Introduction to Whole Numbers and Integers

Numbers can be used for many different purposes. The cost of a house is a number that mathematics can be performed on to obtain some new amount. For example: Vendor wants a quick sale, was $500 000, now $495 000.

However, some numbers are never used mathematically such as your postcode, your telephone number or the level number in a building.

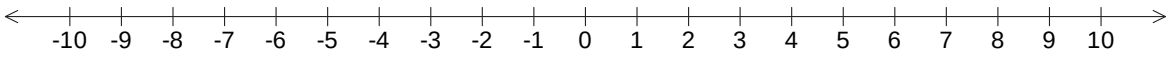
This module is about numbers that we want to do mathematics on. Two groups of numbers are referred to:

The whole numbers are $0, 1, 2, 3, 4, 5, \ldots\ldots$ (where …… means going on forever).

The integers (Pronounced in-te-gers) are all the whole numbers plus the negative numbers, represented as:

$$\ldots\ldots -5,\ -4,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5, \ldots\ldots\ldots$$

A number line representation of integers is below.



Module contents

Introduction

1. **Place value of large numbers**
2. **Some properties of whole numbers**
3. **Adding and subtracting**
4. **Multiplying and dividing**
5. **Order of operations**
6. **Rounding and estimation**

Outcomes

- To say and write large numbers to billions.
- To describe numbers as odd or even, prime or composite.
- To find factors and multiples of numbers.
- Perform all four operations with whole number and integers.
- Perform mixed operations following the correct order of operations.
- Use estimation to obtain approximate answers.

Check your skills

This module covers the following concepts, if you can successfully answer these questions, you do not need to do this module. Check your answers from the answer section at the end of the module.

1. (a) Write 345 109 050 in words
   (b) Write two billion, five hundred and thirty seven thousand, two hundred and fifty in digit form.
2. (a) Describe 51 as odd or even, prime or composite.
   (b) List the factors of 24.
   (c) List the first 5 multiples of 7.
3. Perform the operation indicated.
   (a) 2345 + 94 + 559 (b) 16 145 – 945 (c) 383 x 73
   (d) 4752 ÷ 9
4. Perform the operation indicated.
   (a) $^{-}234 + {}^{-}109$ (b) $405 - {}^{-}259$ (c) $^{-}175 \times {}^{-}7$
   (d) $8283 \div {}^{-}11$
5. Calculate the answer to this question and check using estimation.
   $825 + 32 \times (18 + 42)$

Topic 1: Place value of large numbers

The number system used everyday is the decimal (base 10) number system which uses the ten digits $0,1,2,3,4,5,6,7,8,9$. To represent larger numbers a **place value** system is used.

When there are more than 4 digits in the number, the digits are grouped in threes starting from the right and each group is separated by a small space.

For example:

The number 3841293 should be spaced as 3 841 293. Commas are not used in Australia, but may be used in other countries.

The cost of the house is $ 395 000.
The developer borrowed $ 10 500 000 to complete the project.
The population of Australia is 22 000 000 compared to 240 000 000 in Indonesia.
The predicted annual household income for 2020 is $ 64 539.
The land area of NSW is 800 628 sq. km.

This chart will help in understanding place value.

For example:

Billions			Millions			Thousands					
Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units
					3	8	4	1	2	9	3

This number is said or written in words as – Three million, eight hundred and forty one thousand, two hundred and ninety three.

Billions			Millions			Thousands					
Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units

		5	**0**	**0**	**3**	**7**	**0**	**2**	**6**	**0**	**0**

This number is said or written in words as – Five billion, three million, seven hundred and two thousand, six hundred.

Billions			Millions			Thousands					
Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units
			3	**8**	**5**	**1**	**0**	**0**	**0**	**0**	**2**

This number is said or written in words as – Three hundred and eighty five million, one hundred thousand and two.

Note: the next set of three digits after billions would be trillions, then quadrillions and so on.

Video 'Saying and Writing Large Numbers'

When expressing a written number in digit form, use the words billion, million and thousands as cues to leave a small space.

For example:

Two hundred and thirteen million, five hundred and seventy four thousand, two hundred written in digits is:
213 million, 574 thousand, 200 which gives 213 574 200.

