Introduction to rates and ratios

Have you ever used a heart rate monitor to watch how your heart rate varies with exercise?

Usually your resting heart rate is 70 – 80 bpm and this can rise quickly when exercising. But have you ever noticed the word **rate** in the term **heart rate**? And what does bpm mean?

In this module you will learn that a rate involves two or more quantities. **Heart rate** involves two quantities; number of beats and time measured in minutes. **Heart rate** is measured in the units bpm (b/m) or beats per min.

A **rate** compares quantities with different units. The rate is obtained by dividing one quantity by the other. In the rate km/hr, the rate is obtained by first quantity divided by second quantity. The rate km/hr is known as speed and is calculated by

$\text{speed(km/hr)} = \text{distance(km)} \div \text{time(hr)}$	or	$\text{speed (km/hr)} = \dfrac{\text{distance (km)}}{\text{time (hr)}}$

The flow rate of a stream is given in L/min, so flow rate is calculated by

$\text{Flow rate(L/min)} = \text{Volume (L)} \div \text{Time (min)}$	or	$\text{Flow rate (L/min)} = \dfrac{\text{Volume (L)}}{\text{Time (min)}}$

The pay rate of an employee is given in units of \$/hr, so the pay rate is calculated by

$\text{Pay rate(\$/hr)} = \text{Amount (\$)} \div \text{Time (Hr)}$	or	$\text{Pay rate(\$/hr)} = \dfrac{\text{Amount (\$)}}{\text{Time (Hr)}}$

The end result is called a **unit rate** because it is per 1. A pay rate of $12.45/hr is a **unit rate** because it is expressed as per 1 hour. Unit rates enable to compare different people's pays. For example: it is clear that a person with a pay rate of $22/hr is paid more per hour than a person earning $18/hr.

A flow rate of 128L/min is also a unit rate because it is expressed per min.

A **ratio** compares quantities of the same kind with the same units in a definite order.

If the teacher student ratio is 1:25 this means that there is 1 teacher to 25 students. It also means that there could be 2 teachers to 50 students or 8 teachers to 200 students. In this situation we can say that the proportion of teachers to students is the same because the ratio in each case simplifies to 1:25.

Ratios are normally written with whole numbers although in some instances writing a ratio as 1:? is an effective way to communicate comparisons, for example, a school with a teacher student ratio of 1 : 22.1 is better staffed than a school with a teacher student ratio of 1 : 22.9 . Ratios can also be written as fractions, written as first quantity over the second quantity. This means a ratio is easily converted to a fraction then to a decimal and percentage.

Module contents

Introduction

- **Rates: meaning, unit rates and rate conversions**
- **Calculations based on rates**
- **Ratios: meaning, ratios to other number forms**
- **Simplifying ratios**
- **Equivalent ratios**
- **Sharing an amount in a ratio**

Answers to activity questions

Outcomes

- To demonstrate an understanding of rates and ratios.
- To simplify both rates and ratios.
- To change rates to new units using unit conversions.
- To use equivalent ratios to solve problems including map scales.
- To share an amount using ratios.

Check your skills

This module covers the following concepts, if you can successfully answer these questions, you do not need to do this module. Check your answers from the answer section at the end of the module.

1. (a) The salt solution contained 200g/L. Explain the meaning of this.
   (b) Fertilizer should be applied at 4t/ha. Explain the meaning of this.
2. (a) Make this a unit rate: Jim earned $131.95 for 7 hours work.
   (b) Change to rate $6/kg to cents/gram.
   (c) If water leaves a hose at 20L/min,
      (i) What is the volume of water after a quarter of an hour?
      (ii) How long will it take to fill a 250L drum
3. (a) Write the ratio 4:5 as a fraction, decimal and percentage.
   (b) Simplify the ratios (i) 45:60 (ii) 5mins:1hr (iii) 0.4:1.2
4. (a) Determine the value of p in the equivalent ratios $3:7 = p:49$
   (b) If the ratio of two-stroke fuel is 1:20 (oil: petrol). How much petrol should be mixed with 200 mL of two-stroke oil?
   (c) A map scale is given as 1:2000. If the distance between 2 features on the map is 2.5cm, what is the actual distance in real life?
5. High Brass is a mixture of copper and zinc in the ratio13:7. How much copper is found in 10kg of high brass?

Topic 1: Rates – meaning, unit rates and rate conversion

Rates are used in everyday life to express one quantity in terms of another.

A gardener mixing up weed spray may read an instruction on a label that states: put 5 scoops of herbicide in the volume of water 1litre, as a rate, this is 5 scoops per 1 litre or 5 scoops/litre.

At a greengrocer, a sign on the tomatoes stating the cost is \$3.50/kg means that it costs \$3.50 for the weight 1 kg. This is a rate because it expresses an amount in terms of weight.

A nurse may set a volumetric infusion pump at a rate of 200mL/hr which means that a volume of 200mL of medication is infused every hour (time). This is equivalent to 400mL over 2 hours, 600mL over 3 hours etc.

A farmer has been advised to apply fertilizer at a rate of 750kg/ha. This means that he should apply a weight of 750kg over an area of 1ha. If the farmer has a paddock of 5ha, then he must apply 5 lots of 750kg or 3750kg or 3.75 tonnes.

A worker's pay rate is \$17.35/hr. This means she is paid \$17.35 for one hour's (time) work. For a working week of 35 hours, she would receive a pay of \$17.35 x 35 = \$607.25

Video 'Rates: Meaning'

Obtaining Unit Rates

A truck driver travels the 934km from Sydney to Brisbane in 11hrs. What is his average speed? To answer this question it is important to realise that speed is a rate and in this case the units are km/hr. The speed is calculated by distance divided by time:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$
$$\text{speed} = \frac{934 \text{ km}}{11 \text{ hrs}}$$
$$\text{speed} = 84.9 \text{ km/hr}$$

A shopper notices that \$14.68 was paid for 2.45 kg of grapes. What is the unit price? Many students comment that two rates can be calculated given this information. The rate price/kg or kg/price can be calculated, which one is best? In this situation it is conventional that a price per kg is used, so the rate price/kg is calculated as below.

$$\text{price/kg} = \frac{\text{price paid}}{\text{weight in kilograms}}$$
$$\text{price/kg} = \frac{\$14.68}{2.45\text{ kg}}$$
$$\text{price/kg} = \$5.99\,/\,\text{kg}$$

A nurse gives a patient a tablet containing 500mg of paracetamol every 6 hours. What is the dosage rate in g/day? (500mg = 0.5g and 6 hours= 0.25 or ¼ day)

Using decimals

$$\text{dosage rate (g/day)} = \frac{\text{dose (g)}}{\text{time (day)}}$$
$$\text{dosage rate (g/day)} = \frac{0.5\text{ g}}{0.25\text{ day}}$$
$$\text{dosage rate (g/day)} = 2\text{ g/day}$$

or

Using fractions

$$\text{dosage rate (g/day)} = \frac{\text{dose (g)}}{\text{time (day)}}$$
$$\text{dosage rate (g/day)} = \frac{0.5\text{ g}}{\frac{1}{4}\text{ day}}$$
$$\text{dosage rate (g/day)} = 0.5 \times \frac{4}{1}$$
$$\text{dosage rate (g/day)} = 2\text{ g/day}$$

A painter notices that a 4 litre tin of can cover an area of 64 square metres (m^2). What is the coverage rate? It is not known whether the rate is square metres/litre or litres/square metre. Conventionally it is usually given as square metres/litre because the answer obtained is (hopefully) greater than 1 and people tend to prefer to use rates that are greater than one.

