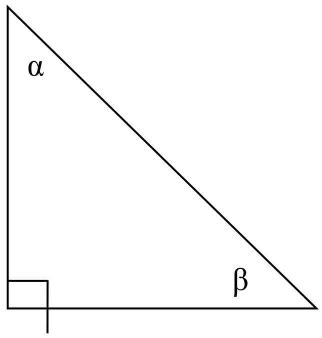
Introduction to Trigonometry

Pythagoras' Theorem and basic Trigonometry use right angle triangle structures. (Advanced Trigonometry uses non-right angled triangles)

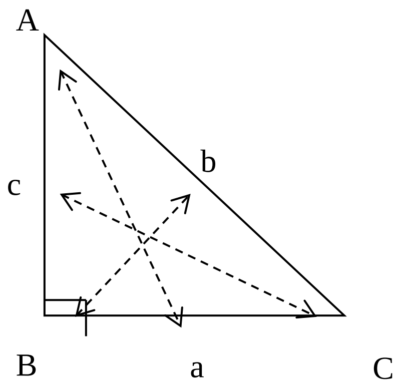
The angle sum of a triangle is 180°, as one angle is 90° the other two angles must add to 90°.



For this triangle, it is possible to write

$\alpha + \beta = 90°$

Conventions for naming triangles involve using capital letters for vertices (corners) and lower case letters for sides. You may have already noticed that letters from the Greek Alphabet are used for naming angles.



You will notice that side *a* is opposite angle *A*, side *b* is opposite angle *B* etc.

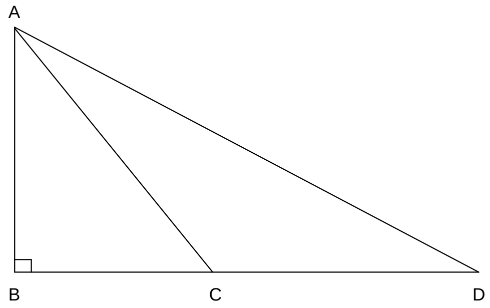
Angles can be labelled;

(a) using Greek letters $\theta = 32^\circ$

(b) using the letter at the vertex $\angle A = 32^\circ$

(c) using three letters $\angle BAC = 32^\circ$

The three letter method can remove ambiguity in more complex situations.



$\angle A$ is ambiguous.

$\angle BAC$, $\angle CAD$ and $\angle BAD$ are expressed clearly.

Note: $\angle BAD = \angle BAC + \angle CAD$

Module contents

- **Introduction**
- **Pythagoras' Theorem**
- **The Trigonometric Ratios – Finding Sides**
- **The Trigonometric Ratios – Finding Angles**
- **Answers to activity questions**

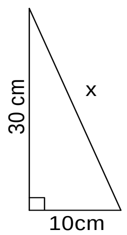
Outcomes

- To use Pythagoras' Theorem to solve right angled triangle problems.
- To solve right angled triangles, ie: missing sides and missing angles.
- To use Pythagoras' Theorem and trigonometry to solve problems.

Check your skills

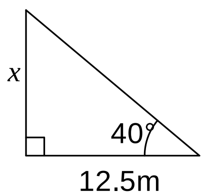
This module covers the following concepts, if you can successfully answer these questions, you do not need to do this module. Check your answers from the answer section at the end of the module.

1. (a) Use Pythagoras' Theorem to calculate x.

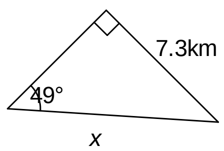


(b) A rectangle has a length of 6.72m and width of 4.83m. Find the length of the diagonal (to 2 d.p.).

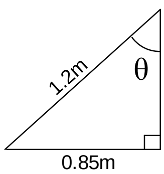
2. (a) Calculate the value of x in this triangle.



(b) Calculate the value of x in this triangle.



(c) Calculate the value of θ in this triangle.



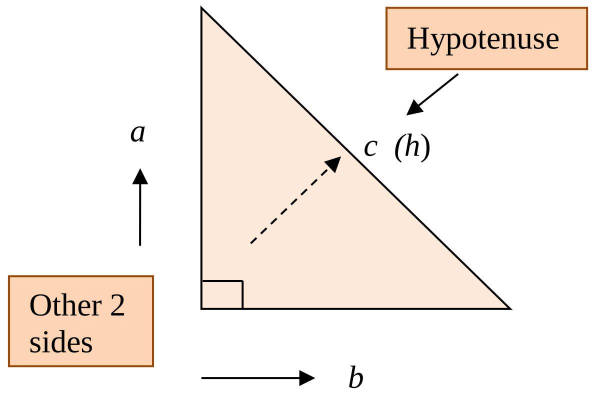
3. A kite on the end of a 30m string is flying at an angle of elevation of 72°. What is the height of the kite directly above the ground?

Topic 1: Pythagoras' Theorem

Pythagoras' Theorem states that in a right angled triangle:

> 'The square of the hypotenuse is equal to the sum of the squares of the other two sides'

Diagrammatically:



The Hypotenuse is the longest side in the triangle and is also opposite the right angle.

$$c^2 = a^2 + b^2$$

The square of the Hypotenuse	=	Sum of the squares of the other 2 sides

Alternatively, because the Hypotenuse is a unique side:

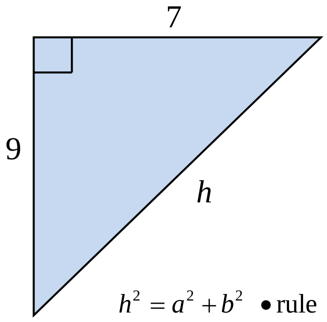
$$h^2 = a^2 + b^2$$

This means that Pythagoras' Theorem can be used to find the length of a missing side in a right angled triangle.

Examples

Hypotenuse unknown

(i)



$h^2 = a^2 + b^2$ • rule

$h^2 = 7^2 + 9^2$ • substitute

$h^2 = 49 + 81$

$h^2 = 130$

$h = \sqrt{130}$ • square root

$h \approx 11.4$

(ii)

In a right triangle: side *a* = 5 cm, *b* = 10 cm and *c* is the hypotenuse. Determine the length of side *c*.

$h^2 = a^2 + b^2$ • rule

$h^2 = 5^2 + 10^2$ • substitute

$h^2 = 25 + 100$

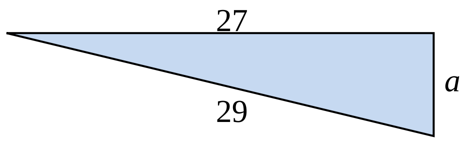
$h^2 = 125$

$h = \sqrt{125}$ • square root

$h \approx 11.2$

Other side unknown

(i)



$h^2 = a^2 + b^2$ • rule

$29^2 = 27^2 + a^2$ • substitute

$841 = 729 + a^2$

$841 - 729 = a^2$ • rearrange

$112 = a^2$ • square root

$a = \sqrt{112}$

$a \approx 10.6$

Or

$h^2 = a^2 + b^2$ • rule

$h^2 - b^2 = a^2$ • rearrange

$29^2 - 27^2 = a^2$ • substitutite

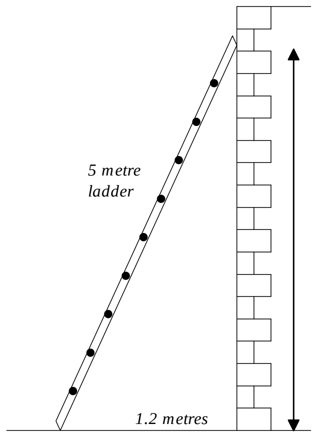
$841 - 729 = a^2$

$112 = a^2$ • square root

$a = \sqrt{112}$

$a \approx 10.6$

(ii)



How far up the wall does the ladder reach?

$h^2 = a^2 + b^2$ • rule

$5^2 = 1.2^2 + b^2$ • substitute

$25 = 1.44 + b^2$

$25 - 1.44 = b^2$ • rearrange

$23.56 = b^2$ • square root

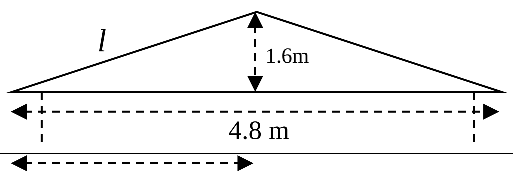
$b = \sqrt{23.56}$

$b \approx 4.85$

The ladder reaches approximately 4.85m up the wall.

Mixed worded questions

(i) A shed has a gable roof as drawn below. Calculate the length of sheets of roofing iron required in its construction.



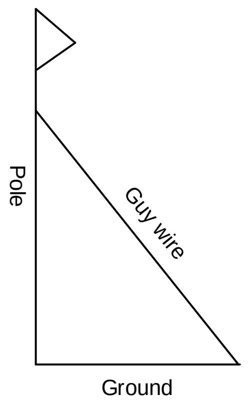
Let the length of the sheets be *l*.

2.4 m

$$h^2 = a^2 + b^2$$
$$l^2 = 1.6^2 + 2.4^2$$
$$l^2 = 2.56 + 5.76$$
$$l^2 = 8.32$$
$$l = \sqrt{8.32}$$
$$l = 2.88$$

The length of the sheets of roofing iron is 2.88m

(ii) A guy (support) wire is attached 3.2 m up a pole and at a point 2.1 m from the pole. The ground and the pole are perpendicular (at right angles). What is the length of the guy wire?

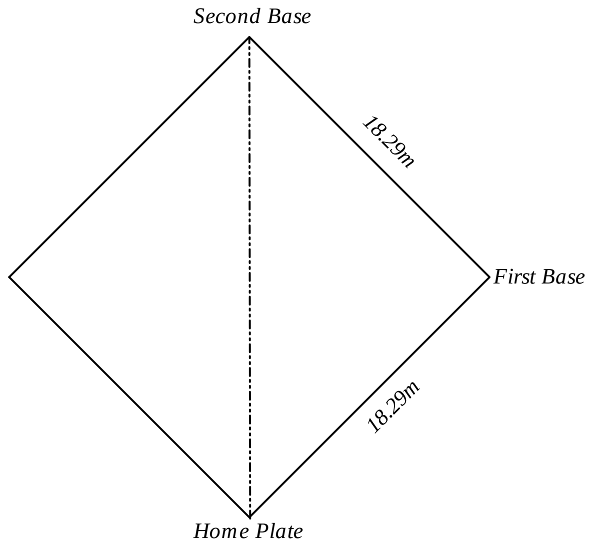


Let the length of the guy wire be *l*.

$$h^2 = a^2 + b^2$$
$$l^2 = 3.2^2 + 2.1^2$$
$$l^2 = 14.65$$
$$l = \sqrt{14.65}$$
$$l = 3.83$$

The length of the guy wire is 3.83m

(iii) On a softball diamond, the distance between bases is 60 feet or 18.29m. How far must the catcher (at home base) throw the ball to the player on second base?



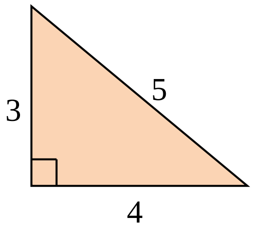
The angle at first base is a right angle. Let the distance from Home plate to second base be *d*.

$$h^2 = a^2 + b^2$$
$$d^2 = 18.29^2 + 18.29^2$$
$$d^2 = 669.05$$
$$d = \sqrt{669.05}$$
$$d = 25.87$$

The distance from home plate to second base is 25.87m

Pythagorean Triple or Triad

Sometimes, the three lengths of a right angled triangle are all **whole numbers**. When this occurs, they are called a Pythagorean Triple or Triad. The most commonly known of these is (3,4,5) representing the triangle below:

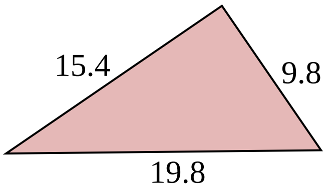


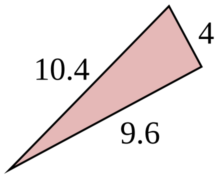
$$h^2 = a^2 + b^2$$
$$5^2 = 3^2 + 4^2$$
$$25 = 9 + 16$$
$$25 = 25$$

Three other Triads are (5,12,13), (7,24,25) and (8,15,17).

Other Triads can be based on multiples of the base Triads; the Triads (6,8,10), (9,12,15), (12,16,20) ... are based the base Triad (3,4,5).

If it is important to decide if a triangle is a right angled triangle, then Pythagoras' Theorem can be used to decide this.





$$\begin{aligned} h^2 &= 19.8^2 = 392.04 \\ a^2 + b^2 &= 15.4^2 + 9.8^2 \\ &= 237.16 + 96.04 \\ &= 333.2 \\ h^2 &\neq a^2 + b^2 \end{aligned}$$

This is not a right angled triangle.

$$\begin{aligned} h^2 &= 10.4^2 = 108.16 \\ a^2 + b^2 &= 4^2 + 9.6^2 \\ &= 16 + 92.16 \\ &= 108.16 \\ h^2 &= a^2 + b^2 \end{aligned}$$

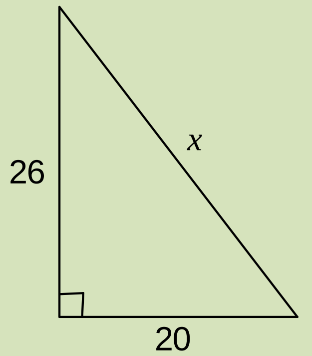
This is a right angled triangle.

