Introduction to percentages

Percentages are commonly used in everyday language to express fractional numbers as whole numbers mostly between zero and one hundred which is the range of numbers that most people are comfortable with using.

For example: A shop offering discounts could advertise 0.1 off, one tenth off or 10% off, the figure 10% is in the range 0 to 100 which most people find easy to understand.

The word gives a strong clue to its meaning. Per means 'out of' and Cent means '100' so percentages are numbers out of 100 or $\frac{\quad}{100}$.

This diagram has 13% shaded, that is, 13 squares shaded out of 100

17% means 17 out of 100. If in a room of 100 people, 17 were left handed, it is possible to say that 17% are left handed.

36% not only means 36 out of 100, but also means 72 out of 200, 108 out of 300 etc. It can also mean 18 out of 50, 9 out of 25.

If the values are suitable, finding a percentage of a number is possible using just the meaning of percentages.

For example: Find 27% of 1500

27% means 27 out of 100,
as 1500 is 15 lots of 100,
27% of 1500 will be 27 x 15 =405

Many shopkeepers are able to work percentages out based on the 10% amount.

10% is special because 10% is 10 out of 100 $\left(\frac{10}{100}\right)$ or 1 out of 10 $\left(\frac{1}{10}\right)$ or one tenth.

Finding one tenth of a number is quite straight forward, a move of a decimal point or dropping a zero can achieve this.

For example:

10% of $45 is $4.50
10% of 340 smarties is 34
10% of 1.25m is 0.125m

Extending this: 25% of $36 can be thought of as 10% + 10% + 5%
10% of $36 is $3.60 → 5% of $36 is $1.80

So 25% of $36 is $3.60 + $3.60 + $1.80 = $9

The other sections are calculations with percentages where it is difficult to do mental calculations due to the nature of the numbers.

Module contents

Introduction

- **Conversions between fractions, decimals and percentages**
- **Using percentages**
- **Making percentages**
- **Harder questions: using equations**

Answers to activity questions

Outcomes

- Convert between fractions, decimals and percentages.
- Solve problems where a percentage is used to determine the answer.
- Solve problems where the answer will be a percentage.
- Use equations to solve harder percentage problems.

Check your skills

This module covers the following concepts, if you can successfully answer these questions, you do not need to do this module. Check your answers from the answer section at the end of the module.

1. Fill in the missing spaces:

Fraction	Decimal	Percentage
$\frac{7}{20}$		
	0.08	
		12.5%

2. (a) Find 45% of $640.
   (b) Increase 345 000 by 22.5%

3. (a) What percentage is 45 out of 75.
   (b) A pair of ear-rings was reduced in price from $90 to $68. Calculate the percentage price reduction.

4. Jane went to a 15% off sale and received a discount of $8.40 on a pair of jeans. How much did the jeans originally cost and how much did Jane pay?

Topic 1: Conversions between fractions, decimals and percentages

Converting from a percentage

Using the meaning of percentages, 'out of 100 $\left(\frac{\quad}{100}\right)$', a percentage can be easily converted to a decimal (divide by 100) or a fraction (over a hundred).

For example:

$12\% \rightarrow \frac{12}{100}$ as a fraction. This fraction can be simplified to its lowest form.

$12\% \rightarrow \frac{12^{\div 4}}{100^{\div 4}} = \frac{3}{25}$

or

$12\% \rightarrow 12 \div 100 = 0.12$ as a decimal.

When solving questions involving percentages, the first step is to change the percentage to either a fraction or a decimal. Using fractions or decimals is largely a personal choice.

$22\% \rightarrow \frac{22}{100} = \frac{11}{50}$ as a fraction

$22\% \rightarrow 22 \div 100 = 0.22$ as a decimal

The fractional percentage $12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100}$ is a problem because the definition of a fraction does not allow for having a fraction within a fraction.