$$\text{coverage rate } (m^2/L) = \frac{\text{area covered } (m^2)}{\text{volume of paint used (L)}}$$
$$\text{coverage rate } (m^2/L) = \frac{64\ m^2}{4\text{ (L)}}$$
$$\text{coverage rate } (m^2/L) = 16m^2/L$$

Video ‘Unit Rates’

Rate Conversions

In science, speed is often measured in m/sec rather than km/hr because both metres (m) and seconds (s) are base SI units of length and time. John has recorded the speed of a car at 80km/h. This must be converted to m/sec before he can substitute into a motion formula.

$$80km/hr = \frac{80\text{ km}}{1\text{ hour}} = \frac{80 \times 1000\text{m}}{1 \times 60 \times 60\text{ sec}} = \frac{80000\text{m}}{3600\text{sec}} = 22.2m/\text{sec}$$

Jamie has obtained an answer to a calculation as 10 m/sec and wants to change it to km/hr.

$$10\text{m/sec} = \frac{10\text{m}}{1\text{sec}} = \frac{10 \div 1000\text{km}}{1 \div 60 \div 60\text{hr}} = \frac{0.01\text{km}}{0.0002777hr} = 36\text{km/hr}$$

In nursing it is common to change the rate mg/hr to mcg/hr. Change 4mg/hr to mcg/hr. Note mg is milligrams and mcg is micrograms, in nursing mcg is used instead of μg because a poorly written μ can look similar to an m.

$$4\ \text{mg/hr} = \frac{4\ \text{mg}}{1\ \text{hr}} = \frac{4 \times 1000\ \text{mcg}}{1\ \text{hr}} = 4000\ \text{mcg/hr}$$

A gardener is given a bag of fertilizer with the application rate stated as 750kg/ha. For him a rate of g/m^2 would be better, convert 750 kg/ha to g/m^2.

$$750\ \text{kg/ha} = \frac{750\ \text{kg}}{1\,\text{ha}} = \frac{750 \times 1000\ \text{g}}{10000\ \text{m}^2} = 75\text{g/m}^2$$

More on rate conversions is in the module Measurement, Units and Conversions.

Video ‘Rate Conversions’

Activity

1. Fill in the gaps in the sentences below.
   (a) In the rate 5mL/hr, the two quantities are volume and _______.
   (b) In the rate 7.5km/L, the two quantities are ______ and volume.
   (c) If the car’s speed is 100km/hr, the two quantities are _____ and _____.
   (d) Old records had to be played at 78 revolutions/min, the two quantities in this rate are _____ and _____.
   (e) Carpet costs $95/square metre, the two quantities in this rate are _____ and _____.

2. Express the following as a unit rate.
   (a) Eric earned $157.75 for 5 hours work.
   (b) A record did 100 revolutions in 3 mins.
   (c) Freda’s heart rate is 18 beats over 15 secs. (heart rate is normally expressed as beats/min)
   (d) Garry cycles 70km over 2.5 hours.
   (e) A 3.25kg bag of oranges costs $2

3. Convert the rates below to the new units given

   (a) Change 15km/hr to m/hr.

   (b) Change 1.5t/day to kg/day.

   (c) Change 60km/hr to m/sec.

   (d) Change 15m/sec to km/hr

   (e) Change 38 words/min to words/hour

Topic 2: Calculations based on rates

Jane works at a restaurant to supplement her income; she is paid $17.50/hr. How much will she earn if she works for 5 hours?
Jane will earn $5 \times 17.50 = 87.50$ This is performed by either rearranging the equation, as shown below, or intuitively knowing to multiply.

$$Rate\ (\$/hr) = \frac{Amount(\$)}{Time\ worked\ (hours)}$$

can be rearranged to

$$Amount(\$) = Rate\ (\$/hr) \times Time\ worked\ (hours)$$

Peter is applying fertilizer to his crop at a rate of 750kg/ha. How much fertilizer should be applied to an 8 ha paddock?

Weight of fertilizer required is: $8 \times 750 = 6000$ kg or 6 tonnes

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A volumetric infusion pump for IV injections is set at 125mL/hr. The pump has been running for 3hrs 15mins, what volume of solution has been injected?

The time 3hrs 15mins must be expressed in hours, 15mins is 15 ÷ 60 hour=0.25, so 3hrs 15mins = 3.25 hours.
Volume injected is $3.25 \times 125 = 406$ mL (to the nearest mL)

A drug must be given to a patient (weight 85kg) at a rate of 0.2mg/kg/day. This means that the drug dose is 0.2 mg per kg of body weight per day. How many mg must be given to the patient if there are 2 doses in the day?
Amount to be given is $0.2 \times 85 = 17$ mg/day. The amount per dose is 17 mg $\div 2 = 8.5$ mg

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Harder questions are: (Rearranging the equation is more important in this type)

Tomatoes are sold for $3.50/kg. What weight (kg) of tomatoes can I obtain for $10?
As

$$\text{Rate }(\$/\text{kg}) = \frac{\text{Amount}(\$)}{\text{Weight(kg)}}$$
$$3.50 = \frac{10}{\text{Weight(kg)}}$$
$$\text{Weight (kg)} \times 3.50 = 10$$
$$\text{Weight (kg)} = \frac{10}{3.50}$$
$$\text{Weight (kg)} = 2.857 \text{ kg}$$

A volumetric injection pump is set at 135mL/hr. How long will it take for 500mL of solution to be injected?

$$\text{Rate mL/hr} = \frac{\text{Volume (mL)}}{\text{Time (hrs)}}$$
$$135 \text{ mL/hr} = \frac{500 \text{ mL}}{\text{Time (hrs)}}$$
$$\text{Time (hrs)} \times 135 = 500$$
$$\text{Time (hrs)} = \frac{500}{135}$$
$$\text{Time (hrs)} = 3.7 \text{ hrs or 3hrs 42min}$$

Video ‘Rate Calculations’

Which is better value: A 250g tin of baked beans for 57 cents or a 900g tin for $2?

The first step is to convert each to a unit rate. Should the rate g/$ or the rate $/g be used? The rate g/$ is probably better but it doesn’t matter as long as the two answers are interpreted properly. With the rate g/$, the high the answer the better because more baked beans are obtained per dollar. If the rate $/g is used, the lower the answer the better because the cost is lower for each gram.

Using g/$:

For the 250g tin, the rate g/$ is:

$$\text{Rate g/\$} = \frac{\text{weight in g}}{\text{Cost in \$}}$$
$$\text{Rate g/\$} = \frac{250\text{g}}{\$0.57}$$
$$\text{Rate g/\$} = 438.60 \text{ g/\$ (to 2 d.p.)}$$

For the 900g tin, the rate g/$ is:

$$\text{Rate g/\$} = \frac{\text{weight in g}}{\text{Cost in \$}}$$
$$\text{Rate g/\$} = \frac{900\text{g}}{\$2}$$
$$\text{Rate g/\$} = 450 \text{ g/\$}$$

The 900g tin is better value because there is a higher weight (more baked beans) per dollar.

Video ‘Comparing Rates’

Activity

1. A typist can type at 18 words/minute. How many words will be typed after:

   (a) 5 minutes

   (b) Half an hour

   (c) 37 minutes

   How long will it take the typist to type: Answer in minutes and seconds?