 Video 'Pythagoras' Theorem'

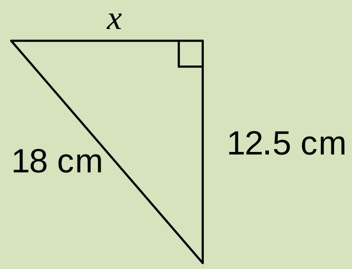
Activity

1. Use Pythagoras' Theorem to find the missing length.

(a)



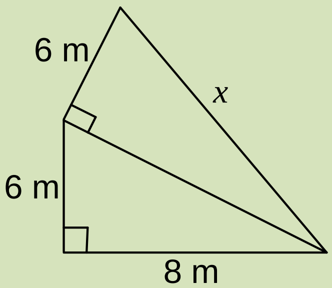
(b)



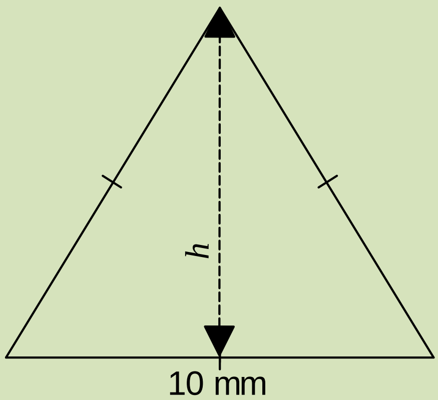
(c) Find the hypotenuse when $a=6.1$ and $b=3.4$

(d) Find the missing side given $a=23.5$ and $h=40$

(e)



(f)



Find the height of the triangle.
Also calculate the area.

(g) A orienteering participant runs 650m north an then turns and runs 1.4 km east. How far from the starting point is the runner?

(h) **A Real Challenge**

A square has a diagonal of 20 cm, what is the side length?

2. Do the following triangles contain a right angle?

(a)	10, 24, 26	(b)	7, 8, 10	(c)	2, 4.8, 5.2
(d)	1.4, 4.8, 5	(e)	6, 6, 8	(f)	5, 6, 7

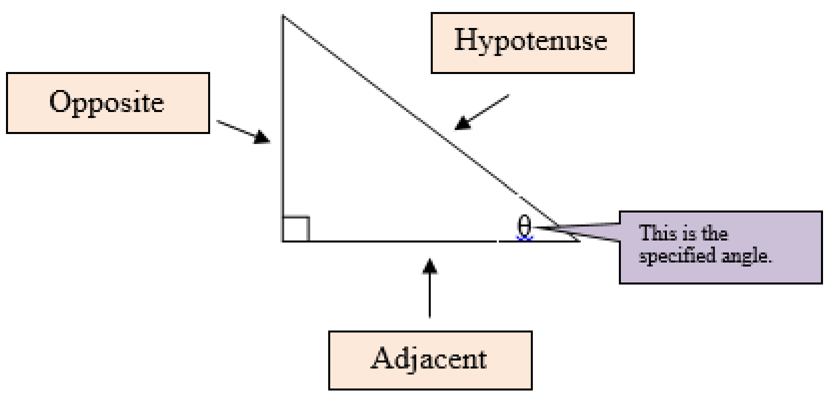
Topic 2: The Trigonometric Ratios – Finding Sides

Labelling sides

To use the Trigonometric Ratios, commonly called the Trig Ratios, it is important to learn how to label the right angled triangle. The hypotenuse of the triangle is still the longest side and located opposite the right angle. The two other sides are named: the opposite and the adjacent. The everyday language meanings of these terms help in using these labels.

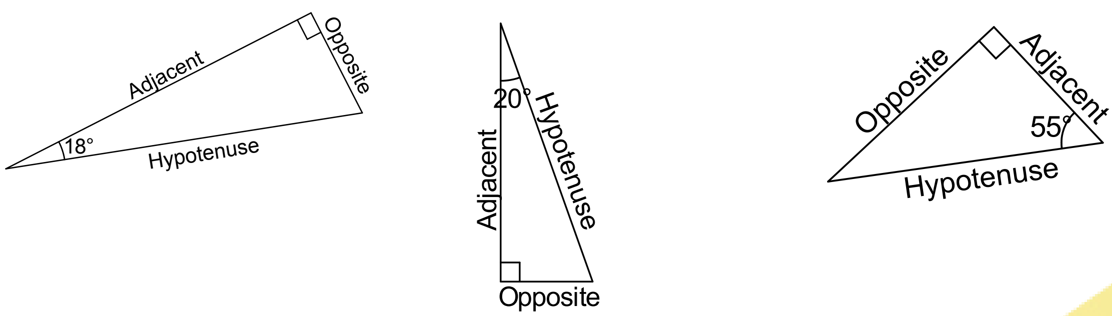
The opposite side is located opposite the specified angle.

The adjacent is located adjacent (next to) the specified angle.

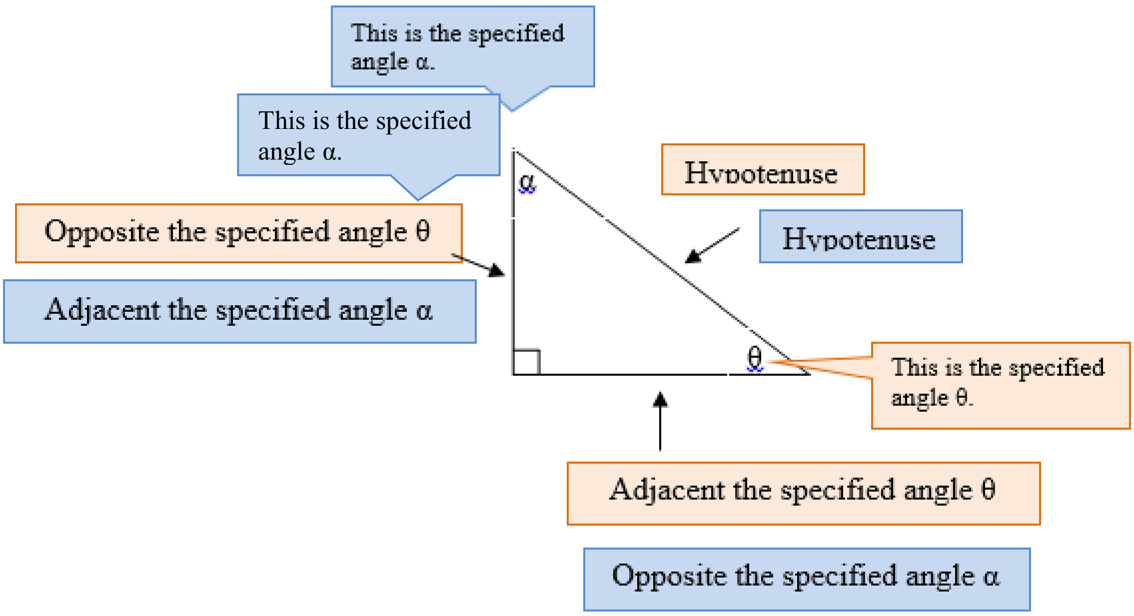


When naming a triangle, always name the Hypotenuse first. The reason for doing this is that both the Hypotenuse and the Adjacent are adjacent (next to) the specified angle. By naming the hypotenuse first there can only be one side that is adjacent to the specified angle.

Example:



If there are two angles, the opposite and adjacent are still relative to the specified angle. The diagram below shows the idea.



The Trigonometric Ratios

With right angled triangles having three sides, it is possible to have 6 ratios. They are:

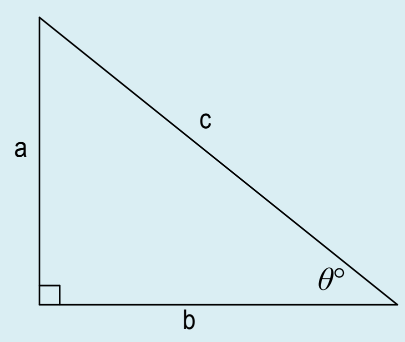
$$\frac{opposite}{adjacent}, \frac{adjacent}{opposite}, \frac{opposite}{hypotenuse}, \frac{hypotenuse}{opposite}, \frac{adjacent}{hypotenuse} \text{ and } \frac{hypotenuse}{adjacent}.$$

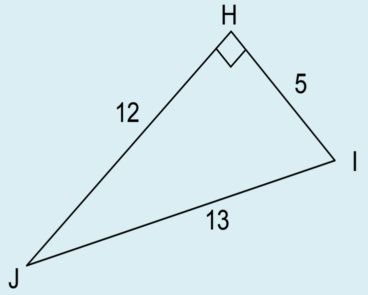
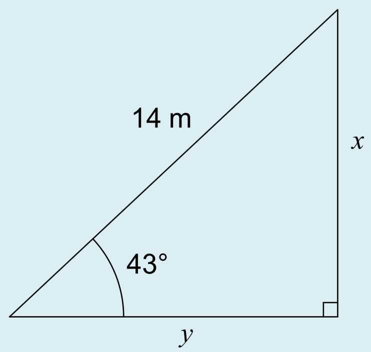
There are basically 3 unique ratios. The 3 trig ratios commonly used are given the names Sine, Cosine and Tangent. These names are abbreviated to Sin, Cos and Tan. If the specified angle is θ, then the ratios are written as:

$Sin\ \theta = \frac{opposite}{hypotenuse}$	$Cos\ \theta = \frac{adjacent}{hypotenuse}$	$Tan\ \theta = \frac{opposite}{adjacent}$

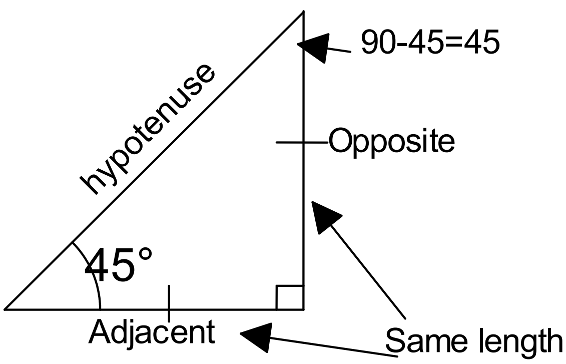
A challenge for you is to devise a way to remember these ratios.

The table below show how the ratios are applied to right angled triangles.

 	For the specified angle θ: $Sin\ \theta = \frac{opposite}{hypotenuse} = \frac{a}{c}$ $Cos\ \theta = \frac{adjacent}{hypotenuse} = \frac{b}{c}$ $Tan\ \theta = \frac{opposite}{adjacent} = \frac{a}{b}$

<table>
<tr>
<td>
</td>
<td>For the specified angle J:
$Sin\ J = \frac{5}{13}$
$Cos\ J = \frac{12}{13}$
$Tan\ J = \frac{5}{12}$</td>
<td>For the specified angle I:
$Sin\ I = \frac{12}{13}$
$Cos\ I = \frac{5}{13}$
$Tan\ I = \frac{12}{5}$</td>
</tr>
<tr>
<td>
</td>
<td colspan="2">For the specified angle of 43°
$Sin\ 43° = \frac{x}{14}$
$Cos\ 43° = \frac{y}{14}$
$Tan\ 43° = \frac{x}{y}$</td>
</tr>
</table>

To help understand trigonometry, the example below gives an indication of how it works. Consider a right angled triangle with an angle of 45°, this means that the missing angle is also 45°. This means that the opposite and adjacent side must be equal, so the triangle is actually an **isosceles right angled triangle**. The triangle is drawn below.



The Trig ratio associated with opposite and adjacent is $\text{Tan}\,\theta = \frac{opposite}{adjacent}$. With the opposite and the adjacent being the same length, the value of this ratio would be 1.

So far:	In a right angled triangle with a specified angle of 45°, the value of the ratio $\frac{opposite}{adjacent}$ is 1. This means $\text{Tan}\,45°$ is always 1.

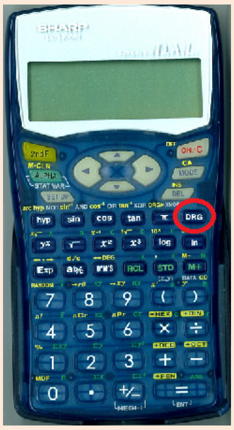
Because the value of the ratio is obtained without actually knowing any measurements, another key idea is:

Key Idea	For any size right angled triangle with a specified angle of 45°, the ratio $\text{Tan } 45° = \dfrac{opposite}{adjacent}$ is always 1. If the specified angle is 45°, the length of the opposite and the adjacent must be the same.

There are other ratios that can be calculated and whatever value they equal, the value will be related the 45° angle. This idea can be applied to right angled triangles with other sizes of specified angles.

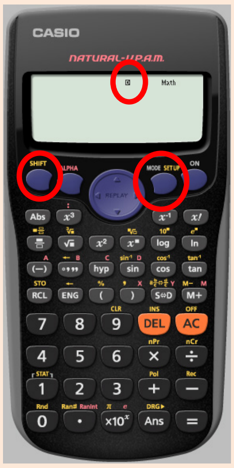
Using your calculator

The next step is learning how to use the Trig functions on your scientific calculator. At this stage the trig ratio associated with a given angle will be found. Scientific calculators allow the user to express the angle in three different ways. In this module, angles are being expressed in degrees. Read the user guide for your calculator to make sure it is always set on degrees.



