Introduction to Measurement

The metric system originates back to the 1700s in France. It is known as a decimal system because conversions between units are based on powers of ten. This is quite different to the Imperial system of units where every conversion has a unique value.

A physical quantity is an attribute or property of a substance that can be expressed in a mathematical equation. A quantity, for example the amount of mass of a substance, is made up of a value and a unit. If a person has a mass of 72kg: the quantity being measured is Mass, the value of the measurement is 72 and the unit of measure is kilograms (kg). Another quantity is length (distance), for example the length of a piece of timber is 3.15m: the quantity being measured is length, the value of the measurement is 3.15 and the unit of measure is metres(m).

A unit of measurement refers to a particular physical quantity. A metre describes length, a kilogram describes mass, a second describes time etc. A unit is defined and adopted by convention, in other words, everyone agrees that the unit will be a particular quantity.

Historically, the metre was defined by the French Academy of Sciences as the length between two marks on a platinum-iridium bar at 0°C, which was designed to represent one ten-millionth of the distance from the Equator to the North Pole through Paris. In 1983, the metre was redefined as the distance travelled by light in free space in $\frac{1}{299\,792\,458}$ of a second.

The kilogram was originally defined as the mass of 1 litre (know as 1 cubic decimetre at the time) of water at 0°C. Now it is equal to the mass of international prototype kilogram made from platinum-iridium and stored in an environmentally monitored safe located in the basement of the International Bureau of Weights and Measures building in Sèvres on the outskirts of Paris.

The term SI is from “le Système international d'unités”, the International System of Units. There are 7 SI base quantities. It is the world's most widely used system of measurement, both in everyday commerce and in science. The system is nearly universally employed.

SI Base Units		
Base quantity	Name	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

For other quantities, units are defined from the SI base units. Examples are given below.

SI derived units (selected examples)		
Quantity	Name	Symbol
Area	square metre	m^2
Volume	cubic metre	m^3
Speed	metre per second	m/s
Acceleration	metre per second squared	$m/\sec^2$

Some SI derived units have special names with SI base unit equivalents.

SI derived units (selected examples)			
Quantity	Name	Symbol	SI base unit equivalent
Force	Newton	N	$m\,kg/\sec^2$
Pressure	Pascal	Pa	N/m^2
Work, Energy	Joule	J	$N\,m$
Power	Watt	W	J/s
Electric Charge	Coulomb	C	$A\,s$
Electric Potential Difference	Volt	V	W/A
Celsius (temperature)	degree Celsius	°C	K
Frequency	Hertz	Hz	$/s$
Capacity	litre	L (or l)	dm^3

If units are named after a person, then a capital letter is used for the first letter. Often, litres is written with a capital (L) because a lowercase (l) looks like a one(1).

An important feature of the metric system is the use of prefixes to express larger and smaller values of a quantity. For example, a large number of grams can be expressed in kilograms, and a fraction of a gram could be expressed in milligrams.

Commonly used prefixes are listed in the table below.

		Multiplication Factor		
Name	Symbol	Word form	Standard form	Power of 10
peta	P	Quadrillion	1 000 000 000 000 000	10^{15}
tera	T	Trillion	1 000 000 000 000	10^{12}
giga	G	Billion	1 000 000 000	10^{9}
mega	M	Million	1 000 000	10^{6}
kilo	k	Thousand	1 000	10^{3}
hecto	h	Hundred	100	10^{2}
deca	da	Ten	10	10^{1}
deci	d	Tenth	0.1	10^{-1}
centi	c	Hundredth	0.01	10^{-2}
milli	m	Thousandth	0.001	10^{-3}
micro	μ, mc	Millionth	0.000 001	10^{-6}
nano	n	Billionth	0.000 000 001	10^{-9}
pico	p	Trillionth	0.000 000 000 001	10^{-12}

The use of prefixes containing multiples of 3 are the most commonly used prefixes.

Using prefixes, conversions between units can be devised.

For example:

1kg = 1000g	On the left hand side the prefix is used. On the right hand side the prefix is replaced with the multiplication factor.
1mg = 0.001g	On the left hand side the prefix is used. On the right hand side the prefix is replaced with the multiplication factor. To make the conversion friendlier to use, multiply both sides by 1000 (Why 1000? Because milli means one thousandth and one thousand thousandths make one whole), so 1000mg = 1g.
1Mm = 1 000 000m	On the left hand side the prefix is used. On the right hand side the prefix is replaced with the multiplication factor.
1 μm = 0.000 001m	On the left hand side the prefix is used. On the right hand side the prefix is replaced with the multiplication factor. To make the conversion friendlier to use, multiply both sides by 1 000 000 (Why 1 000 000? Because micro means one millionth), so 1 000 000 μm = 1 m

Module contents

Introduction

- **Measurement – Length, Perimeter and Circumference**
- **Measurement - Area**
- **Measurement - Volume**

Answers to activity questions

Outcomes

- To demonstrate an understanding of the difference between Perimeter, Area and Volume.
- Identify and name plane and solid shapes.
- Perform calculations involving Perimeter, Area and Volume on a variety of shapes.

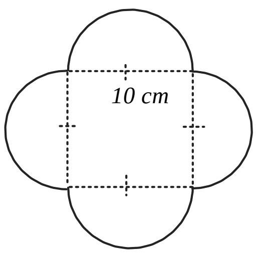
Check your skills

This module covers the following concepts, if you can successfully answer these questions, you do not need to do this module. Check your answers from the answer section at the end of the module.

1. (a) What units are used to measure perimeter?
   (b) What units are used to measure area?
   (c) What units are used to measure volume?

2. (a) Find the perimeter of a soccer ground that is 140 m long and 55 m wide.
   (b) Find the area of a semicircle with diameter 15cm.
   (c) Find the volume in m^3 and L of a square based pyramid with a base side length 1.4m and height 2.1m

3.

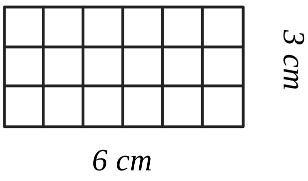
(a) Find the perimeter of this shape (b) Find the area of this shape	

4. The volume of a sphere is $200cm^3$, what is its radius.

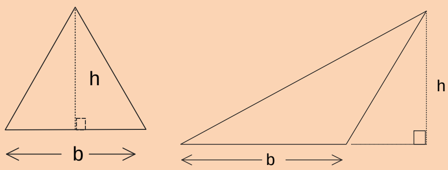
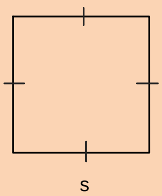
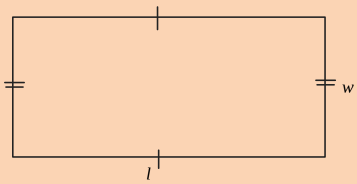
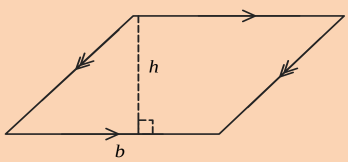
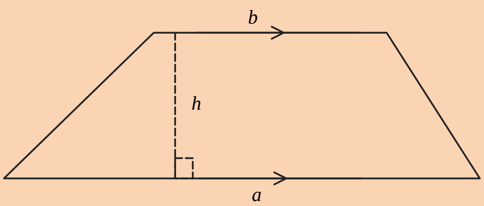
Topic 1: Measurement – area

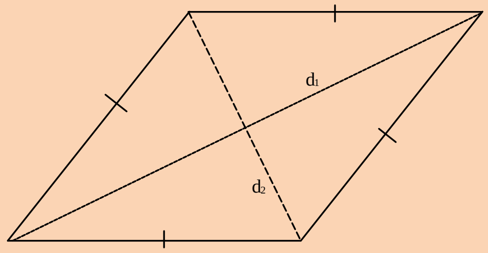
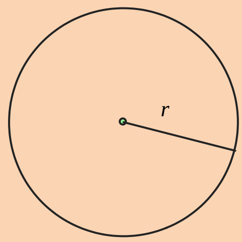
You might recall from the units section of this module that an area of 1 square metre is a square with side length of 1 metre. The purpose of these rules is to provide a simple way of counting the number of square metres, square centimetres or square millimetres within a shape.