Five billion, two hundred and thirteen million, five hundred thousand written in digits is:
5 billion, 213 million, 500 thousand which gives 5 213 500 000. Even though there is no mention of units, tens or hundreds, zeros must be put in those place values.

Three hundred million written in digits is:
300 million which gives 300 000 000. Even though there is no mention of any place values for hundreds, tens and unit thousands and hundreds, ten and units, zeros must be put in those place values.

Video 'Writing Large Numbers in Digits'

The place value of individual digits can also be determined using the chart below.

Billions			Million			Thousands					
Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units
			8	**1**	**7**	**6**	**3**	**9**	**4**	**5**	**2**

For example:

The place value of the 6 is: hundred thousands

The place value of the 4 is: hundreds
The place value of the 1 is: ten millions

Video 'Place Value of Large Numbers'

Activity

1. Write the following in written form

(a)	75 312	(b)	395 085	(c)	1 234
(d)	4 708 020	(e)	203 746 050	(f)	4 375 250 000
(g)	50 230 450	(h)	4 900 000	(i)	6 500 000 000

2. Write the following numbers in digit form.
   a) Twenty two thousand, four hundred and twelve.
   b) Three hundred and fifty thousand and eighty
   c) Four million, six hundred and sixteen thousand, seven hundred and seventy five.
   d) Forty five million
   e) Six billion, two hundred and thirty million, five hundred and seventy thousand, four hundred and two.
   f) One billion, five hundred million.
   g) Three hundred and seventy five million.
   h) Nine hundred million, five thousand and seven.

3. In the number 540 192 368
   a) What is the place value of the 4?
   b) What is the place value of the 6?
   c) What is the place value of the 1?
   d) What is the place value of the 9?
   e) What is the place value of the 5?

Topic 2: Some properties of whole numbers

Odd or Even

Even numbers are numbers that can be divided by 2 with no remainder or another way of saying this is: numbers that are divisible by 2. Even numbers end with the digit 0, 2, 4, 6, or 8. Although 0 is divisible by 2, it is a special case and is not considered as odd or even.

Odd numbers are all the other whole numbers. Odd numbers end with the digit 1, 3, 5, 7 or 9.

Even numbers = {2, 4, 6, 8, 10, 12, 14,….}

Odd numbers = {1, 3, 5, 7, 9, 11, 13,….}

Note: The definition of odd and even can be extended to include integers, so $^{-}204$ is considered to be an even number.

For example:
Are the following numbers odd or even?

(a) 35: 35 ends with a 5, so is an odd number

(b) 49: 49 ends with a 9, so is an odd number

(c) 66: 66 ends with a 6, so is an even number and divisible by 2

(d) 142: 142 ends with a 2, so is an even number and divisible by 2

(e) 1201: 1201 ends with a 1, so is an odd number

(f) $^{-}67$: $^{-}67$ ends with a seven, so is an odd number

It is interesting to note:

even + even = even	even – even = even	even x even = even
even + odd = odd	even – odd = odd	even x odd = even
odd + even = odd	odd – even = odd	odd x even = even
odd + odd = even	odd – odd = even	odd x odd = odd

Think of an example, such as 7 – 3 = 4, as odd – odd = even, to help make sense of these.

Video ‘Odd or Even’

Multiples and Factors

The multiple of a number is the result obtained when that number is multiplied by a natural (counting) number, that is 1, 2, 3, 4, 5….

For example:

Multiples of 5 are: $1\times5,\ 2\times5,\ 3\times5,\ 4\times5,\ 5\times5,$.....ie: 5, 10, 15, 20, 25,.....

The first 3 multiples of 9 are: $1\times9,\ 2\times9,\ 3\times9$ ie: 9, 18, 27

Which of these numbers are multiples of 7? 14, 30, 36, 42, 49, 50

14 can be divided by 7 with no remainder, IS a multiple
30 cannot be divided by 7 with no remainder, NOT a multiple
36 cannot be divided by 7 with no remainder, NOT a multiple
42 can be divided by 7 with no remainder, IS a multiple
49 can be divided by 7 with no remainder, IS a multiple
50 cannot be divided by 7 with no remainder, NOT a multiple

Video 'Multiples'

A factor is a number that will divide into another number without a remainder. Factor pairs are factors that multiply to give the original number. Factors are often listed lowest to highest.