On the Sharp EL 531, there is a key labelled [*DRG*]. Pressing this key allows the user to change between the different angle types. Keep pressing until DEG is found on the upper part of the screen.

Note: on this calculator it is easy to change this setting accidentally. Every time the user intends to use Trig functions, this setting should be checked.



On the Casio fx-82AU, press the SHIFT key then the MODE SETUP key. Press 3 for Degrees. A small D should appear on the top of the display as shown on the left.

Already, it is known from earlier in this topic that Tan 45° = 1. This can be confirmed by pressing [tan] [4] [5] [=], the display on the calculator should read 1. (If not, check the degrees setting and try again.) If you have an older style scientific calculator, you may have to do [4] [5] [tan] [=].

Use your calculator to find the following:

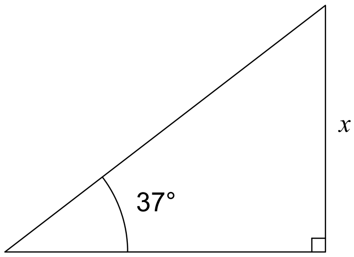
sin 30° (your calculator should display 0.5)
This means that in a right angled triangle with a specified angle of 30°, the opposite side is 0.5 the length of the hypotenuse.

cos 72° (your calculator should display 0.309016994)
This means that in a right angled triangle with a specified angle of 72°, the adjacent side is (approx) 0.31 the length of the hypotenuse.

tan 19° (your calculator should display 0.344327613)
This means that in a right angled triangle with a specified angle of 19°, the opposite side is (approx) 0.34 the length of the adjacent.

Finding missing sides

In this right angled triangle, calculate the length of the side marked *x*.



Step 1: Determine which ratio to use.

In this triangle, one angle and one side are given (37°, 12m).
Based on the 37° (specified) angle, the 12m side is the adjacent and the *x* side is the opposite.

The trig ratio to be used in this question is Tan because it contains the side lengths adjacent and opposite.

Step 2: Write out the ratio and substitute in values.

$$Tan\theta = \frac{opp}{adj}$$

$$Tan 37° = \frac{x}{12}$$

Step 3: Do the necessary rearranging.

$$12 \times Tan 37° = \frac{x}{\cancel{12}} \times \cancel{12}$$ • multiply both sides by 12

$$12 \times Tan 37° = x$$

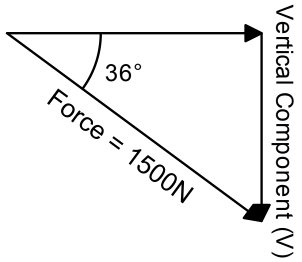
Step 4: Calculate the answer using your calculator.

1 2 X tan 3 7 = 9.042648601
The missing side length (*x*) is approx 9.04m.

Step 5: Try to check the reasonableness

The 12m side is opposite an angle of (90-37) 53°, because x is opposite the 37° angle it should be smaller than 12m, so 9.04 could be correct. (small sides are opposite small angles, large sides are opposite large angles)

A force of 1500N acts at 36° below the right horizontal as shown in the diagram below. Calculate the size of the vertical component of the force (v)



Step 1: Determine which ratio to use.

In this triangle, one angle and one side are given (36°, 1500N).
Based on the 36° (specified) angle, the 1500N side is the hypotenuse and the v side is the opposite.

The trig ratio to be used in this question is Sin because it contains the side lengths hypotenuse and opposite.

Step 2: Write out the ratio and substitute in values.

$$Sin\theta = \frac{opp}{hyp}$$

$$Sin36° = \frac{x}{1500}$$

Step 3: Do the necessary rearranging.

$$1500 \times Sin36° = \frac{v}{\cancel{1500}} \times \cancel{1500}$$ • multiply both sides by 1500

$$1500 \times Sin36° = v$$

Step 4: Calculate the answer using your calculator.

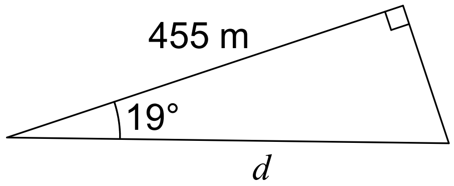
[1] [5] [0] [0] [X] [sin] [3] [6] [=] 881.7

The vertical component of force (v) is approx 881.7N.

Step 5: Try to check the reasonableness

The hypotenuse of this triangle is 1500N. The hypotenuse is also the longest side. Obtaining v as 881.7 N is consistent with this information.

The next problem is slightly different in method. *Step 3: rearranging* is different because the variable is in the denominator.



Step 1: Determine which ratio to use.
In this triangle, one angle and one side are given (19°, 455m). Based on the 19° angle, the 455m side is the adjacent and the d side is the hypotenuse.

The trig ratio to be used in this question is Cos because the question contains the two side lengths adjacent and the hypotenuse.

Step 2: Write out the ratio and substitute in values.

$$Cos\theta = \frac{adj}{hyp}$$

$$Cos19° = \frac{455}{d}$$

Step 3: Do the necessary rearranging.

$$d \times Cos19° = \frac{455}{\not{d}} \times \not{d} \quad \bullet \text{ multiply both sides by } d$$

$$\frac{d \times \cancel{Cos19°}}{\cancel{Cos19°}} = \frac{455}{Cos19°} \quad \bullet \text{ divide both sides by } Cos19°$$

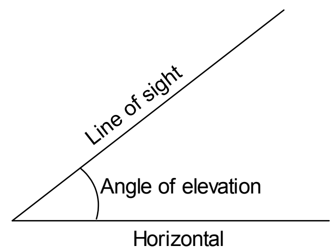
$$d = \frac{455}{Cos19°}$$

Step 4: Calculate the answer using your calculator.
4 5 5 ÷ cos 1 9 = 481.2174099
The missing side length is approx 481.2m

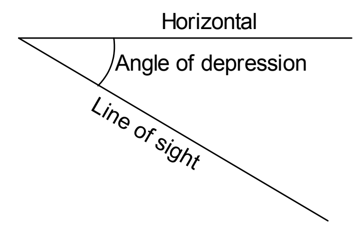
Step 5: Try to check the reasonableness
The missing side is the hypotenuse which is the longest side on the triangle. The answer of 481.2m is longer than the other given side (455m) so the answer could be correct.

Some problem solving questions refer to the **'Angle of Elevation'** or the **'Angle of Depression'**

The **angle of elevation** is the angle formed by a line of sight above the horizontal

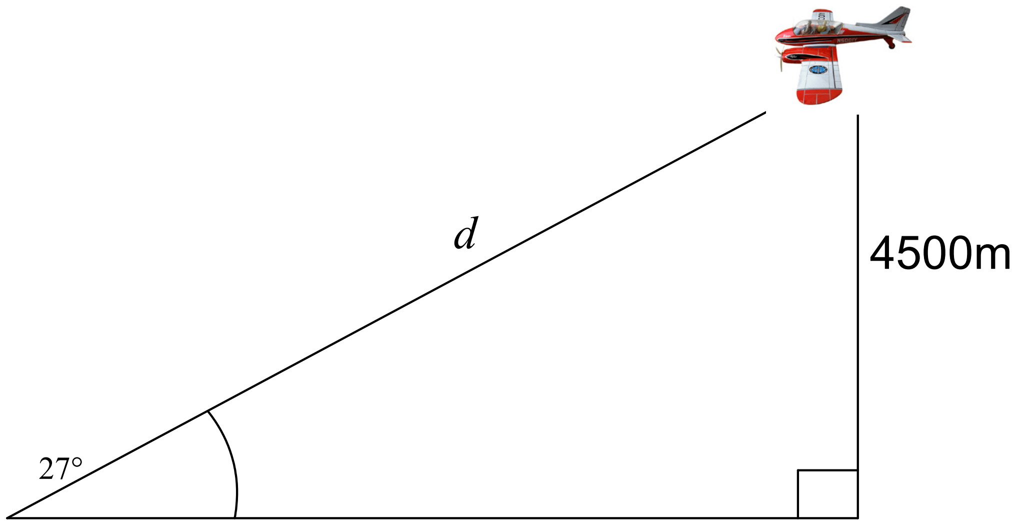


The **angle of depression** is the angle formed by a line of sight below the horizontal



The next problem is a simple problem solving question. The extra skill here is to read the information given and construct a diagram.

The angle of elevation of a plane at an altitude of 4500m is 27°to the horizontal. In a direct line, how far away is the plane.



Step 1: Determine which ratio to use.
In this triangle, one angle and one side are given (27°, 4500m). Based on the 27° angle, the 4550m side is the opposite and the *d* side is the hypotenuse.
The trig ratio to be used in this question is Sin.

tep 2: Write out the ratio and substitute in values.

$$Sin\theta = \frac{opp}{hyp}$$

$$Sin27° = \frac{4500}{d}$$

Step 3: Do the necessary rearranging.

$$d \times Sin27° = \frac{4500}{\cancel{d}} \times \cancel{d} \quad \bullet \text{multiply both sides by} d$$

$$\frac{d \times \cancel{Sin27°}}{\cancel{Sin27°}} = \frac{4500}{Sin27°} \quad \bullet \text{divide both sides by } Sin27°$$

$$d = \frac{4500}{Sin27°}$$

Step 4: Calculate the answer using your calculator.
[4] [5] [0] [0] [÷] [sin] [2] [7] [=] 9912.101691
The plane is 9912m away.

Step 5: Try to check the reasonableness
The missing side is the hypotenuse, which is the longest side on the triangle. The answer of 9912m is longer than the other given side (4500m) so the answer could be correct.

Video ‘Trigonometry Ratios – Finding Side Lengths’

The sections below may or may not be relevant to your studies. Activity questions are located at the end.

Angles less than 1°or containing a part angle

Angles with a part that is smaller than 1 degree can be expressed either in degrees as a decimal or in degrees, minutes and seconds. Angle measurement in degrees, minutes and seconds is just like time (hours, minutes and seconds).

60 seconds equalling 1 minute (60” = 1’)
60 minutes equalling one degree (60’ = 1°)

Finding the tan of 32.7° is [tan] [3] [2] [•] [7] [=], the display should read 0.64198859

Finding the sin of 67°42’ (67 degrees, 42 minutes) is:

On a Casio fx-82: [sin] [6] [7] [°’’’] [4] [2] [°’’’] [=]

The [°’’’] key represents degrees(°), minutes(′) and seconds(″)

Or

On a Sharp EL531: [sin] [6] [7] [D°M'S] [4] [2] [=]

The display should read 0.925209718

Finding the cos of 37°22’41” (37 degrees, 22 minutes, 41 seconds) is:

On a Casio fx-82: [cos] [3] [7] [°’’’] [2] [2] [°’’’] [4] [1] [°’’’]

Or

On a Sharp EL531: [cos] [3] [7] [D°M'S] [2] [2] [D°M'S] [4] [1]

The display should read 0.794647188

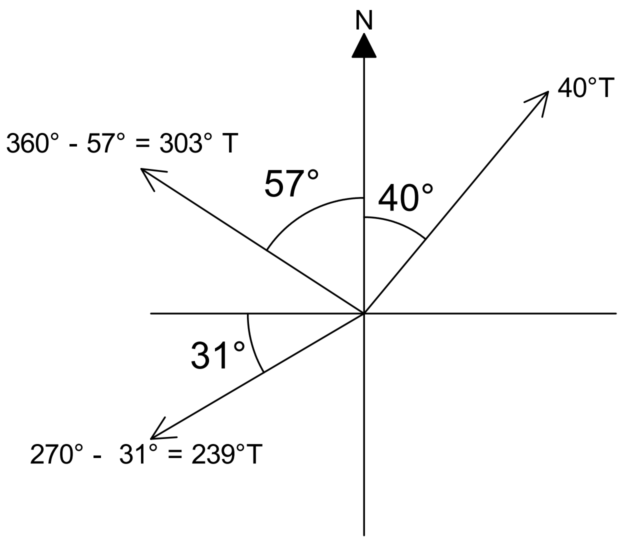
Use your calculator to find the following:

sin 14.5° (your calculator should display 0.250380004)
cos 37°14’ (your calculator should display 0.796178041)
tan 27.1° (your calculator should display 0.511725853)
sin 44°56’7” (your calculator should display 0.706307571)

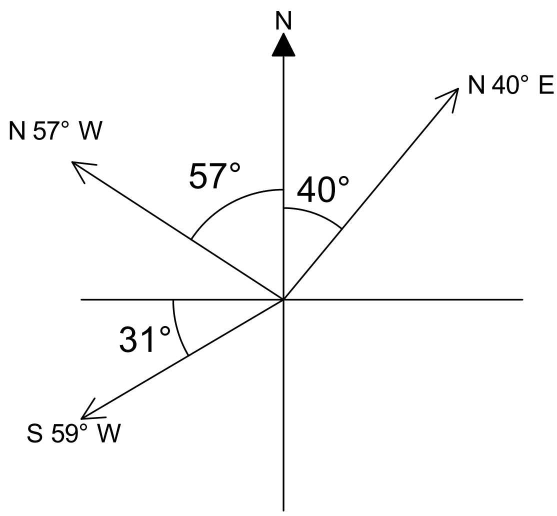
Compass and True Bearings

This problem involves compass directions. There are two methods for expressing directions.