For example:

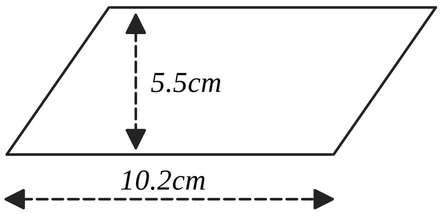


The rectangle is split up into centimetre divisions. The rectangle contains 6 x 3 = 18 squares of area 1 cm^2 each giving an area 18 cm^2. This can be generalised as $A = l \times w$

Plane Shape	Diagram	Rule
Triangle		$A = \frac{1}{2}bh$ Note: h is the perpendicular height
Square		$A = s \times s$ or s^2
Rectangle		$A = lw$
Parallelogram		$A = bh$ h is the perpendicular height (at a right angle to the base).
Trapezium		$A = \frac{1}{2}(a+b)h$ h is the perpendicular height or the separation between the parallel sides.

Rhombus		$A=\frac{1}{2}\times d_1 d_2$ where d_1, d_2 are the length of the diagonals.
Circle		$A=\pi r^2$ Where r is the radius of the circle.

Example: Find the area of this parallelogram.



The area of a parallelogram is given the expression $A=bh$

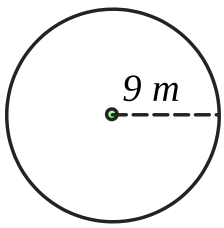
The value of b is *10.2cm* and h is *5.5 cm*

$$A=bh$$

$$A=10.2\times 5.5$$

$$A=56.1cm^2$$

Example: Find the area of this circle.



The area of a circle is given the expression

$$A=\pi r^2$$

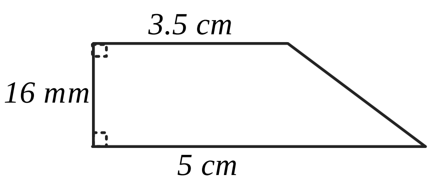
The value of r is *9 m*

$$A=\pi r^2$$

$$A=\pi\times 9^2$$

$$A=254.469m^2 (to 3d.p.)$$

Example: Find the area of this trapezium. The measurements given are in different units, so a conversion is necessary. This is a square trapezium so the separation between the parallel sides is equal to one of the sides. (The 16 mm side)



The area of a trapezium is given by the expression

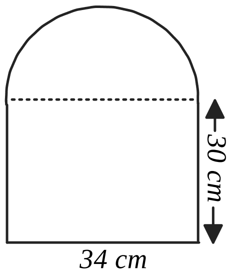
$$A=\frac{1}{2}(a+b)h$$

The value of h is *16 mm*. The lengths of the parallel sides are *3.5 cm* and *5 cm*, converted to *mm* are *35 mm* and *50 mm.*

$$A = \frac{1}{2}(a+b)h$$
$$A = \frac{1}{2}(35+50)\times 16$$
$$A = 680mm^2 \text{ or } 6.8cm^2$$
using the conversion $100mm^2 = 1cm^2$

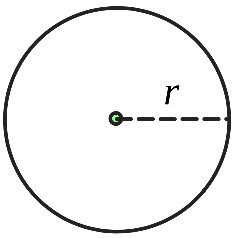
Example: Find the area of this composite shape.

The composite shape is a semicircle plus a rectangle. The dimensions of the rectangle are *30 cm* and *34 cm.* Because the diameter is the same as the length of the rectangle, the radius is *34 cm ÷ 2 = 17 cm.*



$$\begin{aligned}\text{Total Area } &= \frac{1}{2}\pi r^2 + lw \\ &= \frac{1}{2}\times\pi\times 17^2 + 34\times 30 \\ &= 453.96 + 1020 \\ &= 1473.96cm^2\end{aligned}$$

Example: Find the radius of a circle that has an area of $40m^2$?



$$\begin{aligned} A &= \pi r^2 \\ 40 &= \pi r^2 \\ \frac{40}{\pi} &= \frac{\not{\pi}^1 r^2}{\not{\pi}^1} \\ \frac{40}{\pi} &= r^2 \\ \sqrt{\frac{40}{\pi}} &= \sqrt{r^2} \\ \sqrt{\frac{40}{\pi}} &= r \\ r &\approx 3.568 \end{aligned}$$

Example: If fertiliser is applied at a paddock at a rate of 500kg/ha and the paddock is 300m long and 175m wide, what is the amount of fertilizer to be bought?

The area of the paddock is:

$$A = l\times w$$
$$A = 300\times 175$$
$$A = 52500m^2$$

The area units must be in ha, as there are 10 000m^2 in 1 ha, conversion to ha is performed by dividing by 10 000,

$$A = 52500 \div 10000$$
$$A = 5.25ha$$

The rate 500kg/ha means 500kg for 1 ha, so the amount required is 5.25ha x 500kg/ha = 2625 kg or 2.625 t.

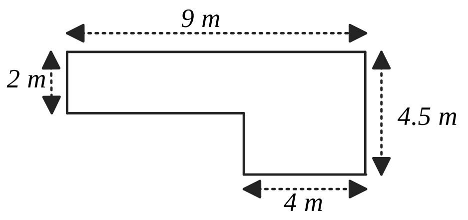
Video 'Area'

Activity

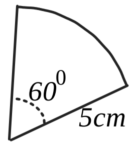
1. Draw a sketch diagram for each and calculate the area.
   (a) A square with a side length of 30 cm.
   (b) A rectangle with length of 0.6m and width 20cm
   (c) An right angled triangle with side lengths 27 cm, 35 cm and 44.2 cm
   (d) A circle with a radius of 45mm
   (e) An isosceles triangle with sides 47mm and 75mm (x2)
   (f) A semicircle with a diameter 1.2m

2. Calculate the area of these shapes.

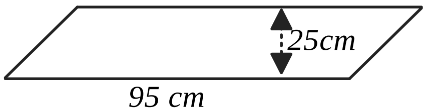
(a)



(b)

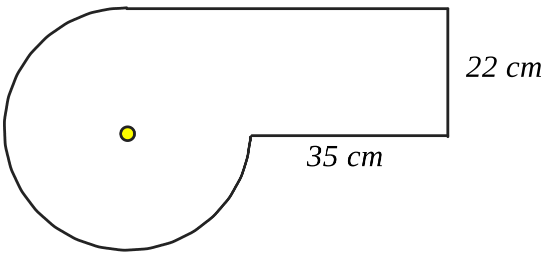


(c)