For example:

Is 4 a factor of 16? 16 can be divided by 4 with no remainder, so 4 is a factor of 16.

List the factor pairs of 12:

$12\div1=12$ so 1 is a factor, 12 is also a factor
$12\div2=6$ so 2 is a factor, 6 is also a factor
$12\div3=4$ so 3 is a factor, 4 is also a factor
$12\div4=3$ this is a repeat of a factor pair above – STOP

The factor pairs of 12 are: 1 and 12, 2 and 6, 3 and 4.

List the factor pairs of 15:

1 and 15 are a factor pair
$15\div2=7$ remainder 1, so 2 is not a factor
$15\div3=5$ so 3 and 5 are a factor pair
$15\div4=3$ remainder 3, so 4 is not a factor
$15\div5=3$ this is a repeat of a factor pair above – STOP

The factor pairs of 15 are: 1 and 15, 3 and 5

List the factors of 15:

In the previous question, two factor pairs 1 and 15 and 3 and 5 were found, these listed in order are 1, 3, 5, 15.

List the factors of 28:

$28 \div 1 = 28$ so 1 is a factor, 28 is also a factor
$28 \div 2 = 14$ so 2 is a factor, 14 is also a factor
$28 \div 3 = 9$ remainder 1, so 3 is not a factor
$28 \div 4 = 7$ so 4 is a factor, 7 is also a factor
$28 \div 5 = 5$ remainder 3, so 5 is not a factor
$28 \div 6 = 4$ remainder 4, so 6 is not a factor
$28 \div 7 = 4$ this is a repeat of a factor pair above – stop
The factors of 28 are 1, 2, 4, 7, 14, 28.

Video 'Factors'

Prime or composite

Whole numbers two or greater can be described as either prime or composite.

A Prime Number is a number that has only two factors, one and itself.

2 is a prime number, it only has factors of 1 and itself (2)
3 is a prime number, it only has factors of 1 and itself (3)
4 is NOT a prime number because it has more than 2 factors
5 is a prime number, it only has factors of 1 and itself (5)
6 is NOT a prime number because it has more than 2 factors
7 is a prime number, it only has factors of 1 and itself (7)
and so on…..

The numbers 4 and 6 were not prime numbers because they had more than 2 factors, these are called Composite Numbers.

Note: 2 is the only even prime number, so, as even numbers are divisible by 2, even numbers (with the exception of 2) will never be prime. Therefore odd numbers are the only possibility to be prime.

For example:

Write the prime numbers between 10 and 20.

10 is even, therefore has more than 2 factors, COMPOSITE
11 has only factors of 1 and itself (11), PRIME
12 is even, therefore has more than 2 factors, COMPOSITE
13 has only factors of 1 and itself (13), PRIME
14 is even, therefore has more than 2 factors, COMPOSITE
15 has factors of 1, 3, 5, 15 COMPOSITE
16 is even, therefore has more than 2 factors, COMPOSITE
17 has only factors of 1 and itself (17), PRIME
18 is even, therefore has more than 2 factors, COMPOSITE
19 has only factors of 1 and itself (19), PRIME
20 is even, therefore has more than 2 factors, COMPOSITE

The prime numbers between 10 and 20 are 11, 13, 17, 19.

Is 32 prime or composite?

32 is even, therefore is COMPOSITE

Is 37 prime or composite?

37 has only factors of 1 and itself (37), therefore is PRIME

Is 51 prime or composite?

51 has factors of 1, 3, 17, 51, therefore is COMPOSITE

Is 438 prime or composite?

Because 438 ends with a 8 it is even, therefore is COMPOSITE

When the number is larger, you may have to use your calculator to decide.

Is 511 prime or composite?

Obviously 1 and 511 are factors, but are there any more?

Is 511 divisible by 3? (Using your calculator)

5 1 1 ÷ 3 = 170.3333333

511 is not divisible by 3.