Change to an improper fraction, then because the denominator of the top fraction is 2, getting an equivalent fraction by multiplying the numerator and denominator by 2 will eliminate the fraction.

$$12\frac{1}{2}\% = \frac{25}{2}\% = \frac{\left(\frac{25}{2}\right)^{\times 2}}{100^{\times 2}} = \frac{25^{\div 25}}{200^{\div 25}} = \frac{1}{8}$$

To change $12\frac{1}{2}\%$ to a decimal, change the fraction part to a decimal first.

$$12\frac{1}{2}\% = 12.5\% = 12.5 \div 100 = 0.125$$

$0.4\% = \frac{0.4^{\times 10}}{100^{\times 10}} = \frac{4^{\div 4}}{1000^{\div 4}} = \frac{1}{250}$ as a fraction, $0.4\% = 0.4 \div 100 = 0.004$ as a decimal.

$75\% = \frac{75^{\div 25}}{100^{\div 25}} = \frac{3}{4}$ as a fraction, $75\% = 75 \div 100 = 0.75$ as a decimal.

Converting to a percentage

Because percentages are 'out of 100', a decimal or fraction has to be multiplied by 100 to become a percentage.

Decimals are easily multiplied by 100.

$$0.65 \rightarrow 0.65 \times 100\% = 65\%$$

$$0.6 \rightarrow 0.6 \times 100\% = 60\%$$

$$0.00352 \rightarrow 0.00352 \times 100\% = 0.352\%$$

When changing fractions to percentages, still multiply by 100 in the form of $\frac{100}{1}$

$$\frac{4}{\cancel{5}^{1}} \times \frac{\cancel{100}^{20}}{1}\% = \frac{80}{1}\% = 80\%$$

$\frac{3}{7} \times \frac{100}{1}\% = \frac{300}{7}\%$ $7\overline{)30^{2}0.^{6}0^{4}0^{5}0}$ $= 42.86\%$ to 2 decimal places.

$$1\frac{1}{4} \times \frac{100}{1}\% = \frac{5}{\cancel{4}^{1}} \times \frac{\cancel{100}^{25}}{1}\% = \frac{125}{1}\% = 125\%$$

An alternative method is changing the fraction to a decimal first and then multiplying by 100.

$$\frac{4}{5} = \frac{8}{10} = 0.8 \times 100\% = 80\%$$

Note: A percentage over 100% means more than one whole.

Video 'Conversions between Fractions, Decimals and Percentages'

Activity

1. Fill in the missing spaces in the table:

Fraction	Decimal	Percentage
$\frac{3}{4}$		
	0.45	
		6%
$\frac{2}{3}$		
	0.625	
		125%

Topic 2: Using percentages

Finding the percentage of a quantity is a very common calculation. It is required to calculate discounts, mark-ups, etc. In these questions the first step is to change the percentage to a fraction or decimal. Using fractions or decimals is a personal choice. The word 'of' is a cue for multiplication.

Examples:

Find 35% of $24.50

As a decimal	or	As a fraction

$$0.35 \times 24.50$$
$$= 8.575$$
$$= \$8.58 \text{ to the nearest cent.}$$

$$\frac{\cancel{35}^{7}}{\cancel{100}^{20}} \times \frac{24.50}{1}$$
$$= \frac{7 \times 24.50}{20}$$
$$= 8.575$$
$$= \$8.58 \text{ to the nearest cent}$$

By Calculator	
0 • 3 5 × 2 4 • 5 = 8.575	3 5 ÷ 1 0 0 × 2 4 • 5 = 8.575

What is 5.8% of $ 195 000

As this percentage contains a decimal, finding a solution by changing to a decimal would be the preferred method.

$$5.8\% \times 195\,000$$
$$= 0.058 \times 195\,000$$
$$= \$11\,310$$

Evaluate $15\frac{1}{4}\%$ of 512 people

This percentage contains a fraction, so there are two options: convert the fraction part of the percentage to a decimal and solve as a decimal or solve by fractions.

As a decimal	or	As a fraction
$15\frac{1}{4}\% \times 512$ $= 15.25\% \times 512$ $= 0.1525 \times 512$ $= 78.08$ $= 78 \text{ people}$		$15\frac{1}{4}\% \times 512$ $= \frac{\left(\frac{61}{4}\right)^{\times 4}}{100^{\times 4}} \times 512$ $= \frac{61}{400} \times 512$ $= 78.08$ $= 78 \text{ people}$
By Calculator		
0 • 1 5 2 5 × 5 1 2 = 78.08		6 1 ▤ 4 0 0 × 5 1 2 = 78.08 or 1 5 (▬음) 1 4 ÷ 1 0 0 × 5 1 2 = 78.08

Increase \$455 by 15%.