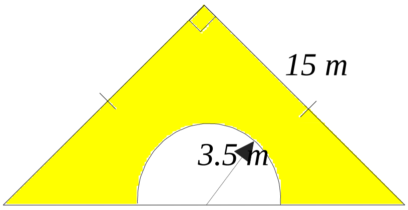


A parallelogram

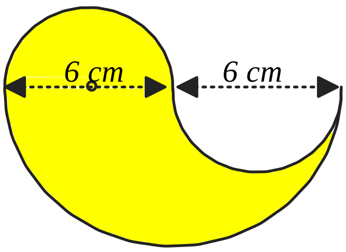
(d)



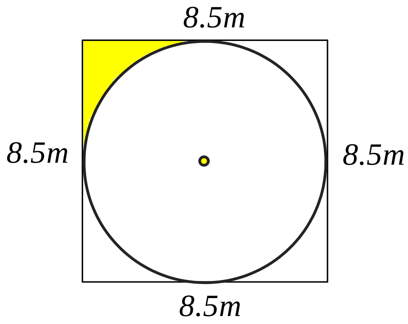
(e)



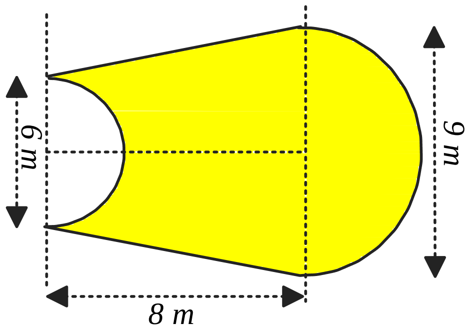
(f)



(g)



(h)



Topic 2: Measurement – length, perimeter and circumference

Naming Plane Shapes (polygons) and their properties

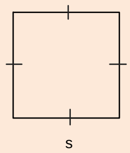
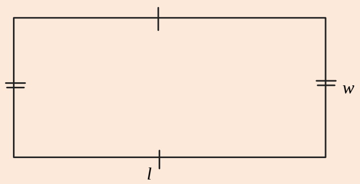
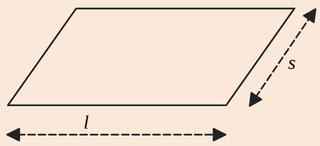
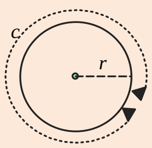
Triangles (3 sides)

- Equilateral – 3 equal sides, 3 equal angles (all 60°)
- Isosceles – 2 equal sides, 2 equal angles.
- Scalene – no equal sides
- Right angled – contains a right angle

Quadrilaterals (4 sides)

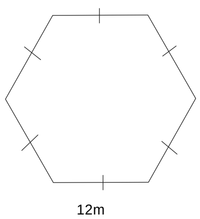
- Square – all sides equal in length, all angles 90° , diagonals equal in length and cut at 90°
- Rectangle – opposite sides equal, all angles 90°, diagonals equal in length
- Parallelogram – opposite sides equal in length and parallel, opposite angles equal
- Rhombus – all sides equal in length and parallel, opposite angles equal , diagonals cut at 90°
- Trapezium – one pair of parallel sides
- Kite – two pairs of adjacent sides equal in length, one pair of opposite angles equal, diagonals cut at 90°

The perimeter of a shape is the distance around the outside of that shape. The shape may consist of straight lines or curves. The perimeter of a circle is given the special name of **circumference**.

Plane Shape	Diagram	Perimeter
Any polygon	s_2 s_1 s_3 s_6 s_5 s_4	General – sum of side lengths $P = s_1 + s_2 + s_3 + s_4 + ... + s_n$
Triangle	s_1 s_1 s_1	For a scalene triangle $P = s_1 + s_2 + s_3$ For an isosceles $P = s_1 + 2s_2$ For an equilateral triangle $P = 3s_1$
Square		$P = 4 \times s$ -where s is the side length.
Rectangle		$P = 2l + 2w$ or $P = 2(l + w)$, -where l is the length and w is the width.
Parallelogram		$P = 2l + 2s$ - where l is the length of the base and s is the sloping height
Circle		$C = 2\pi r$ - where r is the radius of the circle.

Length, perimeter and circumference are all measures of length so the units for answer will be length units such as millimetres, centimetres, metres, kilometres…

Example: Find the perimeter of this regular hexagon.



A regular hexagon has 6 equal length sides (shown by the dashes), so the rule can be written down as;

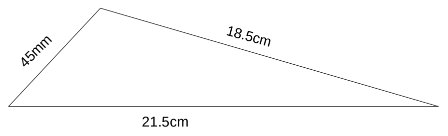
$P = 6s$

$P = 6 \times 12$

$P = 72m$

If units are not the same, conversions must take place before calculating the perimeter. Answers can be expressed either unit.

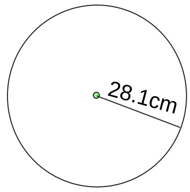
Example: Find the perimeter of this scalene triangle.

	18.5cm = 185mm, 21.5cm = 215mm, in mm, the side lengths are 45,185 and 215. $P = s_1 + s_2 + s_3$ $P = 45 + 185 + 215$ $P = 445mm$ or $44.5cm$

When doing measurement questions, the steps to a solution involve

- Drawing a diagram
- Writing a rule or formula (expression)
- Substituting in value
- Obtaining an answer including units

Example: Find the circumference of the circle below.



The rule for the circumference of circles is:

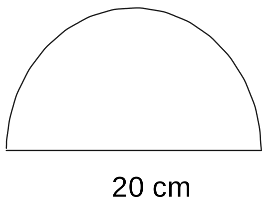
$C = 2\pi r$ Rule

$C = 2 \times \pi \times 28.1$ Substitution

$C = 176.6cm$ (to 1 d.p.) Answer with units

Some shapes are a fraction of a basic shape.

Example: Find the perimeter of the semi-circle below.



The perimeter of this shape is made up of 2 parts: semi-circumference and the diameter. The radius of this circle is a half of 20 cm = 10 cm

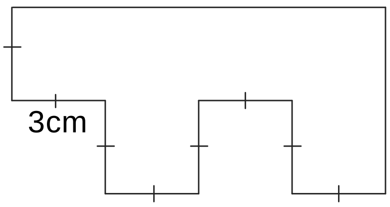
$P = \frac{1}{\not{2}} \times \not{2}\pi r + d$

$P = \pi \times 10 + 20$

$P = 51.42cm$ (to 2 d.p.)

Some shapes are composite shapes, made up of two or more basic shapes. Although not all measurements are given, they can be calculated from the measurements given.

Example: Find the perimeter of the shape below.



This shape has 10 sides, 8 sides measure 3cm. The top side measurement is equal to four 3cm lengths and the right side is equal to two 3cm lengths.

$P = 8s_1 + s_2 + s_3$

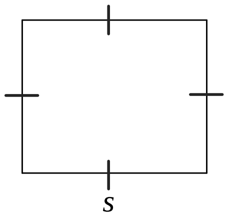
$P = 8 \times 3 + 12 + 6$

$P = 24 + 12 + 6$

$P = 42cm$

Sometimes the perimeter is given and a side length has to be calculated.