Is 511 divisible by 4?

For 4 to be a factor, 511 would have to be even.

511 is **not divisible** by 4.

Is 511 divisible by 5?

If 511 was divisible by 5, the last digit would have to be 0 or 5

511 is **not divisible** by 5.

Is 511 divisible by 6? No, same reason as 4.

Is 511 divisible by 7? Using your calculator,

5 1 1 ÷ 7 = 73

7 is a factor, there are more than two factors, COMPOSITE

Video 'Prime or Composite'

Activity

1. Are the following numbers odd or even?

(a)	12	(b)	85	(c)	234
(d)	1020	(e)	5057	(f)	-351
(g)	5 453	(h)	-12 512	(i)	15 250

2. List the first four multiples of 7.

3. List the factors of

(a)	20	(b)	72	(c)	104

4. Classify the following numbers as either Prime or Composite

(a)	21	(b)	22	(c)	23
(d)	37	(e)	39	(f)	43
(g)	57	(h)	217	(i)	337

5. Answer TRUE or FALSE to the following statements

   a) 36 is a multiple of 6.
   b) 7 is a factor of 42.
   c) 1 is a prime, odd number.
   d) Because 119 is divisible by 7, it is a prime number.
   e) All even numbers are composite numbers.

Topic 3: Adding and subtracting

Adding and subtracting whole numbers

There are four basic operations in mathematics; addition, subtraction, multiplication and division. When adding or subtracting numbers, the numbers of the same place value must be lined up before the operation can be performed. To achieve this, the numbers are placed vertically.

For example:
Add 345, 57 and 224.

Arrange numbers vertically, lining up numbers of the same place value.

Add the numbers in the units column (=16), put the 6 in the units column and the 1 (carry) at the top of the tens column.
Add the numbers in the tens column and the carry(=12), put the 2 in the tens column and the new carry at the top of the hundreds column.
Add the numbers in the hundreds column and the carry (=6), put the 6 is the hundreds column, there is no carry.

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ } \\ 345 \\ 57 \\ +\,224 \\ \hline 626 \end{array}$$

There are different ways of doing subtraction. The method below is based on borrowing, other methods work equally well.

For example:
434 take 172.

Arrange numbers vertically, lining up numbers of the same place value.

In the units column, 4-2=2
In the tens column, 3-7 cannot be done, so borrow 1 from the next column and pay back 10. Now 13-7=6
In the hundreds column, it is now 3-1=2

$$\begin{array}{r} {\scriptstyle 3\ 13\ } \\ \cancel{4}\ \cancel{3}\ 4 \\ -172 \\ \hline 262 \end{array}$$

A harder example:
1024 – 687

Arrange numbers vertically, lining up numbers of the same place value.

In the units column, 4-7 cannot be done, so borrow 1 from the next column and payback 10. Now 14-7=7.
In the tens column, 1-8 cannot be done, so borrow 1 from the next column, the hundreds (0). Because borrowing can't take place, borrow from the next place value, the thousands, (1) and pay back 10 to the hundreds. Now borrow 1 from the hundreds to pay back 10 to the tens, which is now 11-8=3
In the hundreds column, it is now 9-6=3

$$\begin{array}{r} {\scriptstyle 0\ \ 9\ \ 11\ \ 14\ } \\ \cancel{1}\ \cancel{0}\ \cancel{2}\ \cancel{4} \\ -\ 687 \\ \hline 337 \end{array}$$

These examples, performed on a calculator, are below:

Make sure you press the clear key (AC), between examples.

When reading questions involving operations, phrases can give cues for selecting the correct operation. In addition, possible phrases are:

6 add 7	6 added to 7	the sum of 6 and 7
the total of 6 and 7	6 plus 7	increase 6 by 7
I have 6 and gain 7	7 more than 6	

All of these phrases translate to the mathematical expression $6 + 7 = 13$

In subtraction, possible phrases are:

9 minus 5	9 subtract 5	5 less than 9
9 with a reduction of 5	9 decreased by 5	9 less 5
Discount $9 by $5	difference between 9 and 5	Subtract 5 from 9

All of these phrases translate to the mathematical expression $9 - 5 = 4$

For example:

The 125 car parking spaces will increase by 59 when the new car park opens. What is the new number of parking spaces?