The first method is a **True Bearing** where North is 0° and the angle increases in a clockwise direction, giving East as 90°, South as 180° and West as 270°.



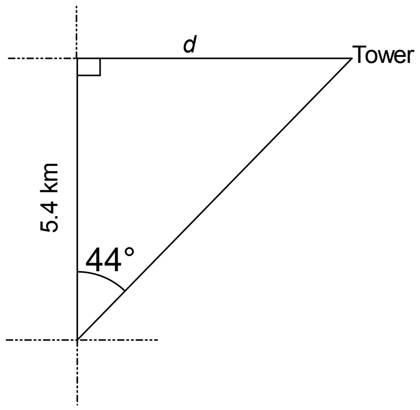
The second method is a **Compass Bearing** such as S40°W. The first compass direction stated is either N or S, followed by an angular measurement in an E or W direction.



 Video 'Bearings'

A group of bushwalkers walk 5.4 km north and then turn. They walk in an easterly direction until they reach a tower. The bearing of the tower from the original point is N44°E. Calculate the distance walked in the easterly direction by the walkers.

The diagram for the problem is drawn below.



1. Determine which ratio to use.
In this triangle, one angle and one side are given (44°, 5.4km). Based on the 44° angle, the 5.4km side is the adjacent and the *d* side is the opposite. The trig ratio to be used in this question is Tan.

2. Write out the ratio and substitute in values.

$$Tan\theta = \frac{opp}{adj}$$

$$Tan44° = \frac{d}{5.4}$$

3. Do the necessary rearranging.

$$5.4 \times Tan44° = \frac{d}{\cancel{5.4}} \times \cancel{5.4}$$

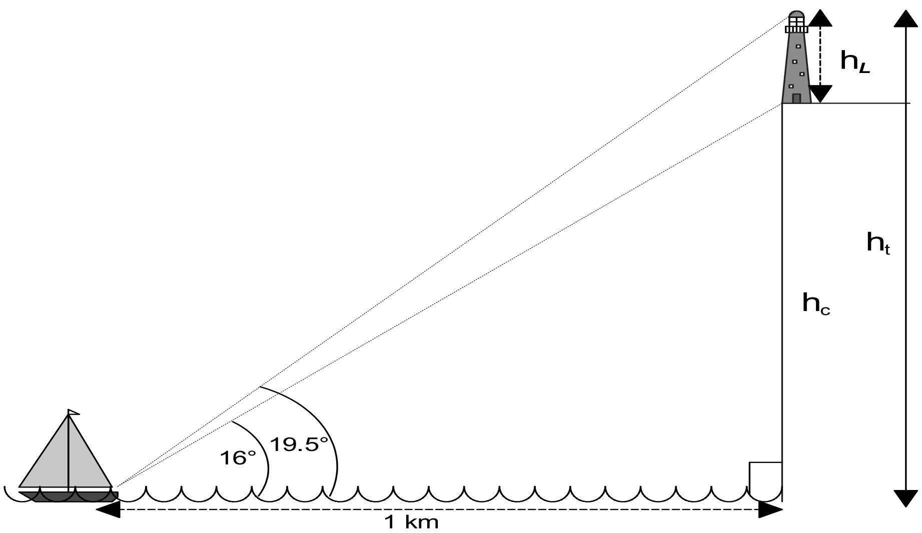
$$5.4 \times Tan44° = d$$

4. Calculate the answer using your calculator.
[5] [•] [4] [×] [tan] [4] [4] [=] 5.214719384
The tower is 5.2km to the east.

5. Try to check the reasonableness
The missing side should be about the same length as the given side.

The next example is a problem solving question that contains multiple steps. Care must be taken to read the question and accurately transfer this information into a diagram.

A ship is 1km out to sea from the base of a cliff. On top of the cliff is a lighthouse. From the ship, the angle of elevation to the base of the lighthouse is 16° and the angle of elevation to the top of the lighthouse is 19.5°. Calculate the height of the lighthouse. The diagram is;



Remember that only right angled triangles can be used to solve this question. The strategy to solve this problem is:

1. Calculate the height of the cliff
2. Calculate the total height (cliff + lighthouse)

3. Calculate the height of the lighthouse by subtracting the height of the cliff from the total height.

Calculating the height of the cliff: (1km = 1000m)

$$Tan16° = \frac{h_c}{1000}$$
$$1000 \times Tan16° = h_c$$
$$286.7 = h_c$$

Calculating the total height:

$$Tan19.5° = \frac{h_t}{1000}$$
$$1000 \times Tan19.5° = h_t$$
$$354.1 = h_t$$

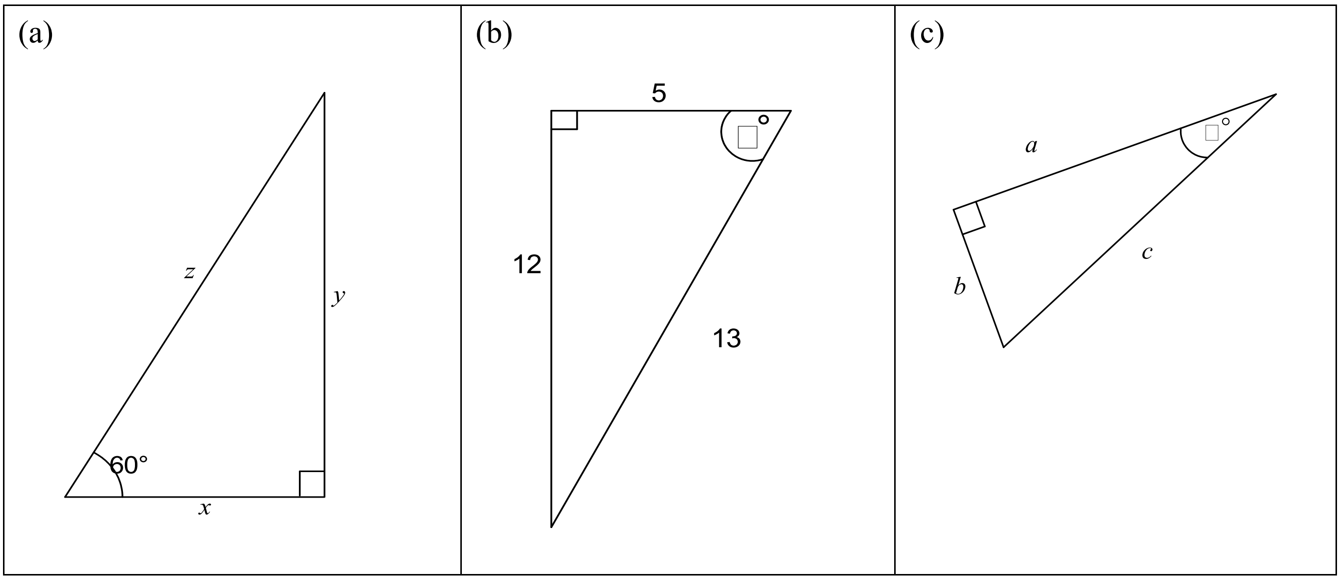
Calculating the height of the lighthouse:

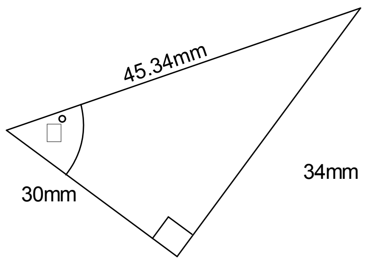
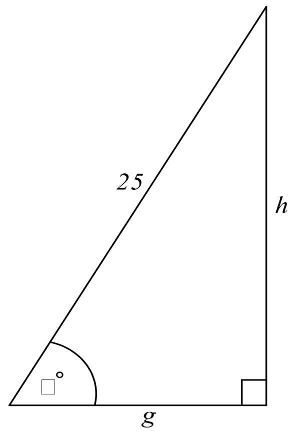
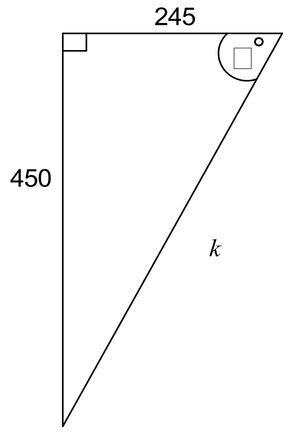
$$h_L = h_t - h_c$$
$$h_L = 354.1 - 286.7$$
$$h_L = 67.4m$$

The height of the lighthouse is 67.4 m. This is approximately equivalent to a 14 level building!

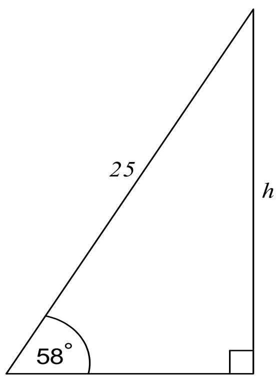
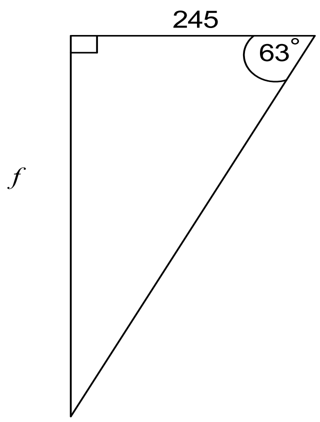
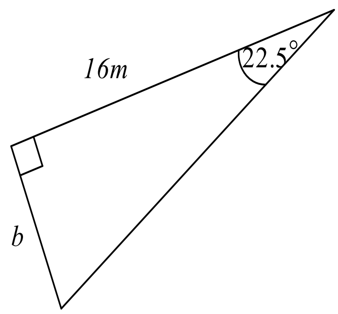
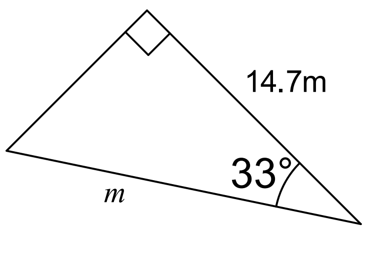
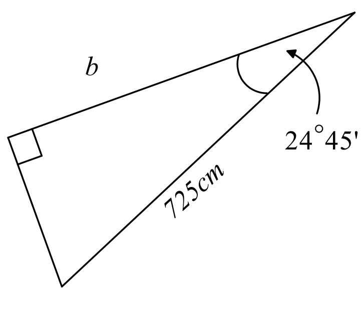
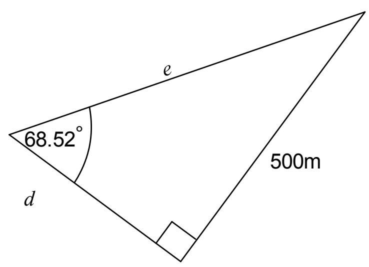
Activity

1. For the following triangles, identify the adjacent, opposite and hypotenuse for each triangle.

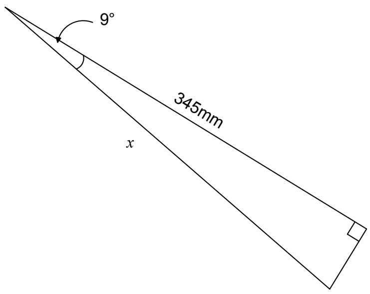


(d)	(e)	(f)
		

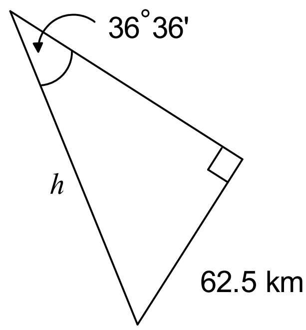
2. For the following triangles, find the value of the missing side(s).

(a)	(b)	(c)
		
(d)	**(e)**	**(f)**
		

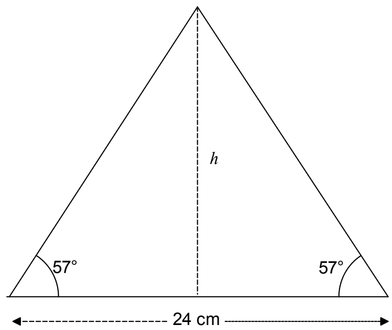
(g)



(h)



(i)



Find *h* and then the area of the triangle.

3. Express the following directions as a true bearing.

(a)	S15°E	(b)	NE	(c)	N45°W
(d)	10° W of N	(e)	SSW	(f)	215°

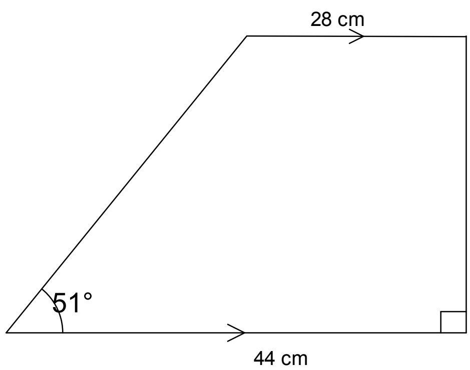
4. A person standing on top of a 25 m cliff sees a rower out to sea. The angle of depression of the boat is 15.3°. How far is the boat from the base of the cliff?