This question can be performed two ways.

First Method	Second Method
The first method is to find 15% of \$455 and then add this amount on to \$455.	The second method takes the amount \$455 as 100% and increases this by 15% to 115%, then calculate 115% of \$455.
$15\% \text{ of } \$455$ $= 0.15 \times 455$ $= \$68.25$ New Amount is $\$455 + \$68.25$ $= \$523.25$	$100\% + 15\% = 115\%$ $115\% \text{ of } \$455$ $= 1.15 \times 455$ $= \$523.25$

Decrease 612 smarties by 8.5%

Like the question above, this question can be performed two ways.

First Method	Second Method
The first method is to find 8.5% of 612 and then subtract this amount from 612.	The second method takes the 612 smarties as 100% and decreases this by 8.5% to 91.5%; then calculate 91.5% of 612.
8.5% of 612 $= 0.085 \times 612$ $= 52.02$ or 52 smarties New Amount is $612 - 52$ $= 560$	$100\% - 8.5\% = 91.5\%$ 91.5% of 612 $= 0.915 \times 612$ $= 559.98$ or 560 smarties

Now a problem solving question:

A car dealer buys a car from a manufacturer for $15 500. The dealer increases the price by 8% to cover costs. After costs are added, the price is then subject to GST of 10%. Melissa negotiates a discount of 2.5%, what does she pay for the car?

The cost price of the car is $15 500.

The selling price is $15 500 + 8%= $16 740 $(1.08 \times 15\,500 = 16\,740)$

The selling price after GST is $16 740 + 10% = $18414 $(1.1 \times 16\,740 = 18\,414)$

Melissa pays $18 414 – 2.5%= $17 953.65 $(0.975 \times 18\,414 = 17\,953.65)$

Always answer problem solving questions in a sentence.

Melissa will pay $17 953.65 for the car.

Video ‘Using Percentages’

Activity

1. Find 25% of $560

2. Calculate 8.5% of $155 000

3. Evaluate $5\frac{2}{5}$% of the population of a town (27450 people)

4. On any particular day, about 12% of students are absent from school. In a school of 840 students, how many would you expect to be absent on a typical day?

5. Decrease 32cm by 12.5%

6. Increase 36 mins by 20%

7. A tiler needs exactly 163 tiles to do a job. He should allow 10% for cutting and breakages, how many boxes will he order if he must buy whole boxes containing ten tiles.

8. After Easter, Easter eggs are discounted by $33\frac{1}{3}\%$. If an egg basket is priced at $12.50, how much discount is obtained and what is the discounted price?

Topic 3: Making percentages

Once again because percentages are commonly in the range of 0 to 100, using percentages to express test scores is preferable to using a raw score. In a question such as 'What percentage is 12 out of 15?' there is a part number and a total number. Make this into a fraction with the part number in the numerator and the total number in the denominator and multiply by a hundred to make a percentage.

$$\text{Percentage\%} = \frac{\text{Part number}}{\text{Total Number}} \times 100$$

Examples: Part Total

What percentage is 12 out of 15?

By simplifying fractions

$$\text{Percentage\%} = \frac{\text{Part number}}{\text{Total Number}} \times 100$$
$$= \frac{\cancel{12}^{4}}{\cancel{15}^{5}_{1}} \times \frac{\cancel{100}^{20}}{1}$$
$$= 80\%$$

Using calculator:

$$\text{Percentage\%} = \frac{\text{Part number}}{\text{Total Number}} \times 100$$
$$= \frac{12}{15} \times \frac{100}{1}$$
$$= 80\%$$

[1] [2] [×] [1] [0] [0] [÷] [1] [5] [=] 80

Total Part

What percentage of 40 is 23?

By simplifying fractions

$$\text{Percentage\%} = \frac{\text{Part number}}{\text{Total Number}} \times 100$$
$$= \frac{23}{\cancel{40}^{2}} \times \frac{\cancel{100}^{5}}{1}$$
$$= 57.5\%$$

Using calculator:

$$\text{Percentage\%} = \frac{\text{Part number}}{\text{Total Number}} \times 100$$
$$= \frac{23}{40} \times \frac{100}{1}$$
$$= 57.5\%$$

[2] [3] [×] [1] [0] [0] [÷] [4] [0] [=] 57.5

Part Total

What percentage is 500g of 2 kg?