Example: Find the side length of a square that has a perimeter of 120 m.



$$P = 4 \times s$$

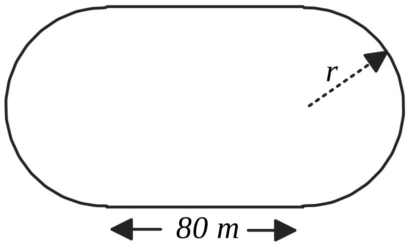
$$120 = 4 \times s$$

$$\frac{\cancel{120}^{30}}{\cancel{4}^{1}} = \frac{\cancel{4}^{1} \times s}{\cancel{4}^{1}}$$

The side length is 30 m

$$30 = s$$

Example: Find the radius of the semicircles on the 400 m running circuit drawn below.



The perimeter is made up 2 straights and 2 semicircles lengths (= 1 full circle circumference)

$$Perimeter = 2 \times 80 + 2\pi r$$

$$400 = 160 + 2\pi r$$

$$400 - 160 = 160 - 160 + 2\pi r$$

$$240 = 2\pi r$$

$$\frac{240}{2\pi} = \frac{\cancel{2\pi}^{1} r}{\cancel{2\pi}^{1}}$$

$$38.2 = r$$

The radius of the semicircles is approx 38.2 m

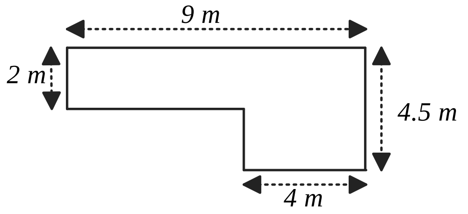
Video ‘Perimeter’

Activity

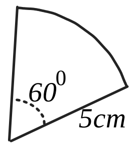
1. Draw a sketch diagram for each and calculate the perimeter.
   - (a) A square with a side length of 30 cm.
   - (b) A rectangle with length of 0.6m and width 20cm
   - (c) An equilateral triangle with side length 27cm
   - (d) A circle with a radius of 45mm
   - (e) An isosceles triangle with sides 47mm and 75mm (x2)
   - (f) A semicircle with a diameter 1.2m

2. Calculate the perimeter of these shapes.

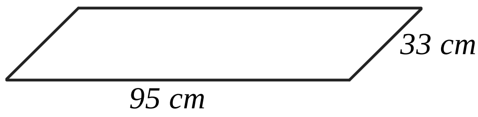
(a)



(b)

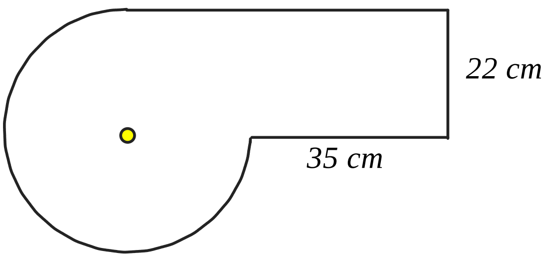


(c)

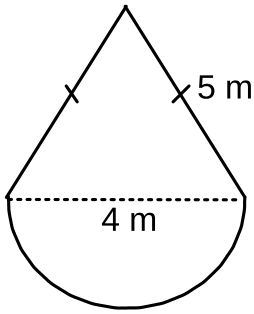


A parallelogram

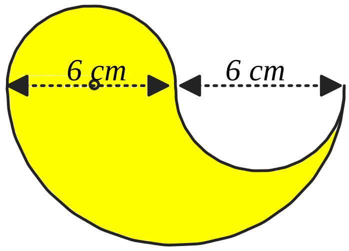
(d)



(e)



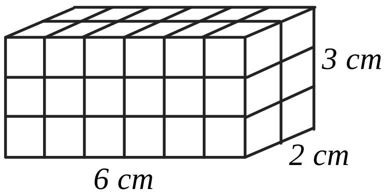
(f)



Topic 3: Measurement – volume

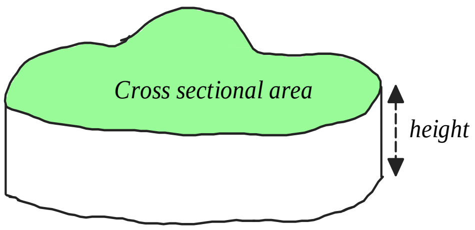
You might recall from the units section of this module that a volume of 1 cubic metre is a cube with side length of 1 metre. The purpose of these rules is to provide a simple way of counting the number of cubic metres, cubic centimetres or cubic millimetres within a shape.

For example:



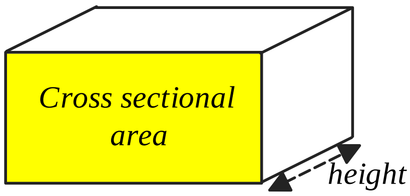
The rectangular prism is split up into centimetre divisions. The rectangular prism contains 6 x 2 x 3 = 36 cubes of volume 1 cm^3 each giving a volume of 36 cm^3. This can be generalised as $V = l \times w \times h$

The first group of solids being considered are the prisms. A prism is a 2 dimensional shape such as a rectangle, triangle, circle …. that is extended into the third dimension. This solid will have the same cross sectional area along its height.



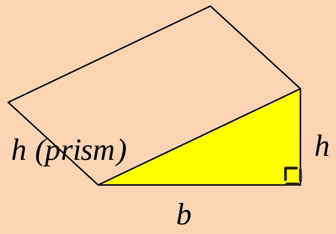
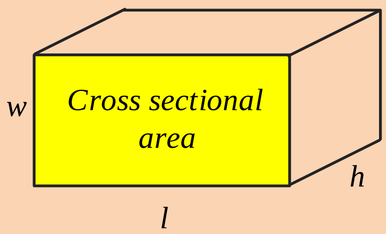
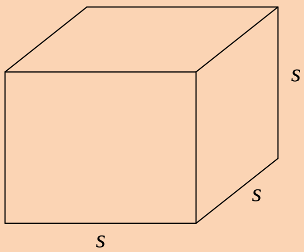
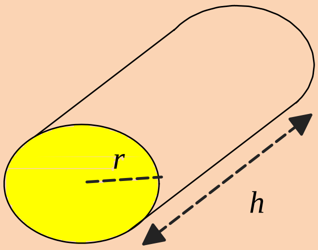
This is a general prism. The cross sectional area is shaded and the height is shown. The volume of any prism is $V_{prism} = A \times h$, where A is the cross sectional area and h is the height of the prism.

The cross sectional area of the prism can be one of the plane (flat) shapes used above.

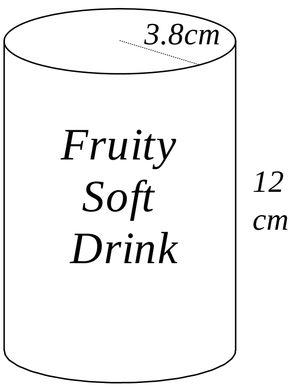


The solid drawn is a rectangular prism because the cross sectional area is a rectangle. Using the volume of a prism expression $V_{prism} = A \times h$ and with the area of a prism $A = l \times w$, the volume of a rectangular prism can be written as $V_{prism} = A \times h$

$$V = l \times w \times h$$

Prism	Diagram	Rule
Triangular Prism		$V = \left(\frac{1}{2}bh\right) \times h_{prism}$ Note: there are two h values: the first is the perpendicular height of the triangle on the end and the second is the height of the prism
Rectangular Prism		$V = A \times h$ $V = l \times w \times h$
Cube		$V = s \times s \times s$ $V = s^3$
Cylinder (circular prism)		$V = Ah$ $V = \pi r^2 h$

Example: Find the volume in cm^3 and mL of the can drawn below.