   (d) 100 words

   (e) 550 words

   (f) 1200 words

2. An orange picker can pick at a rate of 38 kg/hr. How long will it take to pick a tonne (1000kg)

3. Electricity costs 19.2c/unit. What will the charge be for 857 units?

4. Carpet costs $85/square metre. How much will it cost to lay carpet in a room measuring 5.5m by 4.2 ?

5. Which is better value: 550g for $1.74 or 1.2kg for $3.85 ?

6. A truck driver is expected to average 80 km/hr on longer journeys. William drove the 1610km from Brisbane to Cairns in 21 hours, did he exceed the expected rate?

7. Two friends went jogging: Barry covered 8 km in 33 mins, while Bruce covered 5 km in 24mins, which person jogged at the fastest rate?

8. It takes a gang of workers laying railway line 5 days to lay 1250m of line. How long would it take this gang to lay 4500m of line?

9. At present the exchange rate to the English pound is $A1=£0.52

   (a) How many English pounds will $A500 buy?

   (b) How many Australian dollars will £135 buy?

Topic 3: Ratios – meaning, ratios to other number forms

A **ratio** compares quantities of the same kind with the same units in a definite order. The ratio itself has no units.

In a class, there are 15 girls, 20 boys and 1 teacher.
The ratio of girls to boys is 15:20 or 15 to 20.
The ratio of teachers to students is 1:35 or 1 to 35.

To mix concrete, a mix of cement, sand and gravel is mixed in the ratio 1:3:2. The order in which the ingredients are listed correspond to the numbers in the ratio: 1 part cement : 3 parts sand : 2 parts gravel.

Ratios containing two quantities can be written as a fraction.

The ratio of black jelly beans to red jelly beans is 1:4. To write a ratio as a fraction, it is written as first amount / second amount.

The ratio 1:4 can be written as the fraction $\frac{1}{4}$ meaning there is $\frac{1}{4}$ the number of black jelly beans to the number of red jelly beans. Once the ratio is written as a fraction it is then possible for it to be written as a decimal and percentage.

Ratio	Fraction	Decimal	Percentage
1:4	$\frac{1}{4}$	$\frac{1^{\times 25}}{4^{\times 25}} = \frac{25}{100} = 0.25$	$0.25 \times 100 = 25\%$

Extra information can be interpreted from this ratio. With the ratio 1:4 there are 1 + 4 = 5 parts. The black jelly beans are 1 part out of 5 or $\frac{1}{5}$, the red jelly beans are 4 parts out of 5 or $\frac{4}{5}$. If a container holds 25 black and red jelly beans in the ratio 1:4, we know that $\frac{1}{5}$ will be black and $\frac{4}{5}$ will be red. However, this is not what is done when asked to change a ratio into a fraction.

The ratio of nurses to patients is 1:5, meaning that the number of nurses is $\frac{1}{5}$ that of the number of patients.

Ratio	Fraction	Decimal	Percentage
1:5	$\frac{1}{5}$	$\frac{1^{\times 2}}{5^{\times 2}} = \frac{2}{10} = 0.2$	$0.2 \times 100 = 20\%$

The ratio of boys to girls in a class is 4:7, meaning that the number of boys is $\frac{4}{7}$ that of the girls or that the number of girls is $\frac{7}{4}$ that of the boys.

Ratio	Fraction	Decimal	Percentage
4:7	$\frac{4}{7}$	$4 \div 7 = 0.571428$	$0.571428 \times 100 = 57.1\%$

Write 125% as a decimal, fraction and ratio. This would mean that the first quantity is 125% of the second quantity.

Percentage	Decimal	Fraction	Ratio
125%	$125\% \div 100 = 1.25$	$1.25 = \frac{125^{\div 25}}{100^{\div 25}} = \frac{5}{4}$	5:4

Video 'Ratios – Conversions to other numbers'

Activity

1. Complete this table:

Ratio	Fraction	Decimal	Percentage
1:5			
	$\frac{7}{8}$		
		0.65	
			240%

Topic 4: Simplifying ratios

When writing ratios, it is expected that they will be written in their simplest form containing whole numbers. Remember ratios can be written as fractions and simplifying fractions ideas are used to simplify ratios.

Examples:

Simplify 12:20
To simplify, the Highest Common Factor of 12 and 20 is required. Remember the HCF is the highest number that goes into 12 and 20, the HCF is 4. If the common factor used is not the HCF, then simplifying may take two or more stages.

$12^{\div 4} : 20^{\div 4}$
$= 3 : 5$

Simplify 35:75
In this question the HCF is 5.

$35^{\div 5} : 75^{\div 5}$
$= 7 : 15$

Simplify 300mm:1.2m
In this question the units are different, so the first step is make the units the same. Generally, it is better to express both in the smaller unit. (1.2m = 1.2 x 1000 =1200 mm)

$300 : 1200$
$= 300^{\div 300} : 1200^{\div 300}$
$= 1 : 4$

Simplify 0.5:1.25
Because both numbers are expressed as decimals, multiplying both sides by 10, 100, 1000, etc is required to make both whole numbers. The number with the most decimal place is 1.25, that is 2 decimal places, multiplying by 100 is required to remove decimals.

$0.5 : 1.25$
$= 0.5^{\times 100} : 1.25^{\times 100}$
$= 50 : 125$

Now the HCF of 50 and 125 is 25, the ratio can be simplified.

$50 : 125$
$= 50^{\div 25} : 125^{\div 25}$
$= 2 : 5$

Simplify $2\frac{1}{2} : \frac{3}{5}$

With numbers expressed as fractions, it is important that mixed numbers are changed to improper fractions. Both fractions should then be expressed with a common denominator. Each fraction can then be multiplied by that common denominator and then inspected to see if more simplifying can take place.

$2\frac{1}{2}:\frac{3}{5}$ Change any mixed numbers to improper fractions

$=\frac{5^{\times 5}}{2^{\times 5}}:\frac{3^{\times 2}}{5^{\times 2}}$ Express both over a common denominator of 10

$=\frac{25}{10}:\frac{6}{10}$ Multiply both sides by the common denominator 10

$=25:6$ This cannot simplify any further

When doing comparisons, writing ratios as unit ratios (either ... : 1 or 1: …) gives a way of deciding best or worst case situations.

Examples:

At Aisville High School, there are 23 teaching staff for 385 students. At a neighbouring school, Beeston High School, there are 44 teaching staff for 672 students. Use unit ratios to determine the school with the best teacher student ratio.

Aisville High School	Beeston High School
$23:385$ divide both by 23 to obtain a ratio 1: ... $=23^{\div 23}:385^{\div 23}$ $=1:16.74$ (to 2 decimal places)	$44:672$ divide both by 44 to obtain a ratio 1 : ... $=44^{\div 44}:672^{\div 44}$ $=1:15.27$ (to 2 d.p.)

The best teacher student ratio is at Beeston High School because there are fewer students for each teacher.

It is accepted that a doctor patient ratio of 1 : 1200 is acceptable for providing adequate patient care. In an outback district, there are 6 doctors for a population of 10430 people. Is there an acceptable number of doctors in this district?

Acceptable ratio	District ratio
1:1200	$6:10430$ divide both by 6 to obtain a ratio 1 : ... $=6^{\div 6}:10430^{\div 6}$ $=1:1738$ (to nearest whole number)

No, there is an unacceptable number of doctors in this district. The doctor patient ratio for the outback district exceeds the acceptable level.

[Video ‘Simplifying Ratios’]

Activity

1. Simplify the following ratios

(a)	15 : 35	(b)	49 : 84
(c)	0.8 : 2	(d)	0.125 : 0.5
(e)	$\frac{2}{5}:\frac{1}{4}$	(f)	$2\frac{2}{3}:1\frac{1}{6}$
(g)	15 mm : 5 cm	(h)	1.5g : 250mg

2. The Greeks believed that rectangles that had a length to width ratio of 1.618 (approx) : 1 were the most pleasing to the eye. This ratio is known as the Golden Ratio.