The first step in this question is to have both quantities in the same units. 2kg = 2000g.

By simplifying fractions

$$\text{Percentage\%} = \frac{\text{Part number}}{\text{Total Number}} \times 100$$
$$= \frac{\cancel{500}^{25}}{\cancel{2000}^{20^1}} \times \frac{\cancel{100}^{1}}{1}$$
$$= 25\%$$

Using calculator:

$$\text{Percentage\%} = \frac{\text{Part number}}{\text{Total Number}} \times 100$$
$$= \frac{500}{2000} \times \frac{100}{1}$$
$$= 25\%$$

5 0 0 × 1 0 0 ÷ 2 0 0 0 = 25

To solve the following problems, it is important to read the question carefully.

A metal rod 0.1454m long expands to 0.1473m when heated in a high temperature oven. What is the percentage increase in its length?

The question asks for the percentage increase, the first part of the question is to calculate the increase.

1. What is the increase? 0.1473 - 0.1454 = 0.0019

Express the increase as a fraction of the original length. Always use the original quantity unless stated otherwise in the question.

2. What is the fraction increase? $\frac{0.0019}{0.1454}$ $\frac{\text{increase}}{\text{original length}}$

Now make this fraction a percentage by multiplying by 100.

3. Make a percentage (x100) $\frac{0.0019}{0.1454} \times 100 = 1.30674\%$

The length of the rod increased by 1.307% (rounded to 3 d.p.)

Jane has a class of 33 students. On a particular day 5 are absent. What percentage of the class is present?

If 5 students are absent, then 33 – 5 = 28 must be present.

The fraction of the class present is $\frac{28}{33}$.

Making this a percentage

$$\frac{28}{33} \times 100\%$$
$$= 84.\overline{84}\%$$
$$\approx 84.8\% \text{ to 1 decimal place}$$

Jane has approximately 84.8% of her class present.

A store buys a particular type of confectionery for $1.20 and adds on 75 cents to get the selling price of $1.95. What is the percentage mark-up?

The mark up is 75 cents.

The fraction mark-up is $\frac{75}{120}$. Both figures must be in the same units and the denominator is usually the original amount before the mark up.

Making this a percentage

$$\frac{75}{120} \times 100\%$$
$$= 62.5\%$$

The percentage mark –up on the confectionery is 62.5%

Video 'Making Percentages'

Activity

1. What percentage of 60 is 12?

2. What percentage is 45 of 75?

3. Rob scored 53 out of 60 on a statistics test, what was his percentage result on the test.

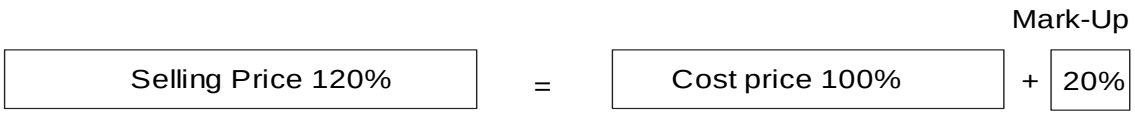
4. On their annual holiday, Tan and Mary will travel 1610km from Brisbane to Cairns. The distance from Brisbane to Gympie is 165km. What percentage of the journey is this? (Answer to 1 decimal place)

5. At an end of financial year sale, a gold necklace originally costing $129 has been reduced by $45. What is the percentage discount that the shop is offering?

6. On a scientific instrument it is known that for a reading of 450 units there is an error of 6 units. Calculate the percentage error in this reading.

Topic 4: Harder questions – using equations

Some percentage questions are the reverse situation of those above. Let's consider the situation where we want to calculate the cost price of an item given that the cost price was marked up by 20% and the selling price is $240.