The volume of a cylinder is given by the expression

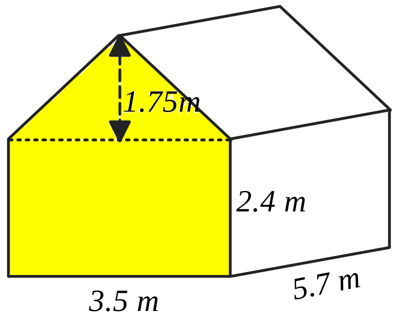
$V = \pi r^2 h$

$r = 3.8cm \quad h = 12cm$

$V = \pi \times 3.8^2 \times 12$

$V = 544.4cm^3$ or $544.4mL$

Example: Find the volume of the shape below. The area of the end of the prism is a composite shape consisting of a triangle and a rectangle.



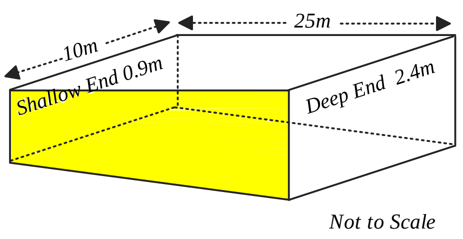
The area of the end is:

$$A=(l\times w)+\left(\frac{1}{2}bh\right)$$
$$A=(3.5\times 2.4)+\left(\frac{1}{2}\times 3.5\times 1.75\right)$$
$$A=8.4+3.0625$$
$$A=11.4625m^2$$

The volume is:

$$V=A\times h$$
$$V=11.4625\times 5.7$$
$V=65.33625$ or $65.34m^3$ to 2 d.p.

Example: Find the volume of water required to fill this pool. Note that the pool has a shallow and deep end adding to the complexity of the question.



The shaded end of the pool is a trapezium, with the parallel sides being 0.9m and 2.4m. The side are separated by 25m.

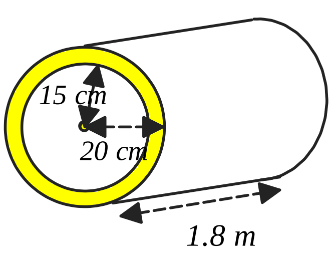
$$A=\frac{1}{2}(a+b)h$$
$$A=\frac{1}{2}(0.9+2.4)\times 25$$
$$A=41.25m^2$$

The volume is

$$V=Ah$$
$$V=41.25\times 10$$
$V=412.5m^3$ or 412.5 klitres or 412 500 litres

(Remember $1m^3=1kL$)

Example: Find the volume of concrete required to construct this pipe.



This end shape is called an annulus.

Finding the area of the annulus:

$$A_{end}=A_{\text{large circle}}-A_{\text{small circle}}$$
$$A=\pi R^2-\pi r^2$$
$$A=\pi\times 20^2-\pi\times 15^2$$
$$A=1256.6-706.9$$
$$A=549.7cm^2$$

To find the volume, the length must be converted to cm, which is 180 cm.

$$V = A \times h$$
$$V = 549.7 \times 180$$
$$V = 98946 cm^3$$

As pre-mix concrete is usually ordered in m^3, a conversion is required. There are $100^3 cm^3 = 1^3 m^3$, that is 1 000 000 cm^3 = $1m^3$, dividing by 1 000 000 is required. Volume required is 0.098 946m^3 or approximately 0.1m^3

Pyramids

A pyramid is a solid, three dimensional shape with a plane shape as its base and the sides coming to a single vertex at the top.

	This is a hexagonal pyramid because the base is a hexagon and the sides come up to a single vertex.		This is a rectangular pyramid.

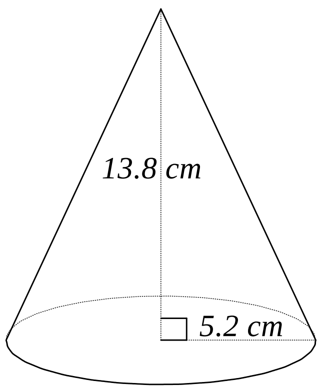
The volume of any pyramid is $V = \frac{1}{3}Ah$ where *A* is the area of the base.

The volume of a square pyramid is $V = \frac{1}{3}s^2h$

The volume of a rectangular pyramid is $V = \frac{1}{3}lwh$

The volume of a cone is $V = \frac{1}{3}\pi r^2 h$

Example: Find the volume of the cone below.



Area of base is:

$$A = \pi r^2$$
$$A = \pi \times 5.2^2$$
$$A = 84.95 cm^2$$

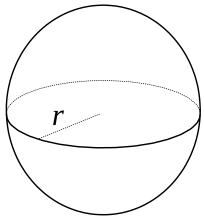
Volume of the cone is :

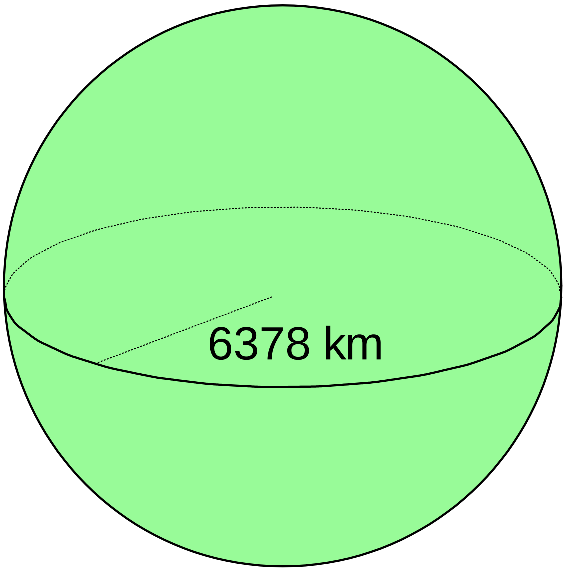
$$V = \frac{1}{3}Ah$$
$$V = \frac{1}{3} \times 84.95 \times 13.8$$
$$V = 390.77 cm^3$$

Sphere

A sphere is a ball like figure.

The volume of a sphere is $V = \frac{4}{3}\pi r^3$





The radius of the Earth is 6378 km, find the volume of the Earth assuming it is perfectly spherical.

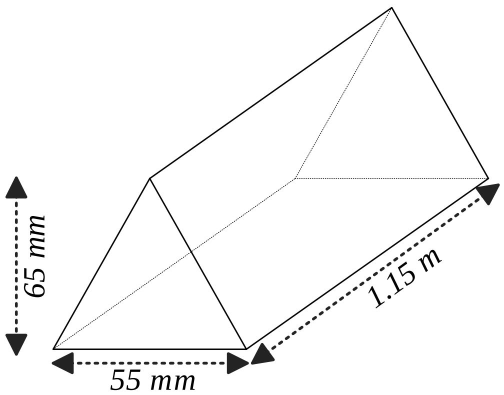
$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3} \times \pi \times 6378^3$$
$$V = 1.09 \times 10^{12}\, km^3$$
$$or \;\; 1\,090\,000\,000\,000\, km^3$$

Video 'Volume'   Video 'Other Ideas'

Activity

1. Draw a sketch diagram for each and calculate the volume.
   (a) A cube with a side length of 20 cm.
   (b) A rectangle prism with length of 0.6m, width of 20cm and height 500mm.
   (c) A prism with a base area of 28 m^2 and a height of 2.7m.
   (d) A cylinder with a diameter of 50 cm and a height of 60 cm. Give your answer in mL and L.
   (e) A prism with a semicircular base (radius = 51 cm) and a height of 63cm