   (a) A computer screen has a length of 37cm and a width of 23cm, how close is it to having the Golden Ratio?

   (b) An envelope measures 21cm by 14cm, how close is it to having the Golden Ratio? To make it closer to the Golden Ratio, would the length or width need to be increased?

Topic 5: Equivalent ratios

If the ratio of nurses to patients is 1:5 then this would be equivalent to:

Nurses	Patients
1	5
2	10
3	15
4	20
5	25
6	30
7	35
8	40
9	45
10	50

The ratios given are all equivalent ratios. It can be said that the number of nurses to the number of patients is in the same proportion. When ratios are in proportion, the ratios are equal, that is, for example:

$$1:5 = 5:25 \text{ or } \frac{1}{5} = \frac{5}{25}$$

This can be used to find missing quantities.

Examples:

Solve for n: $3:8 = n:56$

The equivalent ratios $3:8 = n:56$ can be written in fraction form as $\frac{3}{8} = \frac{n}{56}$.

$$\frac{3}{8} = \frac{n}{56} \quad \text{multiplying boths sides by 56 gives}$$

$$\frac{3 \times 56}{8} = \frac{n \times \cancel{56}^{1}}{\cancel{56}^{1}}$$

$$21 = n$$

$$\text{or } n = 21$$

To make fibreglass, the ratio of resin to glass is 40% to 60%. If James has 10kg of resin, how much glass will be needed?

It may be better to simplify the given ratio. The HCF of 40 and 60 is 20, so the simplified ratio is 2:3. Let g be the amount of glass required. The equivalent ratios are $2:3 = 10:g$.
Changing the ratios into fraction form:

$$\frac{2}{3}=\frac{10}{g} \quad \text{Take reciprocal of both sides.}$$
$$\frac{3}{2}=\frac{g}{10} \quad \text{Now multiply both sides by 10.}$$
$$\frac{3\times 10}{2}=\frac{g\times \cancel{10}^{1}}{\cancel{10}^{1}}$$
$$15=g$$
$$g=15$$

James needs 15kg of glass.

Two stroke fuel is made in the ratio 1:20, that is, 1 part oil to 20 part petrol. If Jamie has a 375mL container of 2 stroke oil, how much petrol will she need? Let the volume of petrol be *p*. The equivalent ratios are $1:20=375:p$.

Changing the ratios into fraction form:

$$\frac{1}{20}=\frac{375}{p} \quad \text{Take reciprocal of both sides.}$$
$$\frac{20\times 375}{1}=\frac{p\times \cancel{375}^{1}}{\cancel{375}^{1}} \quad \text{Multiply both sides by 375}$$
$$7500=p$$
$$p=7500$$

Jamie needs 7500mL of petrol or 7.5L.

Scales (ratios) on maps

The scale on a map is given as (for example) 1:100. Scales on maps represent the ratio:

Map scale = map distance (md) : ground distance (gd)
or
1:100 = map distance : ground distance

On the plan of a house (scale 1:100), the width of the lounge room is 40mm. What is the actual length of the lounge?

Let the length of the lounge room be *gd*.

The equivalent ratios are:

$$\text{Map scale = map distance }(md)\text{ : ground distance }(gd)$$
$$1:100 = 40 : gd$$

Changing the ratios into fraction form:

$$\frac{1}{100}=\frac{40}{gd}$$
$$\frac{100\times 40}{1}=\frac{gd\times \cancel{40}^{1}}{\cancel{40}^{1}}$$
$$40000=gd$$
$$gd=4000mm$$

The length of the lounge room is 4000mm, which converts to 4m.

The scale on a map of NSW is 1 : 1 500 000. The distance from Sydney to Lismore on the map is 490mm, what is the ground distance? Let the distance from Sydney to Lismore be *gd*.
The equivalent ratios are:

$$\text{Map scale} = \text{map distance} : \text{ground distance}$$
$$1 : 1500000 = 490 : gd$$
$$\frac{1}{1500000} = \frac{490}{gd}$$
$$\frac{1500000 \times 490}{1} = \frac{gd \times \cancel{490}^{1}}{\cancel{490}^{1}}$$
$$735000000 = gd$$
$$gd = 735000000\text{mm}$$

The distance from Sydney to Lismore is 735 000 000mm = 735 000m = 735 km.

A draftsman is drawing up plans for a new style of house, she uses a scale of 3:200. The actual length of the house is 15.6m. Let the length of the house on the plan be *md*.
The equivalent ratios are:

$$\text{Map scale} = \text{map distance} : \text{ground distance}$$
$$3 : 200 = md : 15$$
$$\frac{3}{200} = \frac{md}{15}$$
$$\frac{3 \times 15}{200} = md$$
$$md = \frac{45}{200} = 0.225m \times 1000 = 225mm$$

The length of the house on the plan is 225mm.

The map distance between two towns is 4.2 cm, and the actual (ground) distance between the two towns in a straight line is 84 km. From this information calculate the scale of map.

$$\text{Map scale} = \text{map distance} : \text{ground distance}$$
$$\text{Map scale} = 4.2cm : 84km$$
$$\text{Map scale} = \frac{4.2cm}{84km} = \frac{4.2cm}{8400000cm} = \frac{4.2 \div 4.2}{8400000 \div 4.2} = \frac{1}{2000000}$$
$$\text{Map scale} = 1 : 2000000$$

[Video ‘Equivalent Ratios’]

Activity

1. Find the value of the pronumeral in the equivalent ratios below.

   (a) 1 : 2 = a : 12   (b) 8 : 5 = b : 25

   (c) 9 : 2 = 81 : c   (d) 2.5 : 4 = d : 12

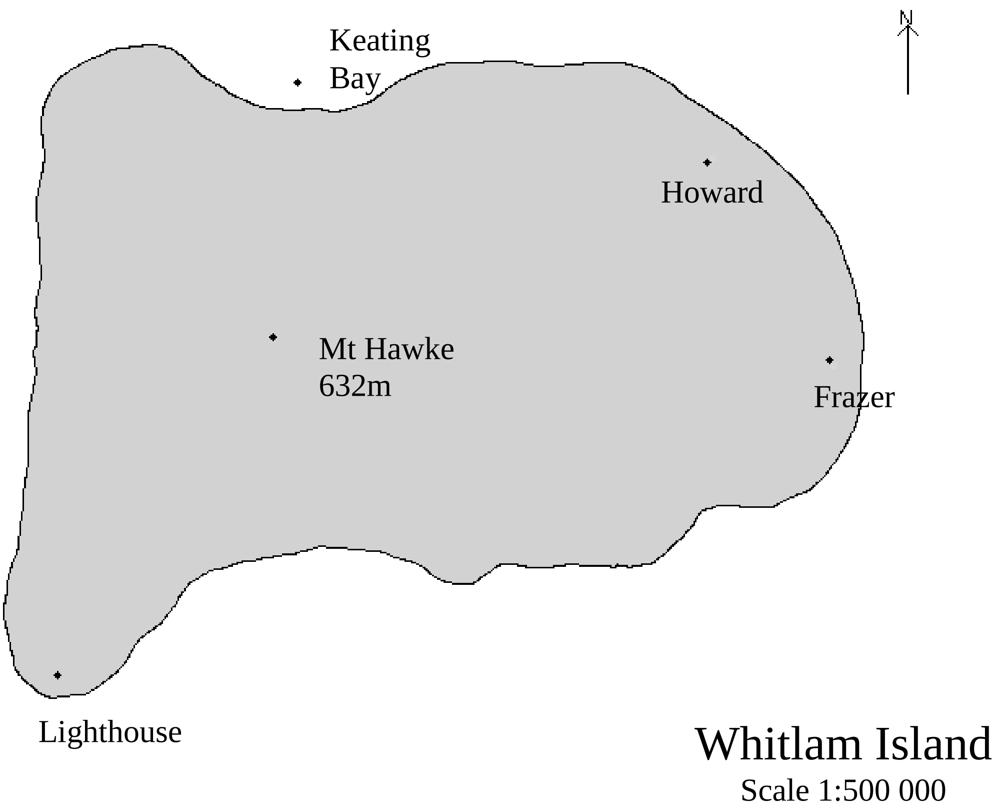
2. In a tile pattern, the ratio of red tiles to green tiles is 3:5. If the tiler has 27 red tiles, how many green tiles will be required?