5. A student wishes to find the height of a building. From a distance of 50m on perfectly level ground, the angle of elevation to the top is 24.6°. Find the height of the building?

6. A plane is flying at altitude of 5000m. The pilot observes a boat at an angle of depression of 12°, calculate the horizontal distance which places the plane directly above the boat.

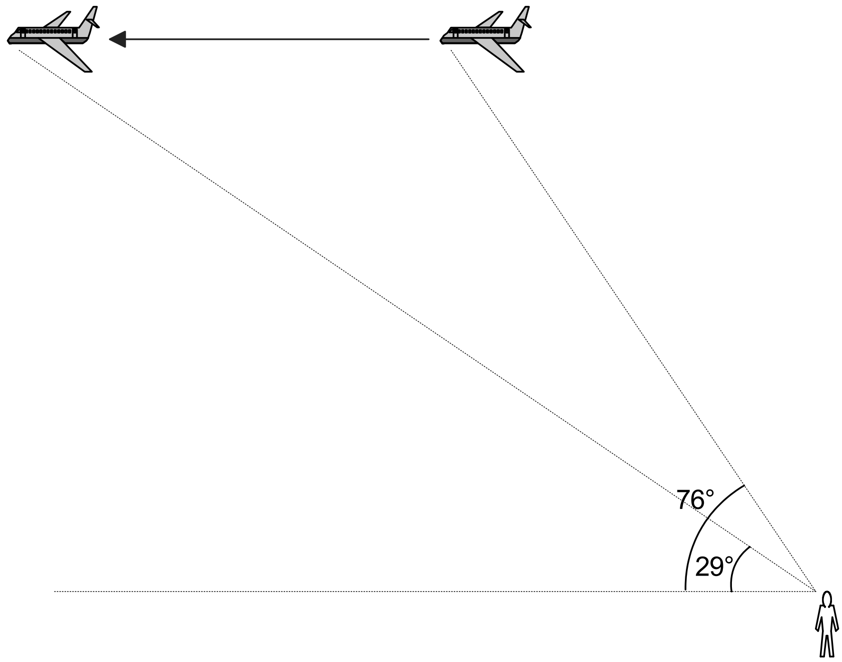
7. A walker decides to take a direct route to a landmark. They walk 1.7 km at a bearing of 78°T. How far did they walk in a northerly and easterly direction?

8. Find the perimeter of this trapezium.



9. A kite is attached to a 45m line. On a windy day, the kite flies at an angle of elevation of 28°. Calculate the height of the kite above the ground.

10. A plane flying at an altitude of 10 000m is flying away from a person. The angle of elevation of the plane is 76° when initially observed. After 1 minute 15 seconds, the plane is at an angle of elevation of 29°. Ignoring the height of the person, what is the speed of the plane in km/hr?



Topic 3: The Trigonometric Ratios – Finding Angles

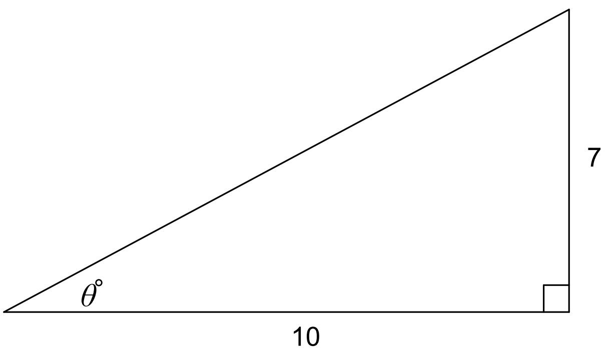
In the previous section, a trig ratio for a given angle was used to find a missing side. In this topic, a trig ratio from information given about sides is used to find the size of an angle.

Earlier in this module, it was discovered the Tan 45° = 1. So far the module has used this fact to find a missing side, either the opposite or the adjacent. In this topic, the thinking is the opposite way round. The value of the Tan ratio is 1(obtained from the lengths of the opposite and adjacent), so the question is 'The Tan of *what angle* is 1?'

In a right angled triangle, it is known that the value of $\frac{opposite}{hypotenuse}$ is equal to 0.5. It follows from this that the Sin of the unknown angle is equal to 0.5 or the sin of '*what angle*' is equal to 0.5?

The example below will help in understanding how to find the unknown angle.

A typical example is:



Step 1: Determine which ratio to use.
In this triangle, two sides are given (7,10). Based on the θ° angle, the 7 side is the opposite and the 10 side is the adjacent.

The trig ratio to be used in this question is Tan.

Step 2: Write out the ratio and substitute in values.

$$Tan\theta = \frac{opp}{adj}$$

$$Tan\theta = \frac{7}{10}$$

$$Tan\theta = 0.7$$

So far it is known that the Tan of θ is 0.7 or the Tan of *'what angle'* is 0.7? How is the angle θ found?
Because this is the opposite process to finding the Tan of a given angle, the ($\tan^{-1}$) key on your scientific calculator is used. The -1on this key means that this is the opposite process to finding the Tan of a given angle.

The next line of working is $\theta = Tan^{-1}0.7$

Step 3: Calculate the answer using your calculator.
On a scientific calculator, the ($\sin^{-1}$) ($\cos^{-1}$) ($\tan^{-1}$) keys are used by first pressing either the [SHIFT] or the [2ndF] first. The key used will depend upon the make and model of calculator used. For example; to get ($\tan^{-1}$) on a Casio fx-82, [SHIFT] [tan] keys are used.

On a Casio fx-82, the angle θ is found by pressing [SHIFT] [tan] [0] [•] [7] [=]
On a Sharp EL-531, the angle θ is found by pressing [2ndF] [tan] [0] [•] [7] [=]

The complete setting out is:

$$Tan\theta = \frac{opp}{adj}$$

$$Tan\theta = \frac{7}{10}$$

$$Tan\theta = 0.7$$

$$\theta = Tan^{-1}0.7$$

$$\theta = 34.992 \approx 35°$$

The angle θ is approximately 35°. Answers found on your calculator will need to be rounded as they are usually expressed to many decimal places. Rounding to one decimal is usually sufficient. Another alternative is to express the answer (angle) in degrees, minutes and seconds.

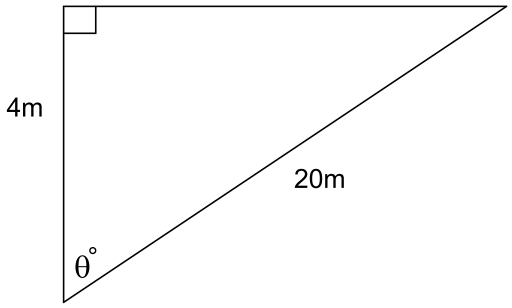
To obtain the answer in degrees, minutes and seconds (° ′ ″);

On a Casio fx-100, pressing [°′″] gives 34° 59′ 31.27″ If [°′″] is pressed again it changes back to the decimal form.

On a Sharp EL-531, pressing [2ndF][D°M'S] gives 34° 59′ 31.27″ If [2ndF][D°M'S] is pressed again it changes back to the decimal form.

Step 4: Try to check the reasonableness
Because smaller sides are opposite smaller angles, the angle opposite the 7 side should be less than 45°. The answer could be correct.

The next example is similar except it uses a different ratio.



Step 1: Determine which ratio to use.
In this triangle, two sides are given (4m, 20m). Based on the θ° angle, the 4m side is the adjacent and the 10 side is the hypotenuse.

The trig ratio to be used in this question is Cos.

Step 2: Write out the ratio and substitute in values.

$$Cos\theta = \frac{adj}{hyp}$$

$$Cos\theta = \frac{4}{20}$$

$$Cos\theta = 0.2$$

$$\theta = Cos^{-1}0.2$$

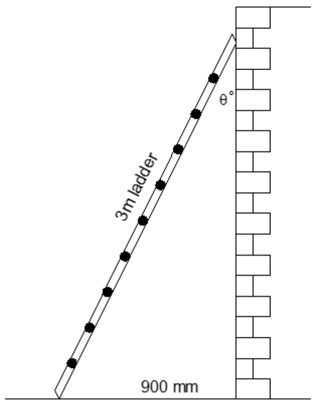
Step 3: Calculate the answer using your calculator.
On a Casio fx-82, the angle θ is found by pressing [SHIFT] [COS] [0] [•] [2] [=] 78.463
On a Sharp EL-531, the angle θ is found by pressing [2ndF] [COS] [0] [•] [2] [=] 78.463

Step 4: Try to check the reasonableness
In this question it is hard to check the reasonableness. The side opposite the angle calculated would be much larger than 4m (could be confirmed by using Pythagoras' Theorem) so the angle would be greater than 45°.

The next example is a problem solving question.

A three metre ladder is placed against a brick wall. The base of the ladder is 900mm from the base of the wall. Find the angle the ladder makes with the wall.

Before starting the process of solving this question, a diagram is required. Read the information carefully. In this question, it would easy to indicate the wrong angle.



Step 1: Determine which ratio to use.
Change 900mm to 0.9m

In this triangle, two sides are given (0.9m, 3m). Based on the $\theta°$ angle, the 0.9m side is the opposite and the 3 side is the hypotenuse.

The trig ratio to be used in this question is Sin.

Step 2: Write out the ratio and substitute in values.

$$Sin\theta = \frac{opp}{hyp}$$

$$Sin\theta = \frac{0.9}{3}$$

$$Sin\theta = 0.3$$

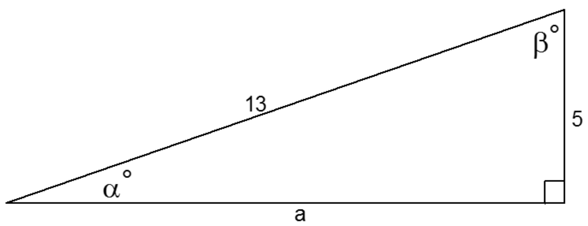
$$\theta = Sin^{-1}0.3$$

Step 3: Calculate the answer using your calculator.
On a Casio fx-82, the angle θ is found by pressing SHIFT sin 0 • 3 = 17.46
On a Sharp EL-531, the angle θ is found by pressing 2ndF sin 0 • 3 = 17.46
The angle between the ladder and the wall is 17.5°

Step 4: Try to check the reasonableness
In this question it is hard to check the reasonableness. This angle is opposite a small side so the angle should be smaller than 45°.

In the next question the instruction is to ‘Complete this Triangle’. This means that all missing angles and sides must be calculated.

Example: Complete this triangle.



As two sides are given, the third side can be found using Pythagoras’ Theorem.

$$a^2 + b^2 = c^2$$
$$a^2 + 5^2 = 13^2$$
$$a^2 + 25 = 169$$
$$a^2 + 25 - 25 = 169 - 25$$
$$a^2 = 144$$
$$a = \sqrt{144} = 12$$

The next step is to find one of the two missing angles. The angle α is found using a Trig ratio. It is always good to use the original sides' measurements given just in case the calculated side is inaccurate.

As the given sides are the opposite and the hypotenuse, the Sin ratio is used.

$$Sin\alpha = \frac{opp}{hyp}$$
$$Sin\alpha = \frac{5}{13}$$
$$Sin\alpha = 0.384615384$$
$$\alpha = Sin^{-1}0.384615384$$
$$\alpha = 22.6° \text{ (to 1 d.p.)}$$

Using the angle properties of a right angled triangle;

$$\angle\alpha + \angle\beta = 90°$$
$$\angle\alpha = 90° - 22.6°$$
$$\angle\alpha = 67.4°$$

In this right angled triangle, the side measurements are 5, 12, 13 and the angles are 22.6° and 67.4°.

The table below summaries how to find all missing sides and angles.

Given	Do
2 Sides	Use Pythagoras' Theorem to calculate the third side. Use a Trig ratio to find one of the angles. Then use your knowledge of right angle triangles to find the other angle.
1 Side and 1 angle	Use a Trig Ratio to calculate one missing side. Then, either Trig or Pythagoras can be used to find the other missing side. Then use your knowledge of right angle triangles to find the other angle.