This translates to the mathematical expression $125 + 59$, which can be solved by pen and paper methods or by calculator.

Calculator

There will be a total of 184 parking spaces available.

Pen and Paper Method

$$\begin{array}{r} {\scriptstyle 1} \\ 125 \\ +\ 59 \\ \hline 184 \end{array}$$

Tom is 184 cm tall and his father is only 176 cm tall, what is the difference between their heights?

This translates to the mathematical expression $184 - 176$, which can be solved by pen and paper methods or by calculator.

Calculator

1 8 4 − 1 7 6 = 8

There is an 8 cm height difference

Pen and Paper Method

$$\begin{array}{r} {}^{7}\not{1}\,{}^{14}\not{8}\not{4} \\ -\,1\,7\,6 \\ \hline 0\,0\,8 \end{array}$$

Video ‘Adding and Subtracting Whole Numbers’

Adding and subtracting integers

Integers are all the whole numbers plus the negative numbers, and are represented as $\ldots\ldots -5,-4,-3,-2,-1,0,1,2,3,4,5,\ldots\ldots$. When writing integers, there are two ways of doing this.

For example: negative five could be written as $^{-}5$ or (-5). You will see the bracketed form when using your calculator.

Normally positive numbers are written with no sign because the assumption is that they are positive, however, there are times they are written with a positive sign. So 8, $^{+}8$ and (+8) are exactly the same number.

The expression: positive nine add negative seven could be written in two ways: $^{+}9+{}^{-}7$ or $(+9)+(-7)$ these will be solved later.

The expression: negative four is increased by positive one can be written as: $^{-}4+{}^{+}1$ or $(-4)+(+1)$

The expression: negative six is subtracted from negative four can be written as: $^{-}4-{}^{-}6$ or $^{-}4-(-6)$

The first step in solving these problems is to replace the double signs between the numbers with a single sign. The rule here is:

Same signs $+^{+}$, $+(+)$ or $-^{-}$, $-(-)$ are replaced with a + (plus)

$^{-}4-{}^{-}6$ becomes $^{-}4+6$

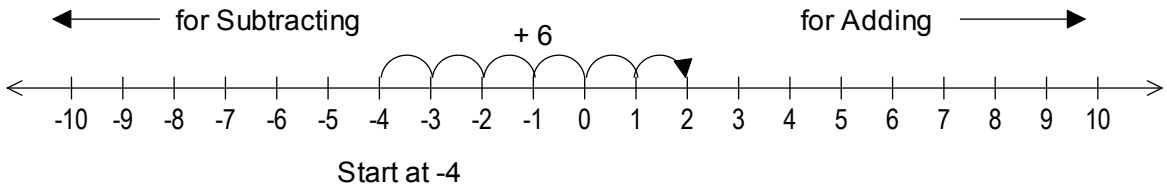
Replace $-^{-}$ with +

Different signs $+^{-}$,$+(-)$ or $-^{+}$,$-(+)$ are replaced with a – (minus)

$^{+}9+{}^{-}7$ becomes $^{+}9-7$

Replace $+^{-}$ with -

Let's follow these two examples through. So far $^{-}4-{}^{-}6$ has been simplified to $^{-}4+6$. The solution to this can be found using a number line. Start at $^{-}4$ and move 6 units to the right (adding).



With setting out:

$$^{-}4-{}^{-}6$$
$$=\ ^{-}4+6$$
$$=2$$

Using a number line, the second example is:

$^{+}9 + {}^{-}7$

$= {}^{+}9 - 7$

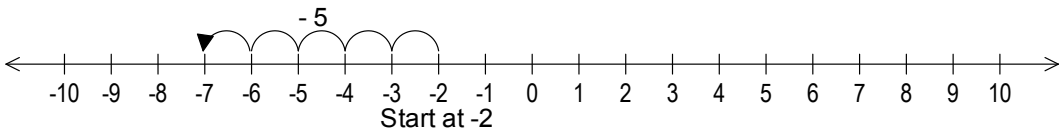
$= 2$

Example: $^{-}2 - {}^{+}5$

$^{-}2 - {}^{+}5$

$= {}^{-}2 - 5$

$= {}^{-}7$



Obviously drawing a number line for every example is difficult so from the examples above some generalisations can be drawn.