2. Calculate the volume of these solids.
   (a)



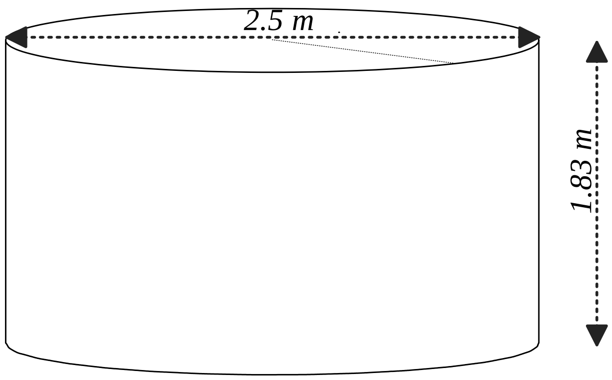
Express the answer in

i mm^3

ii cm^3

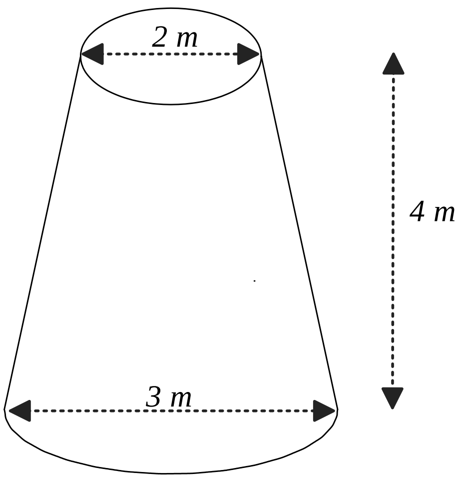
iii m^3

(b)



This is a water tank. Express your answer in L.

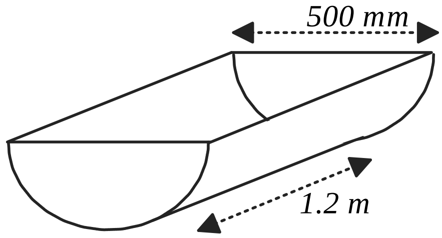
(c)



Hint: Volume of large cone subtract the volume of small cone.

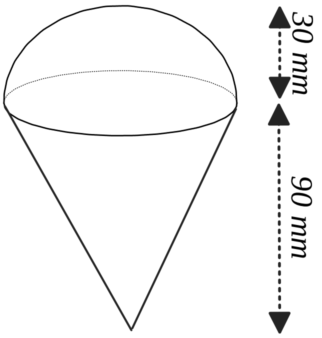
(e) A farmer has a large dam covering 3 ha of land. The average depth when full is 3m. Calculate the volume of water in dam in kL and ML.

(f)



A farmer has built these troughs to hold water for livestock. How many litres of water will each trough hold?

(g)



This is an ice-cream cone. Calculate the volume of soft serve ice cream that would fill the cone and a semi-spherical amount on top of the cone.

(h) A garden hose has an internal diameter of 15 mm and a length of 5 m. What volume of water fills the hose?

(i) A can contains 375 mL of soft drink and must have a height of 120 mm. What will the diameter need to be?

Answers to activity questions

Check your skills

1. (a) Length units such as mm, cm, m, km

   (b) Area units such as $mm^2, cm^2, m^2, ha\ or\ km^2$

   (c) Volume units such as mm^3, cm^3, m^3, km^3

2. (a)
$$P = 2l + 2w$$
$$P = 2 \times 140 + 2 \times 55$$
$$P = 390m$$

   (b)
$$A = \pi r^2 \div 2$$
$$A = \pi \times 7.5^2 \div 2$$
$$A = 88.36m^2$$

   (c)
$$V = \frac{1}{3} \times s^2 h$$
$$V = \frac{1}{3} \times 1.4^2 \times 2.1$$
$$V = 1.372m^3$$
$$or\ \ 1.372kL\ \ or\ \ 1372L$$

3. (a) The perimeter is basically the circumference of two circles with a diameter of 10cm. If the diameter is 10 cm, the radius is 5 cm.
   The circumference C = 2 x 3.142 x 5 = 31.42cm
   The perimeter of the shape is 2 x 31.42 = 62.84 cm

   (b) The area of the shape is 2 circles with a radius of 5 cm and a square with side length 5 cm.
   Area of circle = 3.142 x 5^2 = 78.54 cm^2
   Area of square = 10 x 10 = 100 cm^2

   Total area = 2 x 78.54 + 100 = 257.08 cm^2

4.

$$V = \frac{4}{3}\pi r^3$$
$$200 = \frac{4}{3}\pi r^3$$
$$\frac{200 \times 3}{4 \times \pi} = r^3$$
$$\sqrt[3]{\frac{200 \times 3}{4 \times \pi}} = r$$
$$r = 3.63cm$$

A sphere with radius 3.63cm will have a volume of 200cm^3.

Measurement - length, perimeter and circumference

1. Draw a sketch diagram for each and calculate the perimeter.

(a) A square with a side length of 30 cm.

$$P = 4s$$
$$P = 4 \times 30$$
$$P = 120cm$$

(b) A rectangle with length of 0.6m and width 20cm (0.6m = 60cm)

$$P = 2(l + w)$$
$$P = 2 \times (60 + 20)$$
$$P = 2 \times 80$$
$$P = 160cm$$

(c) An equilateral triangle with side length 27cm

$$P = 3s$$
$$P = 3 \times 27$$
$$P = 81cm$$

(d) A circle with a radius of 45mm

$$C = 2\pi r$$
$$C = 2 \times \pi \times 45$$
$$C = 282.7mm$$

(e) An isosceles triangle with sides 47mm and 75mm (x2)

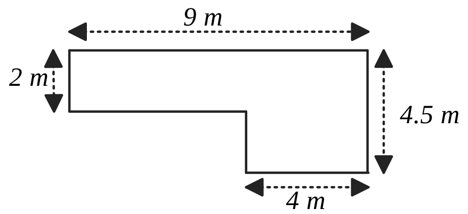
$$P = 2s_1 + s_2$$
$$P = 2 \times 75 + 47$$
$$P = 197mm$$

(f) A semicircle with a diameter 1.2m

$$P = 2\pi r \div 2 + d$$
$$P = 2 \times \pi \times 0.6 \div 2 + 1.2$$
$$P = 3.085m$$

2.

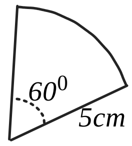
(a)



Unknown sides are $9m-4m=5m$ and $4.5m-2m=2.5m$.