Video 'Trigonometry Ratios – Finding Angles'

Activity

1. Find the size of the angle in the questions below to 2 d.p.

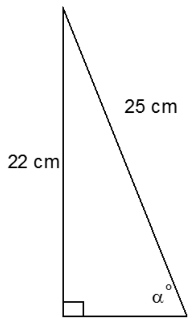
(a)	Sin $\theta = 0.2345$	(b)	Cos $\theta = 0.5736$	(c)	Tan $\theta = 2.4604$
(d)	Cos $\beta = 0.63$	(e)	Tan $\mu = 0.4998$	(f)	Sin $\sigma = 0.9455$

2. Find the size of the angle in the questions below in degrees, minutes and seconds.

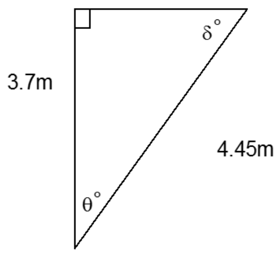
(a)	Sin $\theta = 0.2345$	(b)	Cos $\theta = 0.5736$	(c)	Tan $\theta = 2.4604$
(d)	Cos $\beta = 0.63$	(e)	Tan $\mu = 0.4998$	(f)	Sin $\sigma = 0.9455$

3. For the following triangles, find the size of the missing angle.

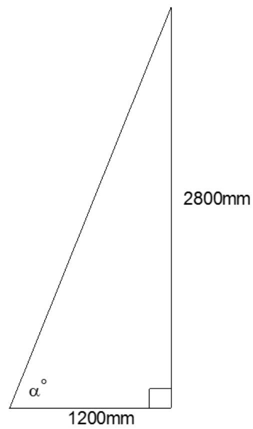
(a)



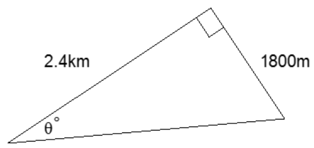
(b)



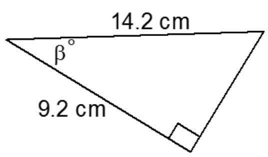
(c)



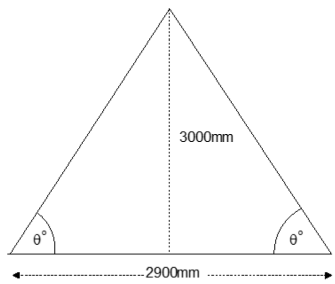
(d)



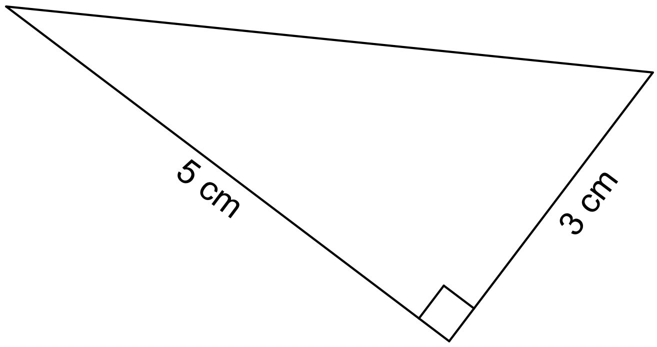
(e)



(f)



4. Complete the triangle below:



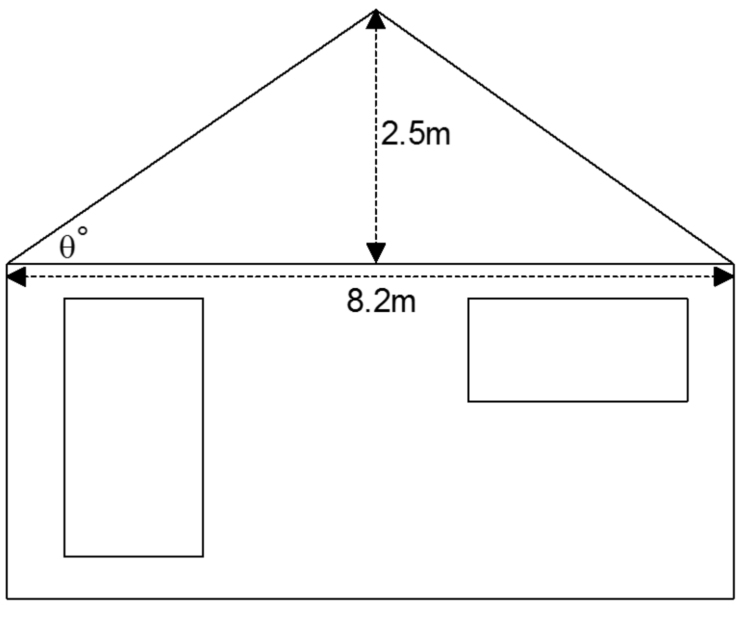
5. A ship sails 400 km north and then 800 km west. How far is it away from the port it started from and at what true bearing?

6. It is recommended that ramps for wheelchairs have a rise to run ratio of 1:15. What angle would the ramp surface make to the horizontal if a ramp follows this specification?

7. Two flag poles are in a park on a level surface 30m apart. One pole is 10m tall and the other is 19m tall. What is the angle of elevation of the top of the tall pole from the top of the smaller pole?

8. A 10m antenna is supported by guide wires on four sides. Each wire is attached to the ground 6 metres from the base of the antenna.
   (i) Find the length of each wire
   (ii) What angle does the wire make with the ground?

9. Calculate the pitch of this roof.

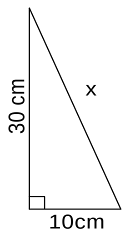


10. In $\Delta ABC: \angle A = 90°, a = 16.9, b = 6.5, calculate \angle B$.

Answers to activity questions

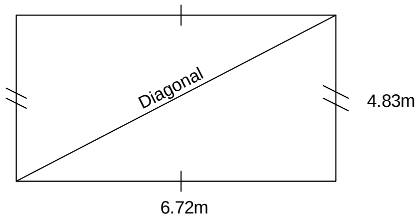
Check your skills

1. (a) Use Pythagoras' Theorem to calculate *x*.



$$h^2 = a^2 + b^2$$
$$x^2 = 30^2 + 10^2$$
$$x^2 = 900 + 100$$
$$x^2 = 1000$$
$$x = \sqrt{1000}$$
$$x = 31.6cm$$

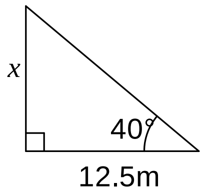
(b) A rectangle has a length of 6.72m and width of 4.83m. Find the length of the diagonal (to 2 d.p.).



Let the length of the diagonal be *d*.

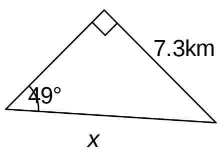
$$h^2 = a^2 + b^2$$
$$d^2 = 6.72^2 + 4.83^2$$
$$d^2 = 45.1584 + 23.3289$$
$$d^2 = 68.4873$$
$$d = \sqrt{68.4873}$$
$$d = 8.28cm$$

2. (a) Calculate the value of *x* in this triangle.



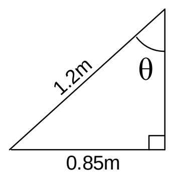
$$Tan\theta° = \frac{opp}{adj}$$
$$Tan40° = \frac{x}{12.5}$$
$$12.5 \times Tan40° = x$$
$$x = 10.49 \text{ (to 2 d.p.)}$$

(b) Calculate the value of *x* in this triangle.



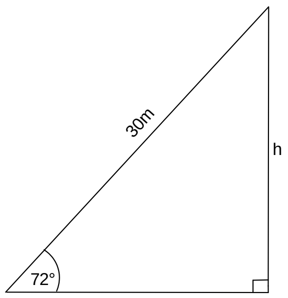
$$Sin\theta° = \frac{opp}{hyp}$$
$$Sin49° = \frac{7.3}{x}$$
$$x \times Sin49° = 7.3$$
$$x = \frac{7.3}{Sin49°}$$
$$x = 9.67km$$

(c) Calculate the value of θ in this triangle.



$$Sin\theta° = \frac{opp}{hyp}$$
$$Sin\theta° = \frac{0.85}{1.2}$$
$$Sin\theta° = 0.7083$$
$$\theta = Sin^{-1}0.7083$$
$$\theta = 45.10° \text{ or } 45°5'48"$$

3. A kite on the end of a 30m string is flying at an angle of elevation of 72°. What is the height of the kite directly above the ground?

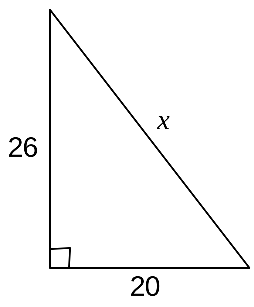


$$Sin\theta° = \frac{opp}{hyp}$$
$$Sin72° = \frac{h}{30}$$
$$30 \times Sin72° = h$$
$$x = 28.53m$$

Pythagoras' Theorem

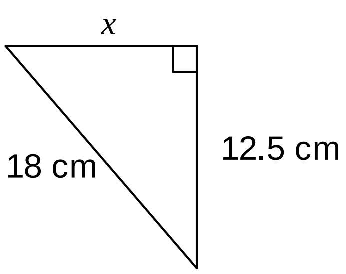
1. Use Pythagoras' Theorem to find the missing length.

(a)



$$h^2 = a^2 + b^2$$
$$x^2 = 26^2 + 20^2$$
$$x^2 = 676 + 400$$
$$x^2 = 1076$$
$$x = \sqrt{1076}$$
$$x = 32.8$$

(b)



$$h^2 = a^2 + b^2$$
$$18^2 = x^2 + 12.5^2$$
$$324 = x^2 + 156.25$$
$$324 - 156.25 = x^2$$
$$x = \sqrt{167.75}$$
$$x = 12.95cm$$

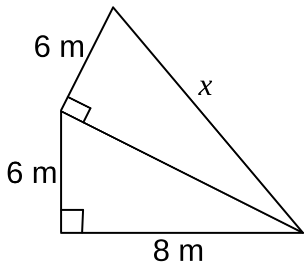
(c) Find the hypotenuse when *a=6.1* and *b=3.4*

$$h^2 = a^2 + b^2$$
$$x^2 = 6.1^2 + 3.4^2$$
$$x^2 = 37.21 + 11.56$$
$$x^2 = 48.77$$
$$x = \sqrt{48.77}$$
$$x = 6.98$$

(d) Find the missing side given *a=23.5* and *h=40*

$$h^2 = a^2 + b^2$$
$$40^2 = 23.5^2 + x^2$$
$$1600 = 552.25 + x^2$$
$$1600 - 552.25 = x^2$$
$$x = \sqrt{1047.75}$$
$$x = 32.37cm$$

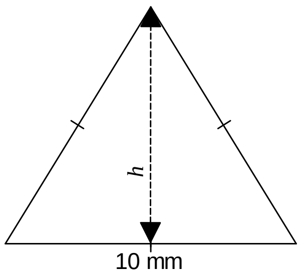
(e)



Let the length of the linking side be *l*.

$$h^2 = a^2 + b^2 \qquad h^2 = a^2 + b^2$$
$$l^2 = 6^2 + 8^2 \qquad x^2 = l^2 + 6^2$$
$$l^2 = 36 + 64 \qquad x^2 = 10^2 + 6^2$$
$$l^2 = 100 \qquad x^2 = 136$$
$$l = \sqrt{100} \qquad x = \sqrt{136}$$
$$l = 10m \qquad x = 11.66m$$

(f)



This is an equilateral triangle. Half the triangle to obtain a right angled triangle.

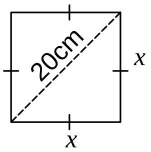
$$h^2 = a^2 + b^2$$
$$10^2 = h^2 + 5^2$$
$$100 = h^2 + 25$$
$$100 - 25 = x^2$$
$$x = \sqrt{75}$$
$$x = 8.66mm$$

(g) A orienteering participant runs 650m north an then turns and runs 1.4 km east. How far from the starting point is the runner?

650m = 0.65km

$$h^2 = a^2 + b^2$$
$$x^2 = 0.65^2 + 1.4^2$$
$$x^2 = 0.4225^2 + 1.96^2$$
$$x^2 = 2.3825$$
$$x = \sqrt{2.3825}$$
$$x = 1.54km$$

(h) **A Real Challenge**
A square has a diagonal of 20 cm, what is the side length?



$$h^2 = a^2 + b^2$$
$$20^2 = x^2 + x^2$$
$$400 = 2x^2$$
$$x^2 = \frac{400}{2}$$
$$x = \sqrt{200}$$
$$x = 14.14cm$$

2. Do the following triangles contain a right angle?

(a) 10, 24, 26 (b) 7, 8, 10 (c) 2, 4.8, 5.2

Remember the hypotenuse is the longest side.

$a^2 + b^2 = 10^2 + 24^2$

$a^2 + b^2 = 100 + 576$

$a^2 + b^2 = 676$

$h^2 = 26^2$

$h^2 = 676$

As $a^2 + b^2 = h^2$ the triangle must contain a right angle.

$a^2 + b^2 = 7^2 + 8^2$

$a^2 + b^2 = 49 + 64$

$a^2 + b^2 = 113$

$h^2 = 10^2$

$h^2 = 100$

As $a^2 + b^2 \neq h^2$ the triangle does not contain a right angle.

$a^2 + b^2 = 2^2 + 4.8^2$

$a^2 + b^2 = 4 + 23.04$

$a^2 + b^2 = 27.04$

$h^2 = 5.2^2$

$h^2 = 27.04$

As $a^2 + b^2 = h^2$ the triangle must contain a right angle.

(d) 1.4, 4.8, 5

$a^2 + b^2 = 1.4^2 + 4.8^2$

$a^2 + b^2 = 1.96 + 23.04$

$a^2 + b^2 = 25$

$h^2 = 5^2$

$h^2 = 25$

As $a^2 + b^2 = h^2$ the triangle does contain a right angle.

(e) 6, 6, 8

$a^2 + b^2 = 6^2 + 6^2$

$a^2 + b^2 = 36 + 36$

$a^2 + b^2 = 72$

$h^2 = 8^2$

$h^2 = 64$

As $a^2 + b^2 \neq h^2$ the triangle does not contain a right angle.