3. In a cocktail the ratio of a spirit to soft drink is 1 : 6. If the person has a 375mL bottle of spirit, how much soft drink will they need?

4. The ratio of workers to instructors must not exceed 8 : 1. If there are 50 workers, what is the minimum number of instructors required?

5. A photograph measuring 15cm by 10cm is being enlarged so that the new length is 36cm. If the photo keeps the same ratio (proportion), what is the new width?

6. The scale on a map is 1: 500 000



   (a) What is the ground distance from Howard to Frazer?

   (b) What is the ground distance from Mt Hawke to the Lighthouse?

   (c) There is a historic site located 36 km west of Howard, how far is this on the map?

(d) There is a ship located on the map 2.6 cm east of the lighthouse. How far is it from the coast? If the ship is travelling west at 5 km/hr, how long before the ship runs aground?

7. The scale on a map is 1:1 200 000. The distance between two features is 46mm, what is the ground distance?

8. Derrick has an old map where the scale has been damaged and is unreadable. However, he knows the actual distance from the picnic area to the cave entrance is 1.6 km. On the map, this distance is 80mm, what is the scale of the map?

Topic 6: Sharing an amount in a ratio

On a busy night in a restaurant, two waitresses decide to share the tips they received in the ratio of the hours they each worked. Ann worked 5 hrs and Bronwyn worked 4 hours and the total of the tips collected that night was $72. Put simply, $72 is to be shared in the ratio of 5:4.

The first step is to add the 5 and 4 to get 9 parts. If the total $72 is divided into 9 parts, then Ann will receive 5 parts and Bronwyn will receive 4 parts.

More mathematically;

The total number of parts is $5+4=9$
Each part is $72 \div 9 = 8$, that is $8.
Ann's share is $5 \times \$8 = \$40$
Bronwyn's share is $4 \times \$8 = \$32$
(The answer can be checked; do the two shares add to $72? Yes, the answer may be correct.)

Candice (aged 15) and Darren (aged 18) decide to share a jar of 198 jelly beans in the ratio of their ages. How many will each receive?

The total number of parts is $15+18=33$
Each part is $198 \div 33 = 6$, that is 6 jelly beans.
Candice's share is $15 \times 6 = 90$
Darren's share is $18 \times 6 = 108$
(Checking: 108+90=198, answer could be correct)

The university has decided to survey 200 students in the ratio of 2:3:4 to reflect the number of students in each of the three faculties (Arts, Business & Science). How many students in each faculty will be chosen for the survey?

The total number of parts is $2+3+4=9$
Each part is $200 \div 9 = 22.2\overline{2}$,
The Arts faculty will survey $2 \times 22.2\overline{2} = 44.4\overline{4}$ which is 44 students.
The Business faculty will survey $3 \times 22.2\overline{2} = 66.6\overline{6}$ which is 67 students.
The Science faculty will survey $4 \times 22.2\overline{2} = 88.8\overline{8}$ which is 89 students.
Because of rounding, it is important to check that the total gives 200.
$44+67+89=200$ so rounding has not caused any problems.

Video 'Sharing an Amount in a Ratio'

Activity

1. Divide the quantity in the given ratio.
   (a) Divide \$75 in the ratio 4:11
   (b) Divide 100cm in the ratio 2:3
   (c) Divide 2 tonnes in the ratio 3:5
   (d) Divide 200mL in the ratio 1:3:4
   (e) Divide \$80 in the ratio 1:5

2. Brass is an alloy of Copper and Tin. The ratio of copper to tin is 88%:12%. If 1500g of brass is used to make a small statue, what was the original quantity of Copper and Tin?

3. On a journey, John and Jane share the driving in the ratio 3:1. What fraction of the journey did Jane drive?

4. A gold chain made from 18 carat gold. In 18 carat gold the ratio of gold to silver is 3:1. If the chain weighs 70g and the price of gold is \$37.41 per gram, what is the value of gold in the chain?

5. A school is calculating measurements for a rectangular soccer ground. The physical education teacher wants the perimeter to be 400m for P.E. students to jog laps. The soccer coach insists that the ratio of length to width is 3:2 just like an international size ground, what will the length and width that will satisfy both people.

Answers to activity questions

Check your skills

1. (a) A salt solution of 200g/L means that salt of weight 200 grams was dissolved in a volume of 1 litre of solution.

   (b) Fertilizer applied at a rate of 4 t/ha means a mass of 4 tonnes of fertilizer was spread over an area of 1 hectare.

2. (a)
$$\text{Pay Rate in } \$ / \text{hr} = \frac{\text{Amount in } \$}{\text{Time worked in hours}}$$
$$\text{Pay Rate } (\$ / \text{hr}) = \frac{\$131.95}{7 \text{ hours}}$$
$$\text{Pay Rate } (\$ / \text{hr}) = \$18.85 / \text{hr}$$

   (b)
$$\text{Rate } \$6/\text{kg} = \frac{\$6}{1 \text{ kg}}$$
$$\text{Rate } \$6/\text{kg} = \frac{600 \text{ cents}}{1000 \text{ g}}$$
$$\text{Rate } \$6/\text{kg} = 0.6 \text{ c/g}$$

   (c) Hose 20L/min

   (i) The rate is given in units of L/min, so a quarter of an hour must be changed to minutes = 15 minutes. The volume after 15 mins is $15 \times 20 = 300$ litres.

   (ii) Time to fill a 250L drum?

$$\text{Rate L/min} = \frac{\text{Volume in Litres}}{\text{Time in minutes}}$$
$$20 \text{ L/min} = \frac{250 \text{ L}}{\text{Time in minutes}} \quad \text{rearranging}$$
$$\text{Time in minutes} = \frac{250 \text{ L}}{20 \text{ L/min}}$$
$$\text{Time in minutes} = 12.5 \text{ min}$$

3. (a)

4:5	$4:5 = \frac{4}{5}$	$\frac{4}{5} = \frac{8}{10} = 0.8$	$0.8 \times 100\% = 80\%$

(b)

(i) $45^{\div 15} : 60^{\div 15} = 3 : 4$	$5\text{ mins} : 1\text{ hr}$ (ii) $5^{\div 5} : 60^{\div 5}$ $1 : 12$	$0.4 : 1.2$ (iii) $= 4 : 12$ $= 1 : 3$

4. (a)

$$3 : 7 = p : 49$$
$$\frac{3 \times \cancel{49}^{7}}{\cancel{7}^{1}} = \frac{p \times \cancel{49}^{1}}{\cancel{49}^{1}}$$
$$21 = p$$
$$p = 21$$

(b)

$$1 : 20 = 200 : p$$
$$\frac{1}{20} = \frac{200}{p}$$
$$\frac{20 \times 200}{1} = \frac{p \times \cancel{200}}{\cancel{200}}$$
$$4000 = p$$
$$p = 4000$$

4000mL or 4 litres of petrol is required.