This diagram indicates that the selling price is 120% of the cost price, which can be written more mathematically as an equation:

$$\text{Selling Price} = 120\% \text{ of Cost Price}$$

Putting in the numbers we have:

$$240 = 120\% \times \text{Cost Price}$$

To find the Cost price, the equation has to be rearranged to make Cost Price the subject. When the 120% is moved to the other side of the equation, it must do the opposite operation, in this case, dividing by 120% (More details on this are in the Algebra module). The equation becomes:

$$\frac{240}{120\%} = \text{Cost Price}$$

$$\frac{240}{1.2} = \text{Cost Price}$$

$$\text{Cost Price} = 200$$

The cost price of the item is $200.

Other similar questions are below:

30% of a number is 9, what is the number?

This question translates easily into a mathematical equation

$$30\% \text{ of a number} = 9$$

$$0.3 \times \text{number} = 9$$

$$\text{number} = \frac{9}{0.3}$$

$$\text{number} = 30$$

In this solution, the word 'number' is often represented by a pronumeral or variable as shown below: Let the number be *n*.

$$30\% \text{ of } n = 9$$
$$0.3 \times n = 9$$
$$n = \frac{9}{0.3}$$
$$n = 30$$

The number is 30.

Over the last year, the population of Lismore has grown by 2.5% to 29500 people. What was the population at the beginning of the year?

The population at the end of the year is 102.5% of the beginning of the year figure.

Let P be the population at the beginning of the year

$$102.5\% \text{ of } P = 29\,500$$
$$1.025 \times P = 29\,500$$
$$P = \frac{29500}{1.025}$$
$$P \approx 28\,780$$

The population at the beginning of the year was approximately 28 780.

Shops have to make their goods for sale with prices that include GST. The rate of GST in Australia is 10%. A large screen television is marked at $1590, what is the GST component of this price?

The marked price is 110% of the non-GST price.

Let N be the non-GST price of the large screen television.

$$110\% \text{ of } N = 1590$$
$$1.1 \times N = 1590$$
$$N = \frac{1590}{1.1}$$
$$N \approx 1445.45 \text{ (to the nearest cent)}$$

The price before GST was added is approximately $1445.45 The GST component was $144.55

A business spends 80% of its income on overheads and makes a profit of $ 27000. What is the total income (revenue) of the business?

The profit of $27 000 represents (100% - 80% =) 20% of the income.

Let I be the total income of the business

$$\begin{aligned} 20\% \text{ of } I &= 27000 \\ 0.2 \times I &= 27000 \\ I &= \frac{27000}{0.2} \\ I &= \$135000 \end{aligned}$$

The total income of the business is $135 000.

 Video 'Harder Percentages - Using Percentages'

Activity

1. 35% of Liam's Grade 4 class is 14 students. How many students are there in the class?

2. Tom gave away 65% of his CD collection and kept 19CDs. What was the size of his original collection.

3. A jacket sold at a 20% off sale for $95. What was the original price?

4. A door to door salesman earns 25% on his total sales. If his commission amounted to $11 500, what were his total sales?

5. A lounge suite was marked up by 35% to sell for $5995. What was the cost price (ignore GST in this question)

Three different types of percentage problems have been presented in this module. In the set below there is a mixed set of questions, you will need to determine the type of question before starting. Some tips to help are:

Read the question carefully – use a highlighter to highlight key information.
If the question is about making a making a percentage – think about the three steps involved.
If the question is using percentages, determine if it is an ordinary question or a working in reverse question where an equation is required.

 Video 'Mixed Questions'

Mixed Activity

1. A used car yard discounted the price of a car with an asking price of $12 500 by 7.5%. What is the discounted price?

2. Jack scored 45 out of 55 on his Algebra test, what percentage did he receive?

3. The price of a DVD player is $359 including GST. What was the price before GST was added? (The rate of GST is 10%)

4. In a packet of 35 jelly beans, there were 5 red ones. What percentage of the packet were 'non' red.

5. Janette went to a sale at a department store advertising 15% off everything. She obtained a $28 discount on a dining set. What were the original price and the discounted price?

6. When water freezes it expands by 4%. John has a container holding 1350mL of water. What is the volume of ice?

7. Jim paid $85 for a jacket after getting a discount of 20%. What was the original price of the item.

8. Engineers working on a new road to link two towns estimate that the new road is 22km compared to the original road length of 18km. Calculate the percentage increase in the length of the road.