Looking at the second step in the solution, the following is noticed:

$^{-}4 + 6$ $= 2$	$^{+}9 - 7$ $= 2$	$^{-}2 - 5$ $= {}^{-}7$	Look at the signs in front of each number • If the signs are the same, add the numbers • If the signs are different, subtract the numbers • The sign of the answer is the sign of the largest number.

For example: $^{-}8 + {}^{-}7$

Step 1	Replace the double sign $+^{-}$ with -	$^{-}8 + {}^{-}7$
		$= {}^{-}8 - 7$
Step 2	The signs in front of both numbers are the same, so add the numbers (8+7=15) The largest number has a – sign in front of it, so the answer has a – sign in front.	$= {}^{-}15$

For example: $^{-}15 + {}^{+}17$

Step 1	Replace the double sign $+^{+}$ with +	$^{-}15 + {}^{+}17$
		$= {}^{-}15 + 17$
Step 2	The signs in front of both numbers are different, so subtract the numbers (17-15=2) The largest number has a + sign in front of it, so the answer has a + sign in front.	$= {}^{+}2$ $= 2$

When subtracting in step 2 always do: Largest number – Smallest number

For example: $105 + {}^{-}217$

Step 1	Replace the double sign $+^{-}$ with -	$105 + {}^{-}217$
		$= 105 - 217$
Step 2	The signs in front of both numbers are different , so subtract the numbers (217-105=112) The largest number has a - sign in front of it, so the answer has a - sign in front.	$= {}^{-}112$ (105 is also $^{+}105$)

For example: $^{-}34 + {}^{-}28 - {}^{-}57$

Step 1	Replace the double signs $+^{-}$ becomes -, $-^{-}$ becomes +	$^{-}34 + {}^{-}28 - {}^{-}57$
		$= {}^{-}34 - 28 + 57$
Step 2	Working from left to right, the signs in front of the first two numbers are the same, so add the numbers (34+28=62). The largest number has a - sign in front of it, so the answer has a - sign in front ($^{-}62$). Now consider the remaining numbers, the signs in front of the numbers are different, so subtract (62-57=5). The largest number has a – sign in front of it, so the answer has a – sign in front.	$= {}^{-}62 + 57$ $= {}^{-}5$

These can be performed on a calculator. Entering a negative sign is achieved by pressing the (−) key. This key may appear on other calculators as Y. Calculators work on the principle that positive numbers are entered without a sign.

Example	Calculator sequence of keys

$^{-}8 + {}^{-}7$	(–) 8 + (–) 7 = -15
$^{-}15 + {}^{+}17$	(–) 1 5 + 1 7 = 2
$105 + {}^{-}217$	1 0 5 + (–) 2 1 7 = -112
$^{-}34 + {}^{-}28 - {}^{-}57$	(–) 3 4 + (–) 2 8 − (–) 5 7 = -5

If many numbers are being added and subtracted, working left to right will obtain an answer. There is another method that reduces the amount and difficulty of calculations. This method is based on the analogy of a bank account where the balance of the account is equal to deposits take withdrawals.

Balance = Sum of Deposits – Sum of Withdrawals

Answer = Sum of Positive numbers – Sum of Negative Numbers

For example: $^{-}105 - 203 - {}^{-}64 + {}^{-}81 + 44$

Step 1	Replace the double signs	$^{-}105 - 203 + 64 - 81 + 44$
Step 2	Think of +numbers as deposits and – numbers as withdrawals. The balance of the account is the sum of the deposits – the sum of the withdrawals.	Deposits – withdrawals $(64 + 44) - (105 + 203 + 81)$ $= 108 - 389$ $= {}^{-}281$