(c)

$$1 : 2000 = 2.5 : d$$
$$\frac{1}{2000} = \frac{2.5}{d}$$
$$\frac{2000 \times 2.5}{1} = \frac{d \times \cancel{2.5}^{1}}{\cancel{2.5}^{1}}$$
$$2000 \times 2.5 = d$$
$$d = 5000$$

The distance in real life is 5000cm which is 50m.

5. 10kg is split in the ratio 13:7
The total number of parts is $13 + 7 = 20$
Each part is $10 \div 20 = 0.5$, that is 0.5kg.
Copper portion is $13 \times 0.5 = 6.5$
Zinc portion is $7 \times 0.5 = 3.5$ (Check: 6.5+3.5=10)

Rates: meaning, unit rates and rate conversions

1. (a) time
   (b) distance
   (c) distance, time
   (d) revolution, time
   (e) cost, area

2. (a)

$$\text{Pay rate (\$/hr)} = \frac{\text{Pay amount(\$)}}{\text{time(hr)}}$$
$$\text{Pay rate (\$/hr)} = \frac{\$157.75}{5\text{ hrs}}$$
$$\text{Pay rate (\$/hr)} = \$31.55 / hr$$

Eric earned $157.75 for 5 hours work.

(b) $\text{Revolutions/min} = \dfrac{\text{Number of revolutions}}{\text{Time (mins)}}$

$\text{Revolutions/min} = \dfrac{100 \text{ revolutions}}{3 \text{ mins}}$

$\text{Revolutions/min} = 33\frac{1}{3} \text{rev/min}$

(c) 15 secs = 0.25 mins

$\text{Heart rate (beats/min)} = \dfrac{\text{beats}}{\text{time (min)}}$

$\text{Heart rate (beats/min)} = \dfrac{18 \text{ beats}}{0.25 \text{ min}}$

Heart rate (beats/min) = 72 beats/min

(d) $\text{speed (km/hr)} = \dfrac{\text{distance (km)}}{\text{time (hrs)}}$

$\text{speed (km/hr)} = \dfrac{70 \text{ km}}{2.5 \text{ hrs}}$

speed (km/hr) = 28 km/hr

(e) $\text{Cost per kg (\$/kg)} = \dfrac{\text{Cost (\$)}}{\text{weight (kg)}}$

$\text{Cost per kg (\$/kg)} = \dfrac{\$ 2}{3.25 \text{ kg}}$

Cost per kg ($/kg) = 0.61538 $/kg

Cost per kg ($/kg) $\approx$ 0.62 $/kg or 62c/kg

3. Convert the rates below to the new units given

(a) $15 \text{ km/hr} = \dfrac{15 \text{ km}}{1 \text{ hr}}$

$15 \text{ km/hr} = \dfrac{15\,000 \text{ m}}{1 \text{ hr}}$

15 km/hr = 15 000m/hr

(b) $1.5 \text{ t/day} = \dfrac{1.5 \text{ t}}{1 \text{ day}}$

$1.5 \text{ t/day} = \dfrac{1500 \text{ kg}}{1 \text{ day}}$

1.5 t/day = 1500 kg/day

(c) $60 \text{ km/hr} = \dfrac{60 \text{ km}}{1 \text{ hour}}$

$60 \text{ km/hr} = \dfrac{60\,000 \text{ m}}{3600 \text{ sec}}$ $\quad [1 \text{ hr} = 60 \text{ min} = 3600\ (60 \times 60) \text{ sec}]$

60 km/hr = 16.7 m/sec (to 1 d.p.)

(d)
$$15 \text{ m/sec} = \frac{15 \text{ m}}{1 \text{ sec}}$$
$$15 \text{ m/sec} = \frac{15 \div 1000 \text{ km}}{1 \div 3600 \text{ hr}}$$
$$15 \text{ m/sec} = \frac{0.015 \text{ km}}{0.00027\overline{7} \text{ hr}}$$
$$15 \text{ m/sec} = 54 \text{ km/hr}$$

(e)
$$38 \text{ words/min} = \frac{38 \text{ words}}{1 \text{ min}}$$
$$38 \text{ words/min} = \frac{38 \text{ words}}{1 \div 60 \text{ hr}}$$
$$38 \text{ words/min} = \frac{38 \text{ words}}{0.016\overline{6} \text{ hr}}$$
$$38 \text{ words/min} = 2280 \text{ words/hr}$$

Calculations based on rates

1. A typist can type at 18 words/minute. How many words will be typed after:
   (a) After 5 minutes $5 \times 18 = 90$ words.

   (b) Half an hour (30 mins) $30 \times 18 = 540$ words.

   (c) After 37 minutes $37 \times 18 = 666$ words.

   How long will it take the typist to type: Answer in minutes and seconds?

   (d)
$$\text{rate (words/min)} = \frac{\text{words}}{\text{time (min)}}$$
$$18 \text{ (words/min)} = \frac{100 \text{ words}}{\text{time (min)}}$$
$$\text{time (min)} = \frac{100 \text{ words}}{18 \text{ (words/min)}}$$
$$\text{time (min)} = 5.5\overline{5} \text{ min or } 5 \text{ min } 33 \text{ sec}$$

   (e)
$$\text{rate (words/min)} = \frac{\text{words}}{\text{time (min)}}$$
$$18 \text{ (words/min)} = \frac{550 \text{ words}}{\text{time (min)}}$$
$$\text{time (min)} = \frac{550 \text{ words}}{18 \text{ (words/min)}}$$
$$\text{time (min)} = 30.5\overline{5} \text{ min or } 30 \text{ min } 33 \text{ sec}$$

   (f)
$$\text{rate (words/min)} = \frac{\text{words}}{\text{time (min)}}$$
$$18 \text{ (words/min)} = \frac{1200 \text{ words}}{\text{time (min)}}$$
$$\text{time (min)} = \frac{1200 \text{ words}}{18 \text{ (words/min)}}$$
$$\text{time (min)} = 66.6\overline{6} \text{ min or } 66 \text{ min } 40 \text{ sec}$$

2. An orange picker can pick at a rate of 38 kg/hr. How long will it to pick a tonne (1000kg)

$$\text{rate (kg/hr)} = \frac{\text{weight (kg)}}{\text{time (min)}}$$
$$38\ \text{(kg/hr)} = \frac{1000\ \text{kg}}{\text{time (hr)}}$$
$$\text{time (min)} = \frac{1000\ \text{kg}}{38\ \text{(kg/hr)}}$$
$$\text{time (min)} = 26.316\ \text{hr or } 26\ \text{hr } 19\ \text{min}$$

3. Electricity costs 19.2c/unit. What will the charge be for 857 units?
Charge = $19.2 \times 857 = 16454.4c = \$164.54$

4. Carpet costs $85/square metre. How much will it cost to lay carpet in a room measuring 5.5m by 4.2 ?

The area of carpet: $A = l \times b$, $A = 5.5 \times 4.2$, $A = 23.1m^2$

Cost is: $Cost = 23.1 \times 85$, $Cost = 1963.5$, Cost is $1963.50

5. Which is better value: 550g for $1.74 or 1.2kg for $3.85?
Let's consider the rate g/$.

For 550g for $1.74

$$\text{Rate g/\$} = \frac{\text{Weight (g)}}{Cost(\$)}$$
$$\text{Rate g/\$} = \frac{550\ \text{g}}{\$1.74}$$
$$\text{Rate g/\$} = 316\ \text{g/\$}$$

For 1.2kg (1200g) for $3.85

$$\text{Rate g/\$} = \frac{\text{Weight (g)}}{Cost(\$)}$$
$$\text{Rate g/\$} = \frac{1200\ \text{g}}{\$3.85}$$
$$\text{Rate g/\$} = 311.7\ \text{g/\$}$$

The 550g container is better value for money.