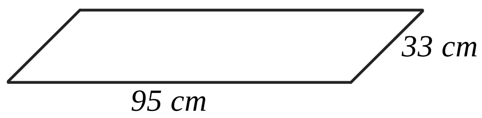
$$P=9+4.5+4+2.5+5+2$$
$$P=27m$$

(b)



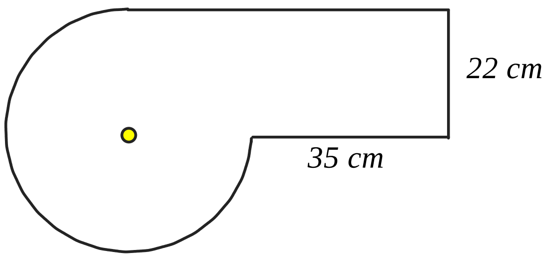
$$P=\frac{60}{360}\times 2\pi r+2r$$
$$P=\frac{1}{6}\times 2\times\pi\times 5+2\times 5$$
$$P=15.236cm$$

(c)



$$P=2l+2s$$
$$P=2\times 95+2\times 33$$
$$P=256$$

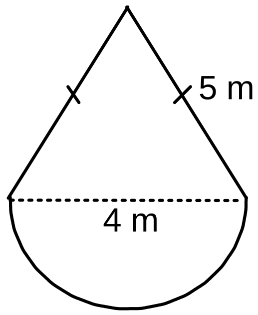
(d)



Missing side is 35+22 (radius) = 57

$$P=\frac{3}{4}\times 2\pi r+35+22+57$$
$$P=217.7cm$$

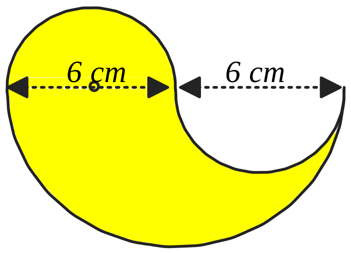
(e)



Radius =2m

$$P=\frac{1}{\not 2}\times\not 2\pi r+2s$$
$$P=\pi\times 2+2\times 5$$
$$P=16.28m$$

(f)



Perimeter is equivalent to the circumference of a small circle (r=3) plus half the circumference of the big circle (r=6)

$$P=2\pi r+\frac{1}{\not 2}\times\not 2\pi R$$
$$P=2\times\pi\times 3+\pi\times 6$$
$$P=37.7cm$$

Area

1. Draw a sketch diagram for each and calculate the area.

(a) A square with a side length of 30 cm.

$A = s \times s$

$A = 30 \times 30$

$A = 900cm^2$

(b) A rectangle with length of 0.6m and width 20cm

$A = l \times w$

$A = 60 \times 20$

$A = 1200cm^2$

(c) An right angled triangle with side lengths 27 cm, 35 cm and 44.2 cm

The base and height are 27cm and 35cm, the 44.2cm side is unnecessary.

$A = \frac{1}{2} \times b \times h$

$A = \frac{1}{2} \times 27 \times 35$

$A = 472.5cm^2$

(d) A circle with a radius of 45mm

$A = \pi r^2$

$A = \pi \times 45^2$

$A = 6361.73mm^2$

$or\ 63.6173cm^2$

(e) An isosceles triangle with base 47mm and height 71.2mm

$A = \frac{1}{2} \times b \times h$

$A = \frac{1}{2} \times 47 \times 71.2$

$A = 1673.2mm^2 = 16.732cm^2$

(f) A semicircle with a diameter 1.2m

(if d=1.2m, then r=0.6m)

$A = \frac{1}{2} \times \pi r^2$

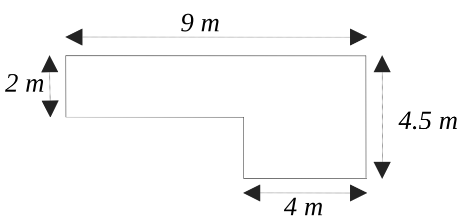
$A = \frac{1}{2} \times \pi \times 0.6^2$

$A = 0.565m^2$

$A = 5650cm^2$

2. Calculate the area of these shapes.

(a)

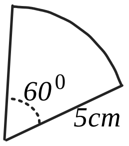


$A_T = l \times w + l \times w$

$A_T = 4 \times 4.5 + 2 \times (9 - 4)$

$A_T = 28m^2$

(b)

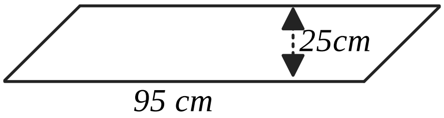


$$A = \frac{\not{60}^{1}}{\not{360}^{6}} \times \pi r^2$$

$$A = \frac{1}{6} \times \pi \times 5^2$$

$$A = 13.1cm^2$$

(c)



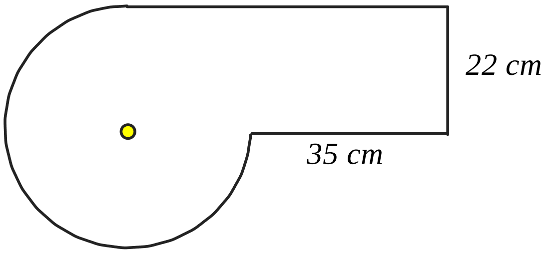
A parallelogram

$$A = b \times h$$

$$A = 95 \times 25$$

$$A = 2375cm^2$$

(d)

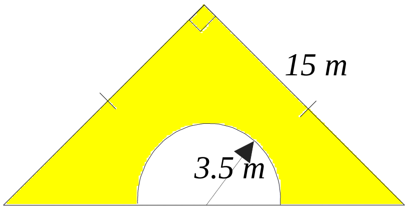


$$A_T = \frac{3}{4} \times \pi r^2 + l \times w$$

$$A_T = \frac{3}{4} \times \pi \times 22^2 + 57 \times 22$$

$$A_T = 2394.4cm^2$$

(e)



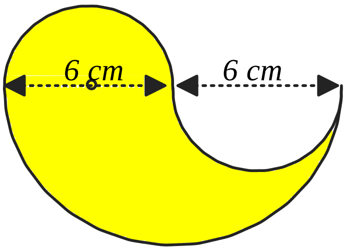
$$A_{shaded} = A_{triangle} - A_{semicircle}$$

$$A_{shaded} = \frac{1}{2}bh - \frac{1}{2}\pi r^2$$

$$A_{shaded} = \frac{1}{2} \times 15 \times 15 - \frac{1}{2} \times \pi \times 1.75^2$$

$$A = 107.7m^2$$

(f)



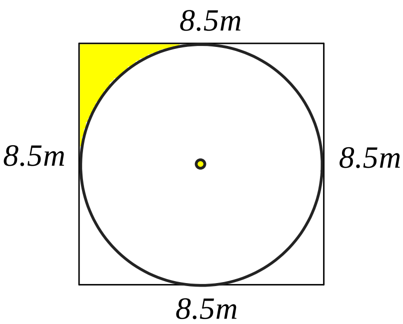
The area of the small semicircle is the same as the area of the semicircle missing from the large semicircle, only the area of the large semicircle is required.

$$A = \frac{1}{2}\pi r^2$$

$$A = \frac{1}{2} \times \pi \times 6^2$$

$$A = 56.55cm^2$$

(g)



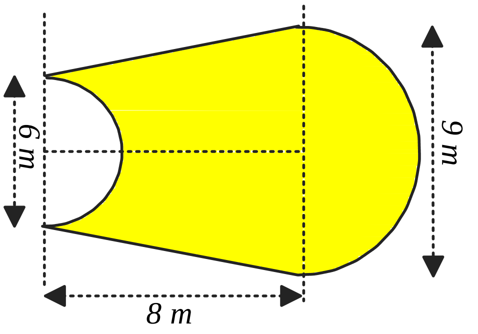
The shaded area is one quarter of the difference is area between the area of the square and the circle.

$$A = \frac{1}{4}\left(s^2 - \pi r^2\right)$$

$$A = \frac{1}{4}\left(8.5^2 - \pi \times 4.25^2\right)$$

$$A = 3.876m^2$$

(h)