9. Michelle's last rate notice was for $1 247.30. The latest media statement from her local council indicates that a 3.6% increase will apply this year. For what amount can Michelle expect her next rates notice be asking for?

10. If a $30 000 car is increased in price by 10% and then discounted by 10%, will the discounted price be:
    (a) less than $30 000
    (b) equal to $30 000
    (c) more than $30 000

    Explain with reasons and/or calculations.

Answers to activity questions

Check your skills

1. Fill in the missing spaces:

Fraction	Decimal	Percentage
$\frac{7}{20}$	$7 \div 20 = 0.35$ Or $\frac{7^{\times 5}}{20^{\times 5}} = \frac{35}{100} = 0.35$	$0.35 \times 100 = 35\%$ Or $\frac{7}{\cancel{20}^{1}} \times \frac{\cancel{100}^{5}}{1} = \frac{35}{1}\% = 35\%$
$\frac{8^{\div 4}}{100^{\div 4}} = \frac{2}{25}$	0.08	$0.08 \times 100 = 8\%$ Or $\frac{2}{\cancel{25}^{1}} \times \frac{\cancel{100}^{4}}{1} = \frac{8}{1}\% = 8\%$
$\frac{12.5^{\times 2}}{100^{\times 2}} = \frac{\cancel{25}^{1}}{\cancel{200}^{8}} = \frac{1}{8}$	$12.5\% \div 100 = 0.125$	12.5%

2. (a) 45% of \$640
$= 0.45 \times 640$
$= 288$

(b) 22.5% of $345\,000$
$= 0.225 \times 345\,000$
$= 77\,625$

New amount is 345 000 + 77 625 = 422 625

3. (a) $\frac{45}{75} \times \frac{100}{1} = 60\%$

(b) The reduction is \$90 - \$68 = \$22

The fraction reduction is $\frac{22}{90}$

The percentage reduction is $\frac{22}{90} \times \frac{100}{1} = 24.44\%$ to 2 decimal places.

4. 15% of the original price is \$8.40

$$0.15 \times \text{original price} = \$8.40$$
$$\text{original price} = \frac{8.40}{0.15}$$
$$\text{original price} = 56$$

The original price of the jeans was \$56 and the discounted price was \$56 - \$8.40 = \$47.60

Conversions between fractions, decimals and percentages

1. Fill in the missing spaces in the table:

Fraction	Decimal	Percentage
$\frac{3}{4}$	$3 \div 4 = 0.75$ Or $\frac{3^{\times 25}}{4^{\times 25}} = \frac{75}{100} = 0.75$	$0.75 \times 100 = 75\%$ Or $\frac{3}{\cancel{4}^{1}} \times \frac{\cancel{100}^{25}}{1} = \frac{75}{1}\% = 75\%$
$\frac{\cancel{45}^{9}}{\cancel{100}^{20}} = \frac{9}{20}$	0.45	$0.45 \times 100 = 45\%$
$\frac{\cancel{6}^{3}}{\cancel{100}^{50}} = \frac{3}{50}$	$6\% \div 100 = 0.06$	6%
$\frac{2}{3}$	$2 \div 3 = 0.666\overline{6}$	$0.666\overline{6} \times 100 = 66.6\overline{6}\%$ or 66.67% to 2d.p. Or $\frac{2}{3} \times \frac{100}{1}\% = \frac{200}{3}\% = 66.67\%$ to 2 d.p.
$\frac{\cancel{625}^{5}}{\cancel{1000}^{8}} = \frac{5}{8}$	0.625	$0.625 \times 100 = 62.5\%$
$\frac{125}{100} = 1\frac{\cancel{25}^{1}}{\cancel{100}^{4}} = 1\frac{1}{4}$	$125\% \div 100 = 1.25$	125%

Using percentages

1. 25% of \$560
   $= 0.25 \times 560$
   $= 140$

2. 8.5% of \$155 000
   $= 0.085 \times 155000$
   $= \$13175$

3.

$5\frac{2}{5}\%$ of 27 450
$= 5.4\% \times 27\,450$
$= 0.054 \times 27\,450$
$= 1482.3$

or

$5\frac{2}{5}\%$ of 27 450
$= \frac{\left(\frac{27}{5}\right)^{\times 5}}{100^{\times 5}} \times 27\,450$
$= \frac{27}{500} \times 27\,450$
$= 1482.3$

There will be approximately 1482 people.

