	-52



By eye: the y-intercept =12,
Using the points (0,12) and (10,-51) the slope is:

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

$$slope = \frac{-51-12}{10-0}$$

$$slope = \frac{-63}{10}$$

$$slope = -6.3$$

The equation is $T = -6.3A + 12$. This means the temperature (T) decreases by 6.3°C for each kilometre in altitude.

Lines of Best Fit – Regression (Excel)

1. Graphs - See Qn 6 from Lines of Best Fit – Pen and Paper Methods (previous section)

Regression Equations:

(a) Power (Excel print out)

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.99861
R Square	0.997222
Adjusted R Square	0.996528
Standard Error	3.270915
Observations	6

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	15364.54	15364.54	1436.088	2.9E-06
Residual	4	42.79554	10.69889		
Total	5	15407.33			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-15.0126	3.93686	-3.81334	0.018886	-25.9431	-4.08212	-25.9431	-4.08212
X Variable 1	66.8314	1.763559	37.89575	2.9E-06	61.93497	71.72782	61.93497	71.72782

The equation is $P = 66.8e - 15$

(b) Torque (Excel print out)

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.997672
R Square	0.995349
Adjusted R Square	0.994187
Standard Error	6.714845
Observations	6

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	38600.98	38600.98	856.1035	8.12E-06
Residual	4	180.3566	45.08915		
Total	5	38781.33			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-16.7868	8.081961	-2.07707	0.106375	-39.2259	5.6523	-39.2259	5.6523
X Variable 1	105.9302	3.620402	29.25925	8.12E-06	95.87839	115.9821	95.87839	115.9821

The equation is $T = 105.9e - 16.8$

(c) Fuel Consumption (Excel print out)

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.983866
R Square	0.967993
Adjusted R Square	0.959991
Standard Error	0.406869
Observations	6

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	20.02616	20.02616	120.9728	0.000388
Residual	4	0.662171	0.165543		
Total	5	20.68833			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	2.849806	0.489706	5.819419	0.004341	1.490164	4.209449	1.490164	4.209449
X Variable 1	2.412791	0.219369	10.99877	0.000388	1.803724	3.021857	1.803724	3.021857

The equation is $FC = 2.4e + 2.85$

2. An archaeologist finds 11 skeletons from an ancient tribe. Their Humerus Bone Lengths and Skeleton Heights are measured.

(a) Excel print out

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.998616
R Square	0.997234
Adjusted R Square	0.996927
Standard Error	1.075599
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	3754.133	3754.133	3244.956	7.97E-13
Residual	9	10.41222	1.156913		
Total	10	3764.545			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	67.3904	1.801987	37.39783	3.47E-11	63.31402	71.46678	63.31402	71.46678
X Variable 1	3.164403	0.05555	56.96451	7.97E-13	3.03874	3.290067	3.03874	3.290067

Equation is $SH = 3.16h + 67.4$

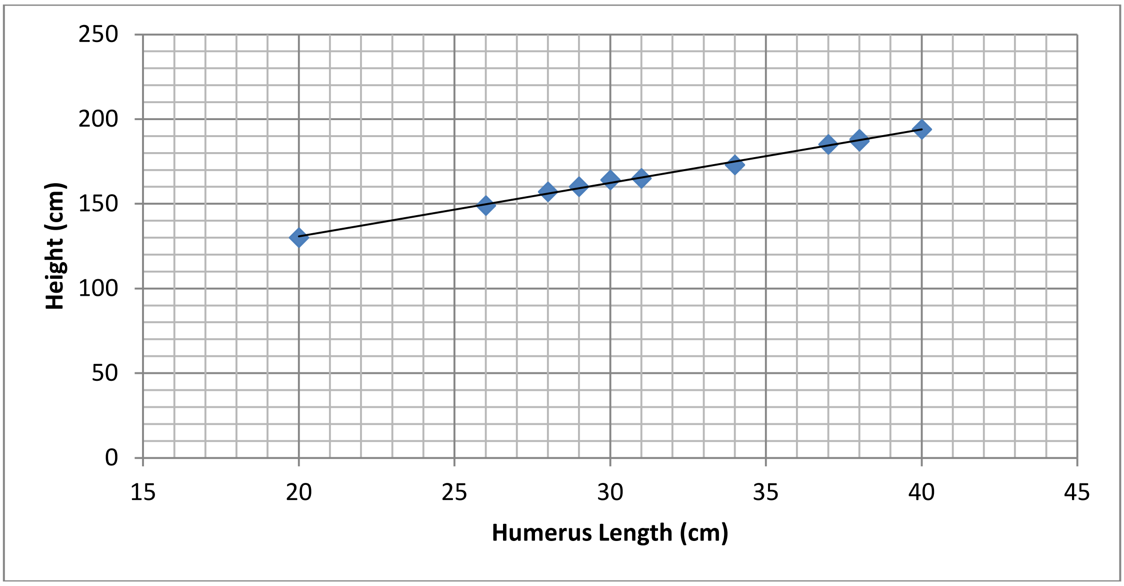
(b) If a 35cm humerus was found, the skeleton height is estimated to be:

$$SH = 3.16h + 67.4$$

$$SH = 3.16 \times 35 + 67.4$$

$$SH = 178cm$$

(c)



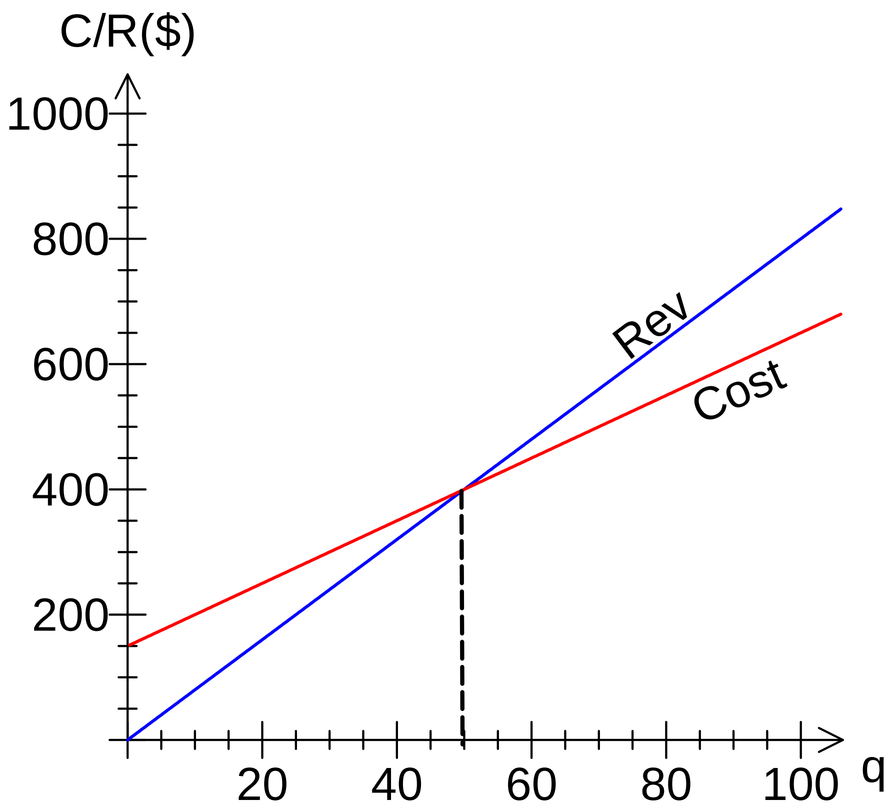
The points are very close to the regression, the prediction in (b) will be close to the actual value.

Applications – Break Even Analysis

1. Algebraically, break even occurs when revenue is equal to costs.

$$R = C$$
$$8q = 5q + 150$$
$$3q = 150$$
$$q = 50$$

Graphically



2. (a) Cost equation is: $C = 5q + 200$ Revenue Equation is: $R = 12.5q$

(b) For break even

$$R = C$$
$$12.5q = 5q + 200$$
$$7.5q = 200$$
$$q = 26.67$$

26 T-shirts makes a slight loss, 27 T-shirts makes a small profit.

(c) When 25 are sold: Rev=$312.50 , Costs=$325, giving a loss of $12.50.

3. The working out for this question is similar to break even analysis situations.

The renter pay using the equation $C = 75$ or $C = 0.2km + 50$

(a) The number of km for equal cost is:

$$75 = 0.2km + 50$$
$$25 = 0.2km$$
$$125 = km$$

The number of kilometres for equal cost is 125.

(b) If you are travelling less than 125 km per day, choose the rate with the km charge. If you are travelling more than 125km, choose the fixed daily rate.