The shaded area is the area of the trapezium in the centre add the area of the large semi-circle subtract the area of the small semicircle. The trapezium has parallel side lengths 6m and 9m separated by a length 8m.

$$A = \frac{1}{2}(a+b)h + \frac{1}{2}\pi R^2 - \frac{1}{2}\pi r^2$$

$$A = \frac{1}{2}(6+9)\times 8 + \frac{1}{2}\pi \times 4.5^2 - \frac{1}{2}\pi \times 3^2$$

$$A = 60 + 31.8 - 14.14$$

$$A = 77.66m^2$$

Volume

1. Draw a sketch diagram for each and calculate the volume.

(a) A cube with a side length of 20 cm.

$$V = s \times s \times s$$
$$V = 20 \times 20 \times 20$$
$$V = 8000cm^3$$

(b) A rectangle prism with length of 0.6m, width of 20cm and height 500mm. (0.6m = 60cm, 500mm = 50cm)

$$V = l \times w \times h$$
$$V = 60 \times 20 \times 50$$
$$V = 60\,000cm^3$$

(c) A prism with a base area of 28 m^2 and a height of 2.7m.

$$V = Ah$$
$$V = 28 \times 2.7$$
$$V = 75.6m^3$$

(d) A cylinder with a diameter of 50 cm and a height of 60 cm. Give your answer in mL and L.

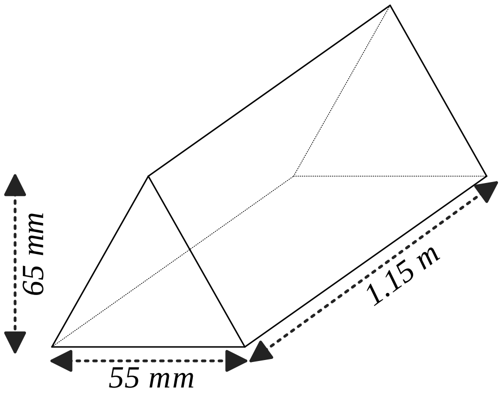
$$V = Ah$$
$$V = \pi r^2 h$$
$$V = \pi \times 25^2 \times 60$$
$$V = 117810cm^3$$
$$V = 117810mL = 117.81L$$

(e) A prism with a semicircular base (radius = 51 cm) and a height of 63cm

$$V = Ah$$
$$V = \frac{1}{2}\pi r^2 h$$
$$V = \frac{1}{2}\pi \times 51^2 \times 63$$
$$V = 257\,395.4cm^3$$

2. Calculate the volume of these solids.

(a)



Express the answer in
(i) mm^3 (ii) cm^3 (iii) m^3

$V = Ah$

$V = \frac{1}{2} bh_{triangle} \times h_{prism}$

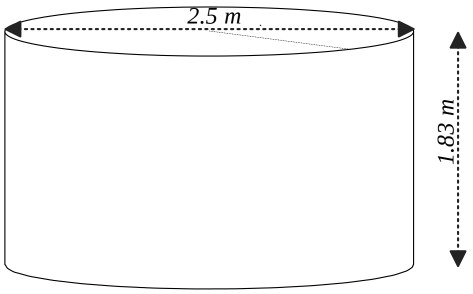
$V = \frac{1}{2} \times 6.5 \times 5.5 \times 115$

$V = 2\,055.625 cm^3$

$V = 2\,055.625 \times 1000 = 2\,055\,625 mm^3$

$V = 2\,055.625 \div 1\,000\,000 = 0.002\,055\,625 m^3$

(b)



This is a water tank. Express your answer in L. $(r = 2.5 \div 2 = 1.25m)$

$V = Ah$

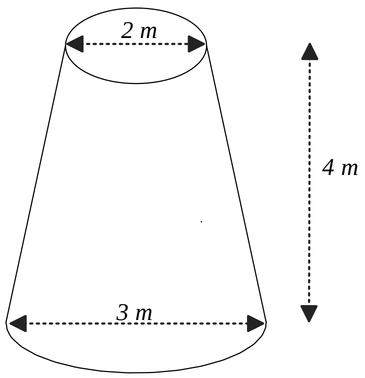
$V = \pi r^2 h$

$V = \pi \times 1.25^2 \times 1.83$

$V = 8.983 m^3$

$V = 8.983 kL = 8\,983 L$

(c)



Volume of large cone (h=12m, d=3m) subtract the volume of small cone (h=8m, d=2m).

$V = \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \times 1.5^2 \times 12 - \frac{1}{3}\pi \times 1^2 \times 8$

$V = 28.27 - 8.38$

$V = 19.89 m^3$

(e) A farmer has a large dam covering 3 ha of land. The average depth when full is 3m. Calculate the volume of water in dam in kL and ML.
(3 ha = 30 000m^2)

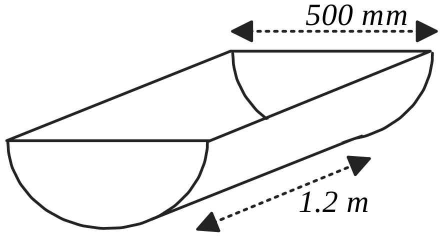
$V = Ah$

$V = 30\,000 m^2 \times 3m$

$V = 90\,000 m^3$

$V = 90\,000 kL = 90 ML$

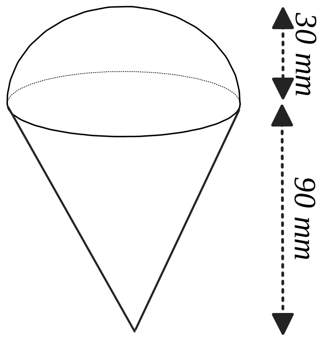
(f)



A farmer has built these troughs to hold water for livestock. How many litres of water will each trough hold?

$$V = \frac{1}{2}\pi r^2 h$$
$$V = \frac{1}{2}\pi \times 0.25^2 \times 1.2$$
$$V = 0.118m^3$$
$$V = 0.118kL = 118L$$

(g)



This is an ice-cream cone. Calculate the volume of soft serve ice cream that would fill the cone and a semi-spherical amount on top of the cone. Using cm,

$$V_T = V_{cone} + V_{hemisphere}$$
$$V_T = \frac{1}{3}\pi r^2 h + \frac{1}{\not{2}} \times \frac{\not{4}^2}{3}\pi r^3$$
$$V_T = \frac{1}{3}\pi \times 3^2 \times 9 + \frac{2}{3}\pi \times 3^3$$
$$V_T = 84.8 + 56.5$$
$$V_T = 141.3cm^3$$

(h) A garden hose has an internal diameter of 15 mm and a length of 5 m. What volume of water fills the hose?

The hose is a cylinder
r=7.5mm=0.75cm, h=500cm

$$V = \pi r^2 h$$
$$V = \pi \times 0.75^2 \times 500$$
$$V = 883.6cm^3$$
$$V = 883.6cm^3$$
$$V = 883.6mL$$

(i) A can contains 375 mL of soft drink and must have a height of 120 mm. What will the diameter need to be?
(375 mL=375cm^3)
(120mm=12cm)

$$V = \pi r^2 h$$
$$\frac{V}{\pi h} = r^2$$
$$\sqrt{\frac{V}{\pi h}} = r$$
$$r = \sqrt{\frac{V}{\pi h}}$$
$$r = \sqrt{\frac{375}{\pi \times 12}}$$
$$r = 3.154cm$$
$$\rightarrow d = 6.31cm$$