(f) 5, 6, 7

$a^2 + b^2 = 5^2 + 6^2$

$a^2 + b^2 = 25 + 36$

$a^2 + b^2 = 61$

$h^2 = 7^2$

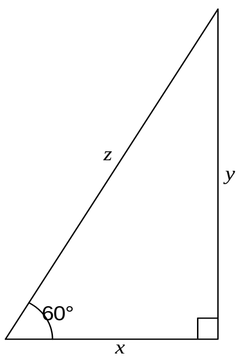
$h^2 = 49$

As $a^2 + b^2 \neq h^2$ the triangle does not contain a right angle.

The Trigonometric Ratios – Finding Sides

1. For the following triangles, identify the adjacent, opposite and hypotenuse for each triangle.

(a)

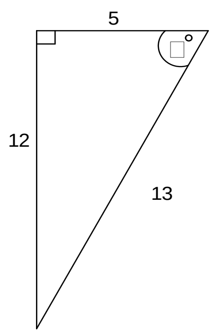


x – adjacent

y - opposite

z - hypotenuse

(b)

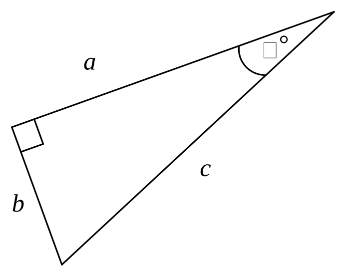


5 – adjacent

12 – opposite

13 – hypotenuse

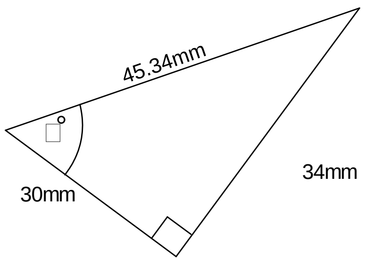
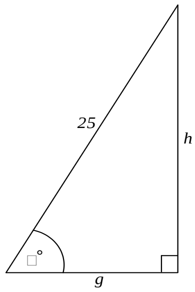
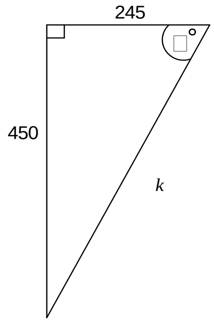
(c)



a – adjacent

b – opposite

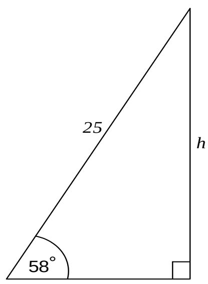
c – hypotenuse

(d)	(e)	(f)
 30mm – adjacent 34mm – opposite 45.34mm – hypotenuse	 g – adjacent h – opposite 25 – hypotenuse	 245 – adjacent 450 – opposite k – hypotenuse

2. For the following triangles, find the value of the missing side(s).

<table>
<tr>
<td>

(a)



$Sin\theta° = \frac{opp}{hyp}$

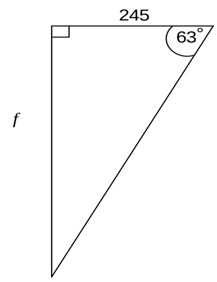
$Sin58° = \frac{h}{25}$

$25 \times Sin58° = h$

$x = 21.2$

</td>
<td>

(b)



$Tan\theta° = \frac{opp}{adj}$

$Tan63° = \frac{f}{245}$

$245 \times Tan63° = f$

$f = 480.8$

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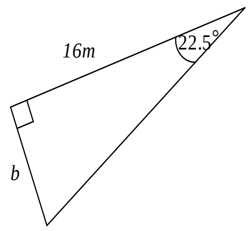
(c)



$Tan\theta° = \frac{opp}{adj}$

$Tan22.5° = \frac{b}{16}$

$16 \times Tan22.5° = b$

$b = 6.63m$

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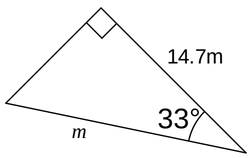
(d)



$Cos\theta° = \frac{adj}{hyp}$

$Cos33° = \frac{14.7}{m}$

$m \times Cos33° = 14.7$

$m = \frac{14.7}{Cos33°}$

$m = 17.5m$

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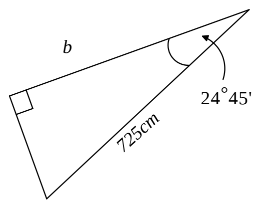
(e)



$Cos\theta° = \frac{adj}{hyp}$

$Cos24°45' = \frac{b}{725}$

$725 \times Cos24°45' = b$

$b = 658.4cm$

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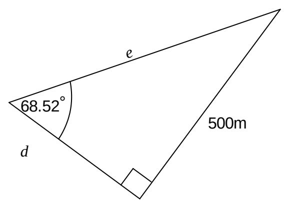
(f)



$Sin\theta° = \frac{opp}{hyp}$

$Sin68.52° = \frac{500}{e}$

$e \times Sin68.52° = 500$

$e = \frac{500}{Sin68.52°}$

$e = 537.3m$

Using Pythagoras

$e^2 = 500^2 + d^2$

$537.3^2 = 500^2 + d^2$

$288691.3 - 250000 = d^2$

$d^2 = 38691.3$

$d = \sqrt{38691.3}$

$d = 196.7$

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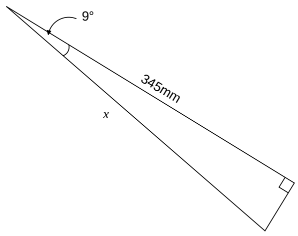
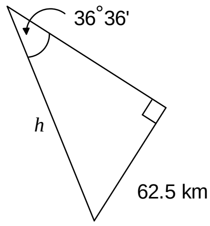
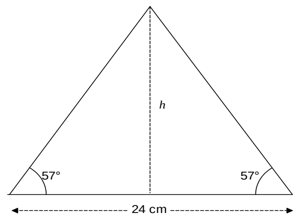
<tr>
<td>(g)</td>
<td>(h)</td>
<td>(i)</td>
</tr>
</table>



$$Cos\theta° = \frac{adj}{hyp}$$
$$Cos9° = \frac{345}{x}$$
$$x \times Cos9° = 345$$
$$x = \frac{345}{Cos9°}$$
$$x = 349.3mm$$



$$Sin\theta° = \frac{opp}{hyp}$$
$$Sin36°36' = \frac{62.5}{h}$$
$$h \times Sin36°36' = 62.5$$
$$h = \frac{62.5}{Sin36°36'}$$
$$h = 104.8km$$



Half the shape is a right angled triangle.

$$Tan\theta° = \frac{opp}{adj}$$
$$Tan57° = \frac{h}{12}$$
$$12 \times Tan57° = h$$
$$h = 18.48cm$$

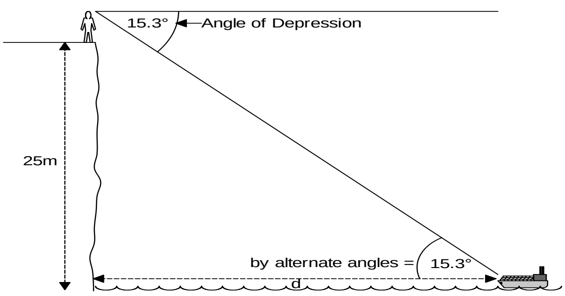
Area:

$$A = \frac{1}{2}bh$$
$$A = 0.5 \times 24 \times 18.48$$
$$A = 221.76cm^2$$

3. Express the following directions as a true bearing.

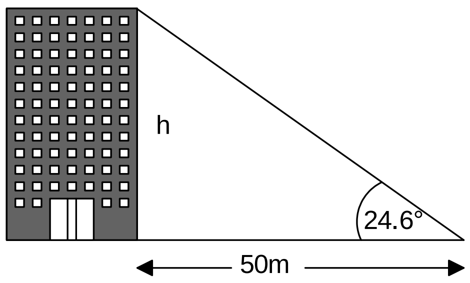
(a)	S15°E = 165°T	(b)	NE = 45°T	(c)	N45°W = 315°T
(d)	10° W of N = 350°T	(e)	SSW = 202.5°	(f)	215° = 215°T

4. A person standing on top of a 25 m cliff sees a rower out to sea. The angle of depression of the boat is 15.3°. How far is the boat from the base of the cliff?



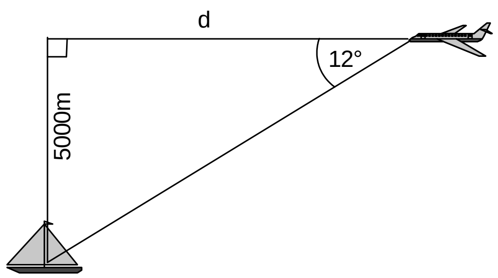
$$Tan\theta° = \frac{opp}{adj}$$
$$Tan15.3° = \frac{25}{d}$$
$$d \times Tan15.3° = 25$$
$$d = \frac{25}{Tan15.3°}$$
$$d = 91.4m$$

5. A student wishes to find the height of a building. From a distance of 50m on perfectly level ground, the angle of elevation to the top is 24.6°. Find the height of the building?



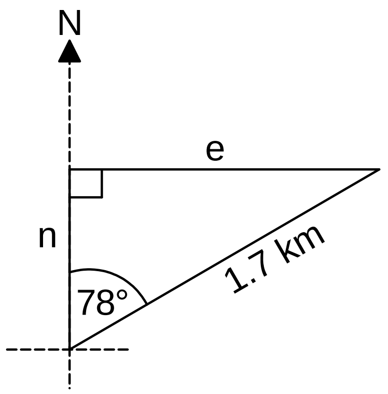
$$Tan\theta° = \frac{opp}{adj}$$
$$Tan24.6° = \frac{h}{50}$$
$$50 \times Tan24.6° = h$$
$$h = 22.9m$$

6. A plane is flying at altitude of 5000m. The pilot observes a boat at an angle of depression of 12°, calculate the horizontal distance which places the plane directly above the boat.



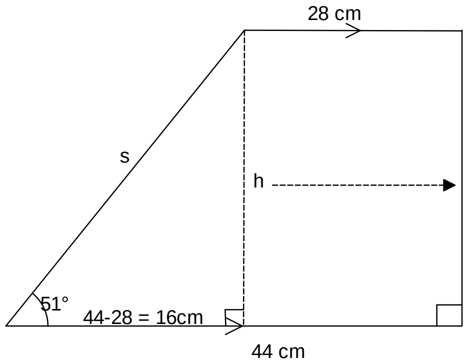
$$Tan\theta° = \frac{opp}{adj}$$
$$Tan12° = \frac{5000}{d}$$
$$d \times Tan12° = 5000$$
$$d = \frac{5000}{Tan12°}$$
$$d = 23523m \text{ or } 23.5km$$

7. A walker decides to take a direct route to a landmark. They walk 1.7 km at a bearing of 78°T. How far did they walk in a northerly and easterly direction?



$$Cos\theta° = \frac{adj}{hyp}$$
$$Cos78° = \frac{n}{1.7}$$
$$1.7 \times Cos78° = n$$
$$n = 0.353km \text{ or } 353m$$
$$Sin\theta° = \frac{opp}{hyp}$$
$$Sin78° = \frac{e}{1.7}$$
$$1.7 \times Sin78° = e$$
$$e = 1.663km$$

8. Find the perimeter of this trapezium.



$$Tan\theta° = \frac{opp}{adj}$$
$$Tan51° = \frac{h}{16}$$
$$16 \times Tan51° = h$$
$$h = 19.8cm$$

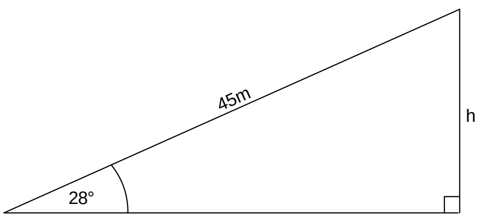
$$Cos\theta° = \frac{adj}{hyp}$$
$$Cos51° = \frac{16}{s}$$
$$s \times Cos51° = 16$$
$$s = \frac{16}{Cos51°}$$
$$s = 25.4cm$$