For example: $73 - {}^{+}29 - {}^{-}121 - 43 + {}^{-}28$

Step 1	Replace the double signs	$73 - 29 + 121 - 43 - 28$
Step 2	Think of +numbers as deposits and – numbers as withdrawals. The balance of the account is the sum of the deposits – the sum of the withdrawals.	Deposits – withdrawals $(73 + 121) - (29 + 43 + 28)$ $= 194 - 100$ $= 94$

Video 'Adding and Subtracting Integers'

Activity

1. Perform the operation indicated by both pen and paper and calculator methods.
   (a) Add 4709, 864 and 21 109
   (b) Take 672 from 2709
   (c) Discount $37 606 by $488
   (d) Increase $294 529 by $57 294

2. Perform the operation indicated by both pen and paper and calculator methods.

(a)	$^{-}3 + {}^{-}4$	(b)	$46 - 56$
(c)	$^{+}27 - {}^{-}36$	(d)	$^{-}37 + {}^{+}16$
(e)	$^{-}411 - {}^{+}276$	(f)	$123 + {}^{-}204$
(g)	$^{-}11 + {}^{-}8 - {}^{-}15 + 12 - {}^{+}9$	(h)	$^{-}234 + {}^{-}76 - {}^{-}127$

3. A pilot is flying at an altitude of 3680 metres when she is instructed by the control tower to increase her altitude by 550m. What is her altitude after the increase?

4. Write a mathematical expression for this scenario and solve. Represent deposits as positive values and withdrawals as negative. On Monday, Jim opens a bank account with a deposit of $500, on Tuesday he withdraws $137, on Wednesday he withdraws $67, on Thursday he deposits $36 and on Friday he withdraws $247. What is the balance at the end of the week?

5. Joe is reading his superannuation statements for the past 3 years. In 2007 he had a balance of $127 456.28, in 2008 a balance of $153 291.74 and in 2009 a balance of $ 151 875.28.
   (a) How much did his superannuation increase from 2007 to 2008?
   (b) How much did his superannuation increase from 2008 to 2009?

Topic 4: Multiplying and dividing

Multiplying whole numbers

Multiplication is really just repeated addition, so $3+3+3+3+3=5$ lots of $3=5\times3$. When multiplying numbers, knowledge of multiplication tables up to 10x will make calculations much faster. Firstly, multiplication by a single digit number will be covered.

For example: 317×6

Arrange numbers vertically, lining up numbers of the same place value.
Multiply the units number by 6. 6x7=42, so the 2 will go in the units column and the 4 will be carried over to the tens column.
Multiply the tens number by 6. 6x1=6, then the carry from the units must be added 6+4=10. The 0 will go in the tens column and the 1 is carried over to the hundreds column.
Multiply the hundreds number by 6. 6x3=18 plus the carry from the tens18+1=19. The 9 will go in the hundreds column and the 1 must go in the thousands column

$$\begin{array}{r} {\scriptstyle 1\ 4\ \ } \\ 317 \\ \times\ 6 \\ \hline 1902 \end{array}$$

For example: 2407×5

Arrange numbers vertically, lining up numbers of the same place value.
Multiply the units number by 5. 5x7=35, so the 5 will go in the units column and the 3 will be carried over to the tens column.
Multiply the tens number by 5. 5x0=0, then the carry from the units must be added 0+3=3. The 3will go in the tens column and there is no carry over to the hundreds column.
Multiply the hundreds number by 5. 5x4=20 plus no carry equals 20. The 0 will go in the hundreds column and the 2 must go in the thousands column
Multiply the thousands number by 5. 5x2=10 plus the carry from the hundreds must be added 10+2=12. The 2 will go in the thousands column and the 1 must go in the ten thousands column

$$\begin{array}{r} {\scriptstyle 2\ \ 3\ \ } \\ 2407 \\ \times\ \ 5 \\ \hline 12\,035 \end{array}$$

Multiplying by multiples of 10

Before multiplying by numbers containing 2 or more digits, knowledge of multiplying by multiples of 10 must be covered.
For example: $7\times10, 100, 1000, 10\,000$ etc.