4. 12% of 840
$= 0.12 \times 840$
$= 100.8$
There will be approximately 101 absent on a typical day

5. 12.5% of 32cm
$= 0.125 \times 32$
$= 4\text{cm}$

$32 - 4 = 28\text{cm}$

or

$100\% - 12.5\% = 87.5\%$

87.5% of 32cm
$= 0.875 \times 32$
$= 28\text{cm}$

6. 20% of 36
$= 0.2 \times 36$
$= 7.2$

New amount $= 36 + 7.2 = 43.2$ mins

or

$100\% + 20\% = 120\%$

120% of 36
$= 1.2 \times 36$
$= 43.2$

7. 10% of 163
$= 16.3$
New amount $= 163 + 16.3 = 179.3$
The tiler will purchase 18 boxes of 10 tiles.

8. Discount $= 33\frac{1}{3}\%$ of \$12.50
$= \frac{\left(\frac{100}{3}\right)^{\times 3}}{100^{\times 3}} \times 12.5$
$= \frac{100}{300} \times 12.5$
$= \frac{1}{3} \times 12.5$
$= 4.17$ (rounded to the nearest cent)
Discounted price = \$12.50 - \$4.17 = \$8.33

Making percentages

1. 12 is the part, 60 is the total.

$$\frac{\cancel{12}^{4}}{\cancel{60}_{1}^{3}} \times \frac{\cancel{100}^{5}}{1} \quad \text{or} \quad 12*100/60é20$$
$$= 20\%$$

2. 45 is the part, 75 is the total.

$$\frac{45}{\cancel{75}^{3}} \times \frac{\cancel{100}^{4}}{1}$$
$$= \frac{180}{3}\% \quad \text{or} \quad 45*100/75é60$$
$$= 60\%$$

3. Rob's percentage is:

$$\frac{53}{\cancel{60}^{3}} \times \frac{\cancel{100}^{5}}{1}$$
$$= \frac{265}{3}\% \quad \text{or} \quad 53*100/60é88.33333333$$
$$= 88.3\% \text{ (to 1 d.p.)}$$

Rob scored 88.3% on his statistics test.

4. Percentage of journey travelled is:

$$\frac{165}{1610} \times \frac{100}{1} \text{ or } 165*100/1610é10.2484472$$
$$= 10.2\%$$

The distance to Gympie is 10.2% of the journey (to 1 d.p.).

5. Discount = $45

Fraction discount $= \frac{45}{129}$

Percentage discount $= \frac{45}{129} \times \frac{100}{1} = 34.9\%$

By calculator: 45*100/129é34.88372093

The percentage discount was 34.9% (to 1 d.p.)

6. Error = 6

Fraction error $= \frac{6}{450}$

Percentage error

$$\frac{\cancel{6}^{2}}{\cancel{450}^{\cancel{9}^{3}}} \times \frac{\cancel{100}^{2}}{1}$$

$$= \frac{4}{3}\% \qquad \text{or } 6*100/450é1.33333333$$

$$= 1.333\%$$

The percentage error in the reading is 1.333%

Harder questions – using equations

1. 35% of Liam's Class is 14 students.

$$35\% \times \text{Number in Class} = 14$$
$$0.35 \times \text{Number in Class} = 14$$
$$\text{Number in Class} = \frac{14}{0.35}$$
$$\text{Number in Class} = 40$$

There are 40 students in Liam's class.

2. Tom gave away 65% of his CD collection, therefore, he kept 35%.

So 35% of his collection is 19 CDs

$$35\% \text{ of Collection is } 19$$
$$0.35 \times \text{Collection} = 19$$
$$\text{Collection} = \frac{19}{0.35}$$
$$\text{Collection} = 54.29$$

Tom's CD collection originally contained 54 (or 55) CDs.