6. A truck driver is expected to average 80 km/hr on longer journeys. William drove the 1610km from Brisbane to Cairns in 21 hours, did he exceed the expected rate?

$$\text{average speed (km/hr)} = \frac{\text{distance (km)}}{\text{time (hr)}}$$
$$\text{average speed (km/hr)} = \frac{1610\ \text{(km)}}{21\ \text{(hr)}}$$
$$\text{average speed (km/hr)} = 76.7\ \text{km/hr (to 2 d.p.)}$$

William has not exceeded the expected rate.

7. Two friends went jogging: Barry covered 8 km in 33 mins, while Bruce covered 5 km in 24mins, which person jogged at the fastest rate?

Let's consider the distance covered per minute (km/min).

For Barry

$$\text{Rate (km/min)} = \frac{\text{distance jogged (km)}}{\text{time taken (min)}}$$
$$\text{Rate (km/min)} = \frac{8\ \text{km}}{33\ \text{min}}$$
$$\text{Rate (km/min)} = 0.242\ \text{km/min}$$

For Bruce

$$\text{Rate (km/min)} = \frac{\text{distance jogged (km)}}{\text{time taken (min)}}$$
$$\text{Rate (km/min)} = \frac{5\ \text{km}}{24\ \text{min}}$$
$$\text{Rate (km/min)} = 0.208\ \text{km/min}$$

Barry jogged at a faster rate (242m/min compared to 208 m/mim)

8. It takes a gang of workers laying railway line 5 days to lay 1250m of line. How long would it take this gang to lay 4500m of line?

Rate of laying railway line is:

$$\text{Rate (m/day)} = \frac{\text{distance laid (m)}}{\text{time(days)}}$$

$$\text{Rate (m/day)} = \frac{1250\text{ m}}{5\text{ days}}$$

Rate (m/day) = 250 m/day

or Using equivalent ratios (see Topic 5)

$$4500:1250 = d:5$$

$$\frac{4500\times 5}{1250} = \frac{d\times \cancel{5}}{\cancel{5}^{1}}$$

$$18 = d$$

$$d = 18$$

Using the rate equation again:

$$\text{Rate (m/day)} = \frac{\text{distance laid (m)}}{\text{time(days)}}$$

$$250\text{ m/day} = \frac{4500\text{ m}}{\text{Time (days)}}$$

$$\text{Time (days)} = \frac{4500\text{ m}}{250\text{ m/day}}$$

Time (days) = 18 days

9. At present the exchange rate to the English pound is \$A1=£0.52
Change this into a rate:

Rate is £0.52 / \$A

(a) How many English pounds will \$A500 buy?

\$A500 will buy 0.52×500=£260

(b) How many Australian dollars will £135 buy?

$$\text{Rate £0.52/\$A}=\frac{\text{Pounds}}{\text{dollars}}$$

$$\text{Rate } 0.52=\frac{\text{£135}}{\text{dollars}}$$

$$\text{dollars}=\frac{\text{£135}}{\text{\$A0.52}}$$

dollars = \$A 259.62

Ratios: meaning, ratios to other number forms

1. Complete this table:

Ratio	Fraction	Decimal	Percentage
1:5	$\frac{1}{5}$	$\frac{1^{\times 2}}{5^{\times 2}} = \frac{2}{10} = 0.2$	$0.2\times 100 = 20\%$
7:8	$\frac{7}{8}$	$8\overline{)7.0^{6}0^{4}0}$ = 0.875	$0.875\times 100 = 87.5\%$
13:20	$\frac{\cancel{65}^{13}}{\cancel{100}^{20}} = \frac{13}{20}$	**0.65**	$0.65\times 100 = 65\%$
12:5	$\frac{24}{10} = \frac{12}{5}$	$240\% \div 100 = 2.4$	**240%**

Simplifying ratios

1. Simplify the following ratios

(a) $15^{\div 5} : 35^{\div 5}$
$= 3 : 7$

(b) $49^{\div 7} \; : \; 84^{\div 7}$
$= 7 : 12$

(c) $0.8^{\times 10} : \; 2^{\times 10}$
$= 8^{\div 4} : 20^{\div 4}$
$= 2 : 5$

(d) $0.125^{\times 1000} : \; 0.5^{\times 1000}$
$= 125^{\div 125} : 500^{\div 125}$
$= 1 : 4$

(e) $\frac{2}{5} : \frac{1}{4}$
$= \frac{8}{20} : \frac{5}{20}$
$= 8 : 5$

(f) $2\frac{2}{3} : 1\frac{1}{6}$
$= \frac{8}{3} : \frac{7}{6}$
$= \frac{16}{6} : \frac{7}{6}$
$= 16 : 7$

(g) 15 mm : 5 cm
=15 mm : 50 mm
= 3 :10

(h) 15g : 250mg
=1500mg : 250mg
= 6 : 1

2. The Greeks believed that rectangles that had a length to width ratio of 1.618 (approx) : 1 were the most pleasing to the eye. This ratio is known as the Golden Ratio.

(a) Length : Width
$= \; 37^{\div 23} \; : \; 23^{\div 23}$ This is quite close to a Golden Ratio
$= \; 1.609 : 1$

(b) Length:Width
$= 21^{\div 14} : 14^{\div 14}$
$= 1.5 : 1$
This somewhat close to a Golden Ratio.

The length would need to be increased, to calculate the ideal length the width would need to be multiplied by the Golden Ratio.

14x1.618 : 14
= 22.65 : 14 If the length was 22.65, the envelope would be in the Golden Ratio.

Equivalent ratios

1. Find the value of the pronumeral in the equivalent ratios below.

(a) $1 : 2 = a : 12$

$$\frac{1 \times 12}{2} = \frac{a \times \cancel{12}^{1}}{\cancel{12}^{1}}$$

$$6 = a$$

(b) $8 : 5 = b : 25$

$$\frac{8 \times 25}{5} = \frac{b \times \cancel{25}^{1}}{\cancel{25}^{1}}$$

$$40 = b$$

(c) $9 : 2 = 81 : c$

$$\frac{9}{2} = \frac{81}{c}$$

$$\frac{2 \times 81}{9} = \frac{c \times \cancel{81}^{1}}{\cancel{81}^{1}}$$

$$18 = c$$

(d) $2.5 : 4 = d : 12$

$$\frac{2.5 \times 12}{4} = \frac{d \times \cancel{12}^{1}}{\cancel{12}^{1}}$$

$$7.5 = d$$

2. In a tile pattern, the ratio of red tiles to green tiles is 3:5. If the tiler has 27 red tiles, how many green tiles will be required?

$$3 : 5 = 27 : g$$

$$\frac{3}{5} = \frac{27}{g}$$

$$\frac{5 \times 27}{3} = \frac{g \times \cancel{27}^{1}}{\cancel{27}^{1}}$$

$$45 = g$$

The tiler will need 45 green tiles.