Perimeter $= 44 + \text{h} + 28 + \text{s}$

$= 44 + 19.8 + 28 + 25.4$

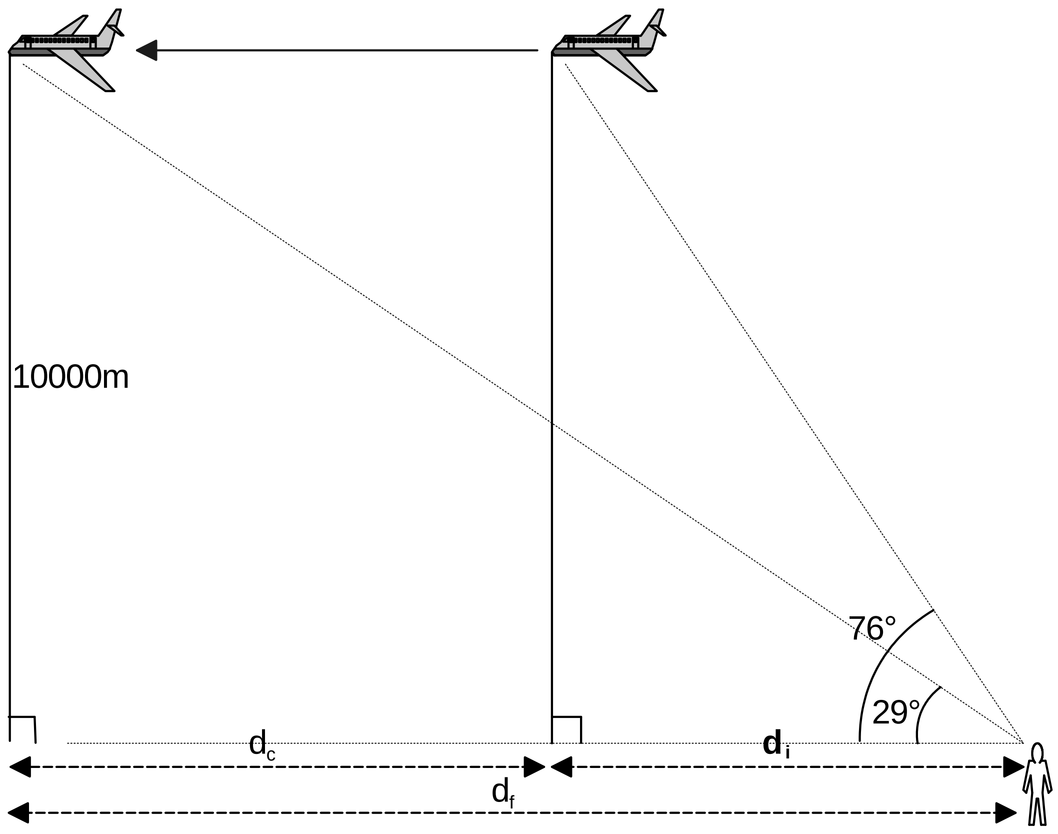
$= 117.2cm$ or 1.172m

9. A kite is attached to a 45m line. On a windy day, the kite flies at an angle of elevation of 28°. Calculate the height of the kite above the ground.



$$Sin\theta° = \frac{opp}{hyp}$$
$$Sin28° = \frac{h}{45}$$
$$45 \times Sin28° = h$$
$$h = 21.13m$$

10. A plane flying at an altitude of 10 000m is flying away from a person. The angle of elevation of the plane is 76° when initially observed. After 1 minute 15 seconds, the plane is at an angle of elevation of 29°. Ignoring the height of the person, what is the speed of the plane in km/hr?



Initially, the plane is d_i away from the observer.
After 1minute 15 seconds, the plane is d_f away from the observer.
The distance covered in 1 minute 15 seconds is $d_f - d_i$.

$Tan\theta° = \frac{opp}{adj}$ $Tan76° = \frac{10000}{d_i}$ $d_i \times Tan76° = 10000$ $d_i = \frac{10000}{Tan76°}$ $d_i = 2493m$	$Tan\theta° = \frac{opp}{adj}$ $Tan29° = \frac{10000}{d_f}$ $d_f \times Tan29° = 10000$ $d_f = \frac{10000}{Tan29°}$ $d_f = 18040m$	$d_f - d_i$ $= 18040 - 2493$ $= 15547m$ or $15.547km$

1 minute 15 seconds is 1.25 minutes. 1.25 minutes ÷ 60 =0.0208333 hours.

$$speed(km/hr) = \frac{\text{distance covered (km)}}{\text{time taken(hr)}}$$
$$= \frac{15.547km}{0.0208333hr}$$
$$= 746km/hr \text{ (rounded)}$$

The Trigonometric Ratios – Finding Angles

1. Find the size of the angle in the questions below to 2 d.p.

(a)	Sin θ = 0.2345	(b)	Cos θ = 0.5736	(c)	Tan θ = 2.4604

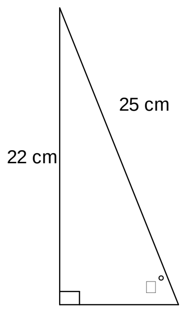
	θ = 13.56°		θ = 55°		θ = 67.88°
(d)	Cos β = 0.63 β = 50.95°	(e)	Tan μ = 0.4998 μ = 26.56°	(f)	Sin σ = 0.9455 σ = 71°

2. Find the size of the angle in the questions below in degrees, minutes and seconds.

(a)	Sin θ = 0.2345 θ = 13°33'44"	(b)	Cos θ = 0.5736 θ = 54°59'54"	(c)	Tan θ = 2.4604 θ = 67°52'53"
(d)	Cos β = 0.63 β = 50°57'	(e)	Tan μ = 0.4998 μ = 26°33'21"	(f)	Sin σ = 0.9455 σ = 70°59'48"

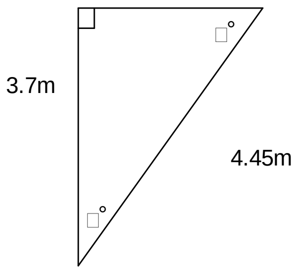
3. For the following triangles, find the size of the missing angle.

(a)



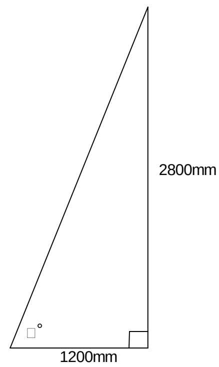
$$Sin\alpha° = \frac{opp}{hyp}$$
$$Sin\alpha° = \frac{22}{25}$$
$$Sin\alpha° = 0.88$$
$$\alpha° = 61.64° \text{ or } 61°38'33''$$

(b)



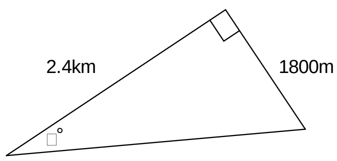
$$Cos\theta° = \frac{adj}{hyp}$$
$$Cos\theta° = \frac{3.7}{4.45}$$
$$Cos\theta° = 0.8315$$
$$\theta = 33.75° \text{ or } 33°45'3''$$

(c)



$$Tan\theta° = \frac{opp}{adj}$$
$$Tan\theta° = \frac{2800}{1200}$$
$$Tan\theta° = 2.33\overline{3}$$
$$\theta° = 66.8° \text{ or } 66°48'5''$$

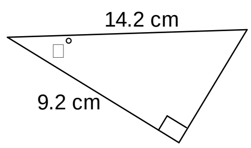
(d)



2.4km = 2400 m

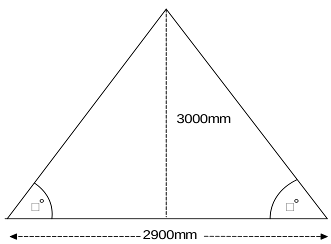
$$Tan\theta° = \frac{opp}{adj}$$
$$Tan\theta° = \frac{1800}{2400}$$
$$Tan\theta° = 0.75$$
$$\theta° = 36.9° \text{ or } 36°52'12''$$

(e)



$$Cos\beta° = \frac{adj}{hyp}$$
$$Cos\beta° = \frac{9.2}{14.2}$$
$$Cos\beta° = 0.6479$$
$$\beta = 49.62° \text{ or } 49°37'3''$$

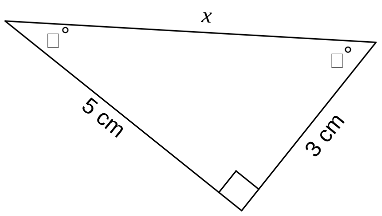
(f)



This is an isosceles triangle.
Half the base is 1450mm.

$$Tan\theta° = \frac{opp}{adj}$$
$$Tan\theta° = \frac{3000}{1450}$$
$$Tan\theta° = 2.0690$$
$$\theta° = 64.2° \text{ or } 64°12'14''$$

4. Complete the triangle below:



Calculate x using Pythagoras.

$$h^2 = a^2 + b^2$$
$$x^2 = 5^2 + 3^2$$
$$x^2 = 25 + 9$$
$$x^2 = 34$$
$$x = \sqrt{34}$$
$$x = 5.83$$

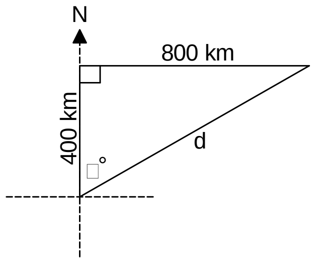
Calculating α

$$Tan\alpha° = \frac{opp}{adj}$$
$$Tan\alpha° = \frac{3}{5}$$
$$Tan\alpha° = 0.6$$
$$\alpha° = 30.96° \text{ or } 30°57'50''$$

Calculating β

$$\beta = 90 - \alpha$$
$$\beta = 90 - 30.96°$$
$$\beta = 59.04°$$
$$or$$
$$\beta = 90 - 30°57'50''$$
$$\beta = 59°2'10''$$

5. A ship sails 400 km north and then 800 km west. How far is it away from the port it started from and at what true bearing?



Find d using Pythagoras.

$$h^2 = a^2 + b^2$$
$$d^2 = 400^2 + 800^2$$
$$d^2 = 160000 + 640000$$
$$d^2 = 800000$$
$$d = \sqrt{800000}$$
$$d = 894.4km$$

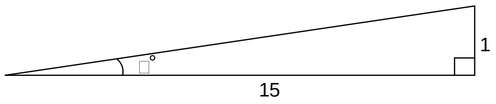
Finding the true bearing

$$Tan\theta° = \frac{opp}{adj}$$
$$Tan\theta° = \frac{800}{400}$$
$$Tan\theta° = 2$$
$$\theta° = 63.4° \text{ or } 63°26'6''$$

The ship is located 894km away at a true bearing of 63.4°.

6. It is recommended that ramps for wheelchairs have a rise to run ratio of 1:15. What angle would the ramp surface make to the horizontal if a ramp follows this specification?

Units are not required to answer this question.



The surface of the ramp should make an angle of about 4° to the horizontal to be wheelchair friendly.

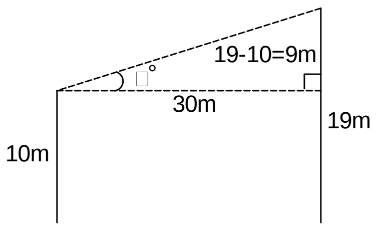
$$Tan\theta° = \frac{opp}{adj}$$

$$Tan\theta° = \frac{1}{15}$$

$$Tan\theta° = 0.066\overline{6}$$

$$\theta° = 3.8° \text{ or } 3°48'51''$$

7. Two flag poles are in a park on a level surface 30m apart. One pole is 10m tall and the other is 19m tall. What is the angle of elevation of the top of the tall pole from the top of the smaller pole?



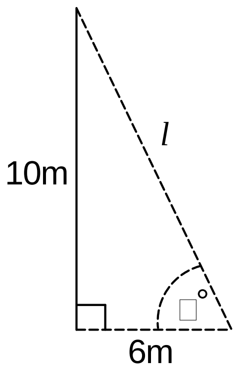
$$Tan\theta° = \frac{opp}{adj}$$

$$Tan\theta° = \frac{9}{30}$$

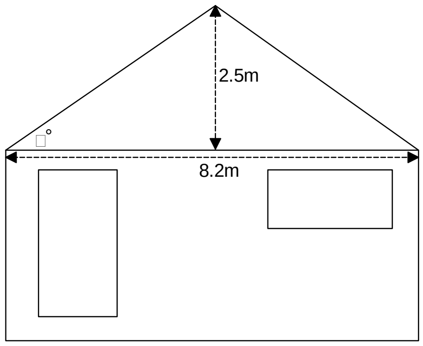
$$Tan\theta° = 0.3$$

$$\theta° = 16.7° \text{ or } 16°41'57''$$

8. A 10m antenna is supported by guide wires on four sides. Each wire is attached to the ground 6 metres from the base of the antenna.

	Find the length of each wire $h^2 = a^2 + b^2$ $d^2 = 6^2 + 10^2$ $d^2 = 36 + 100$ $d^2 = 136$ $d = \sqrt{136}$ $d = 11.66m$	What angle does the wire make with the ground? $Tan\theta° = \frac{opp}{adj}$ $Tan\theta° = \frac{10}{6}$ $Tan\theta° = 1.66\overline{6}$ $\theta° = 59.04° \text{ or } 59°2'10''$

9. Calculate the pitch of this roof. (Assuming symmetry)



This is an isosceles triangle. Half the large triangle is a right angled triangle.

$$Tan\theta^\circ = \frac{opp}{adj}$$

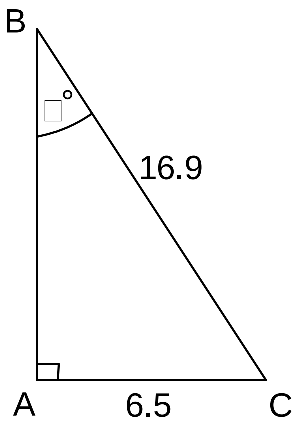
$$Tan\theta^\circ = \frac{2.5}{4.1}$$

$$Tan\theta^\circ = 0.6098$$

$$\theta^\circ = 31.4^\circ \text{ or } 31^\circ 22'23''$$

10. In $\Delta ABC : \angle A = 90^\circ, a = 16.9, b = 6.5, calculate \angle B$.

The first step is to draw the diagram to match the information given.



$$Sin\theta^\circ = \frac{opp}{hyp}$$

$$Sin\theta^\circ = \frac{6.5}{16.9}$$

$$Sin\theta^\circ = 0.3846$$

$$\angle B = \theta^\circ = 22.62^\circ \text{ or } 22^\circ 37'12''$$