3. At the sale, the purchaser paid 80% of the original price which was $95

So, 80% of the Original Price is $95

$$80\% \text{ of Original Price} = 95$$
$$0.8 \times \text{Original Price} = 95$$
$$\text{Original Price} = \frac{95}{0.8}$$
$$\text{Original Price} = 118.75$$

The original price is $118.75

4. 25% of Total Sales is his commission

25% of Total Sales is $11 500

$$25\% \text{ of Total Sales} = 11500$$
$$0.25 \times \text{Total Sales} = 11500$$
$$\text{Total Sales} = \frac{11500}{0.25}$$
$$\text{Total Sales} = 46000$$

The total sales figure is $46 000

5. The cost price was increased by 35% to give the sale price of $5995.

So, 135% of the cost price is the sale price.

135% of the Cost Price is $5995.

$$135\% \times \text{Cost Price} = 5995$$
$$1.35 \times \text{Cost Price} = 5995$$
$$\text{Cost Price} = \frac{5995}{1.35}$$
$$\text{Cost Price} = 4440.74$$

The cost price of the lounge suite is $4440.74

Mixed Questions

1. Using percentages – ordinary type

$$7.5\% \text{ of } 12500$$
$$= 0.075 \times 12500$$
$$= \$937.50$$

$$\text{Discounted Price}$$
$$= 12500 - 937.50$$
$$= 11562.50$$

or

$$100\% - 7.5\% = 92.5\%$$
$$92.5\% \times 12500$$
$$= 0.925 \times 12500$$
$$= 11562.50$$

The discounted price of the car is $11 562.50

2. Making a percentage

Jack got $\frac{45}{55}$ of the questions correct.

Percentage result $= \frac{45}{55} \times \frac{100}{1} = 81.8\%$

3. Using percentages – using an equation

The price $359 represents110% of the pre-GST price.

110% of the pre-GST price is $359

$$1.1 \times \text{pre-GST Price} = 359$$
$$\text{pre-GST Price} = \frac{359}{1.1}$$
$$\text{pre-GST Price} = 326.36$$

The price before GST was $326.36

4. Making a percentage

Number of ‘non red’ jelly beans is 35-5=30

Fraction of ‘non red’ jelly beans in the packet $= \frac{30}{35}$

Percentage of ‘non red’ jelly beans is $\frac{30}{35} \times \frac{100}{1} = 85.7\%$

5. Using percentages – using an equation

15% off the original price is equivalent to $28

$$15\% \text{ of the original price} = 28$$
$$0.15 \times \text{original price} = 28$$
$$\text{original price} = \frac{28}{0.15}$$
$$\text{original price} = 186.67$$

The original price of the dining set was approximately \$186.67, the discounted price is \$186.67 – 28 = \$158.67.

6. Using percentages – ordinary type

The water volume of 1350mL expands by 4%

Volume of ice will be 1350mL + 4% of 1350mL

$$\begin{aligned} &104\% \text{ of } 1350 \\ &= 1.04 \times 1350 \\ &= 1404 \end{aligned}$$

The volume of ice is 1404mL

7. Using percentages – using an equation

Jim received a discount of 20%, therefore he paid 80% of the original price.

80% of the original price is \$85.

$$\begin{aligned} 0.8 \times \text{original price} &= 85 \\ \text{original price} &= \frac{85}{0.8} \\ \text{original price} &= 106.25 \end{aligned}$$

Jim's jacket was originally priced at \$106.25

8. Making a percentage

The increase in length is 4km.

The fraction increase is $\frac{4}{18}$

The percentage increase is $\frac{4}{18} \times \frac{100}{1}$

$= 22.22\%$

The new road is 22.22% (to 2 d.p.) longer than the existing.

9. Using percentages – ordinary type

The amount of the rise is 3.6% of \$1247.30

$$\begin{aligned} &3.6\% \text{ of } \$1247.30 \\ &= 0.036 \times 1247.30 \\ &= 44.90 \end{aligned}$$

The next rates notice will be for \$1247.30 + \$44.90 = \$1292.20

10. The answer is (a)

Reason: the car is increased by 10% based on the initial amount, then is discounted by 10% based on the higher increased price. This means the discount is higher than the original increase. The car will cost less than \$30000.

Calculations:

The cost of the car will increase by 10%, which is 10% of \$30000=\$3000.
The new price is \$33000. The discount will be based on the new price, so the discount is 10% of \$33000, which is \$3300. The final price will be \$33000-\$3300=\$29700.