3. In a cocktail the ratio of a spirit to soft drink is 1 : 6. If the person has a 375mL bottle of spirit, how much soft drink will they need?

$$1 : 6 = 375 : s$$

$$\frac{1}{6} = \frac{375}{s}$$

$$\frac{6 \times 375}{1} = \frac{s \times \cancel{375}^{1}}{\cancel{375}^{1}}$$

$$2250 = s$$

2250 mL or 2.25L is required to make the cocktails.

4. The ratio of workers to instructors must not exceed 8 : 1. If there are 50 workers, what is the minimum number of instructors required?

$$8 : 1 = 50 : i$$

$$\frac{8}{1} = \frac{50}{i}$$

$$\frac{1 \times 50}{8} = \frac{i \times \cancel{50}^{1}}{\cancel{50}^{1}}$$

$$6.25 = i$$

7 instructors are needed. If 6 are chosen the 8:1 ratio will be exceeded.

5. A photograph measuring 15cm by 10cm is being enlarged so that the new length is 36cm. If the photo keeps the same ratio (proportion), what is the new width?

$$15 : 10 = 36 : w$$

$$\frac{15}{10} = \frac{36}{w}$$

$$\frac{10 \times 36}{15} = \frac{w \times \cancel{36}^{1}}{\cancel{36}^{1}}$$

$$24 = w$$

The dimensions of the new picture are 36cm x 24cm

6. The scale on a map is 1: 500 000

(a) What is the distance from Howard to Frazer?
The distance on the map is 32mm

$$1:500000 = 32:gd$$
$$\frac{1}{500000} = \frac{32}{gd}$$
$$\frac{500000 \times 32}{1} = \frac{gd \times \cancel{32}^{1}}{\cancel{32}^{1}}$$
$$16\,000\,000 = gd$$

The ground distance is 16 000 000mm =16 000m = 16km

(b) What is the distance from Mt Hawke to the Lighthouse?

$$1:500000 = 55:gd$$
$$\frac{1}{500000} = \frac{55}{gd}$$
$$\frac{500000 \times 55}{1} = \frac{gd \times \cancel{55}^{1}}{\cancel{55}^{1}}$$
$$27\,500\,000 = gd$$

The ground distance is 27 500 000mm = 27 500m = 27.5km

(c) There is a historic site located 36 km west of Howard, how far is this on the map?

$$1:500000 = md:36$$
$$\frac{1 \times 36}{500000} = \frac{md \times \cancel{36}^{1}}{\cancel{36}^{1}}$$
$$0.000072 = md$$

The map distance is 0.000072 km = 0.072m = 72mm

(d) There is a ship located on the map 2.6 cm east of the lighthouse. How far is it from the coast? If the ship is travelling west at 5 km/hr, how long before the ship runs aground?

The coast is 1.9 cm or 19mm away from the ship.

$$1:500000 = 19:gd$$
$$\frac{1}{500000} = \frac{19}{gd}$$
$$\frac{500000 \times 19}{1} = \frac{d \times \cancel{19}^{1}}{\cancel{19}^{1}}$$
$$9\,500\,000 = gd$$

The ship is 9 500 000mm = 9.5 km from the coast.

At 5 km/hr, the time taken to cover 9.5km is:

$$\text{speed (km/hr)} = \frac{\text{distance (km)}}{\text{time (hrs)}}$$
$$\text{5 km/hr} = \frac{\text{9.5 km}}{\text{time (hrs)}}$$
$$\text{time (hrs)} = \frac{\text{9.5 km}}{\text{5 km/hr}}$$
$$\text{time (hrs)} = \text{1.9 hrs or 1hr 54 min}$$

The ship will run aground after 1hr 54min

7. The scale on a map is 1:1 200 000. The distance between two features is 46mm, what is the ground distance?

$$1:1\,200\,000 = 46:gd$$
$$\frac{1}{1200000} = \frac{46}{gd}$$
$$\frac{1200000 \times 46}{1} = \frac{gd \times \cancel{46}^{1}}{\cancel{46}^{1}}$$
$$55\,200\,000 = gd$$

Ground distance is 55 200 000mm = 55.2 km

8. Derrick has an old map where the scale has been damaged and is unreadable. However, he knows that the actual distance from the picnic area to the cave entrance is 1.6 km. On the map this distance is 80mm, what is the scale of the map?

Scale =map distance : ground distance
Scale= 80mm:1.6km
Scale= 80mm:1 600m
Scale= 80mm:1 600 000mm $(\div 80)$
Scale= 1:20 000

The scale on the map was 1 : 20 000

Sharing an amount in a ratio

1. Divide the quantity in the given ratio.

(a) Divide \$75 in the ratio 4:11

The total number of parts is $4+11=15$
Each part is $75 \div 15 = 5$, that is \$5.
First share is $4 \times \$5 = \$20$
Second share is $11 \times \$5 = \$55$

(b) Divide 100cm in the ratio 2:3

The total number of parts is $2+3=5$
Each part is $100 \div 5 = 20$, that is 20cm.
First share is $2 \times 20 = 40cm$
Second share is $3 \times 20 = 60cm$

(c) Divide 2 tonnes in the ratio 3:5

Change 2 tonnes to 2000kg
The total number of parts is $3+5=8$
Each part is $2000 \div 8 = 250$, that is 250kg.
First share is $3 \times 250 = 750kg$
Second share is $5 \times 250 = 1250kg$

(d) Divide 200mL in the ratio 1:3:4

The total number of parts is $1+3+4=8$
Each part is $200 \div 8 = 25$, that is 25mL.
First share is $1 \times 25 = 25mL$
Second share is $3 \times 25 = 75mL$
Third share is $4 \times 25 = 100mL$

(e) Divide \$80 in the ratio 1:5

The total number of parts is $1+5=6$
Each part is $80 \div 6 = 13.3\overline{3}$, that is \$13.33.
First share is $1 \times 13.3\overline{3} = \$13.33$
Second share is $5 \times 13.3\overline{3} = \$66.67$

2. Brass is an alloy of Copper and Tin. The ratio of copper to tin is 88%:12%. If 1500g of brass is used to make a small statue, what was the original quantity of Copper and Tin?

The ratio 88%:12% simplifies to 22:3.

The total number of parts is $22+3=25$
Each part is $1500 \div 25 = 60$, that is 60g.
Copper share is $22 \times 60 = 1320g$
Tin share is $3 \times 60 = 180g$

3. On a journey, John and Jane share the driving in the ratio 3:1. What fraction of the journey did Jane drive?

The total number of parts is $3+1=4$

Jane will drive $\frac{1}{4}$ of the journey.

4. A gold chain made from 18 carat gold. In 18 carat gold the ratio of gold to silver is 3:1. If the chain weighs 70g and the price of gold is $37.41 per gram, what is the value of gold in the chain?

Fraction of gold is $\frac{3}{4}$

Amount of gold is $\frac{3}{4}\times 70 = 52.5g$

Value of gold is 52.5 x 37.42 =$ 1964.03

5. A school is calculating measurements for a rectangular soccer ground. The physical education teacher wants the perimeter to be 400m for P.E. students to jog laps. The soccer coach insists that the ratio of length to width is 3:2 just like an international size ground, what will the length and width that will satisfy both people.

As the length + width is 200m (half the perimeter)
200m is divided up in the ratio 3:2 (length:width)
The total number of parts is $3+2=5$
Each part is $200\div 5 = 40$, that is 40m.
Length is $3\times 40 = 120m$
Width is $2\times 40 = 80m$