Question	Mental calculation	Answer
7×10	7 x1 attach 0	70
7×100	7 x1 attach 00	700
7×1000	7 x1 attach 000	7000
$7\times10\,000$	7x1 attach 0 000	70 000

Extending this idea:

Question	Mental calculation	Answer
3×30	3x3 attach 0	90
20×50	2x5 attach 00	1000
70×300	7x3 attach 000	21 000

$400 \times 20\,000$ 4x2 attach 000 000 8 000 000

Video ‘Multiplying by Multiples of 10’

When multiplying by a two digit number, the method used consists of two multiplications. If the question is 34 x 26, first 34 is multiplied by 6 then 34 is multiplied by 20. We know already that when a number is multiplied by a multiple of ten, the number will have a 0 in the units place. The method for doing this is often called Long Multiplication.

Example: 34 x 26

Arrange numbers vertically, lining up numbers of the same place value.

6 x 4 = 24 put down the 4 and carry the 2
6 x 3 = 18 plus carry of 2 gives 20, put down the 0 and the 2 must

go in the next place.
Before multiplying by 20, go and cross out the carry numbers to

avoid confusion.
Put a 0 in the units column from multiplying by a multiple of ten..
2x4 = 8 put this in the next place value
2x3=6 put this in the next place value

Add the two part answers

$$\begin{array}{rl} & \overset{2}{\not{3}}\,4 \\ \times & 2\,6 \\ \hline & 2\,0\,4 \quad \leftarrow \text{This is the answer to } 34 \times 6 \\ + & 6\,8\,0 \quad \leftarrow \text{This is the answer to } 34 \times 20 \\ \hline & 8\,8\,4 \quad \leftarrow \text{This is the answer to } 34 \times 26 \end{array}$$

Example: 619 x 45

Arrange numbers vertically, lining up numbers of the same place value.

5 x 9 = 45 put down the 5 and carry the 4
5 x 1 = 5 plus carry of 4 gives 9, put down the 9
5 x 6 = 30 put down the 0 and the 3 in the next place value
Before multiplying by 40, go and cross out the carry numbers to avoid confusion. Put a 0 in the units column from multiplying by a multiple of ten.
4 x 9 = 36 put down the 6 and carry the 3
4 x 1 =4 plus the carry 3 gives 7 put this in the next place value
4 x 6 = 24 put down the 4 and the 2 in the next place value
Add the two part answers

$$\begin{array}{rl} & \overset{3}{\not{6}}\,1\,9 \\ \times & 4\,5 \\ \hline & 3\,0\,9\,5 \quad \leftarrow \text{This is the answer to } 619 \times 5 \\ + & 2\,4\,\overset{1}{7}\,6\,0 \quad \leftarrow \text{This is the answer to } 619 \times 40 \\ \hline & 2\,7\,8\,5\,5 \quad \leftarrow \text{This is the answer to } 619 \times 45 \end{array}$$

These examples, performed on a calculator, are below.

3 1 7 × 6 = 1902
2 4 0 7 × 5 = 12035
3 4 × 2 6 = 884
6 1 9 × 4 5 = 27855

When reading questions involving operations, phrases can give cues for selecting the correct operation. In multiplication, possible phrases are:

the product of 6 and 7	6 times 7	6 multiplied by 7
6 lots of 7	of	6 groups of 7

These phrases translate to $6 \times 7 = 42$.

For example:

There were 7 groups of 23 students through the visitor information centre today. How many is this altogether?

Calculator

2 3 × 7 = 161

There were 161 students through the centre.

Pen and Paper Method

$$\begin{array}{r} {}^{2}23 \\ \times\ 7 \\ \hline 161 \end{array}$$

Video 'Multiplying Whole Numbers'

Multiplying integers

If the signs of the integers are the same, the answer is positive.

If the signs of the integers of are different, the answer is negative.

	Example	
positive x positive ➔ positive	$6 \times 4 = 24$	Same signs
negative x negative ➔ positive	$^{-}6 \times {}^{-}4 = {}^{+}24$	Same signs
positive x negative ➔ negative	$6 \times {}^{-}4 = {}^{-}24$	Different signs
negative x positive ➔ negative	$^{-}6 \times 4 = {}^{-}24$	Different signs

For example: $^{-}3813 \times 8$

Do the multiplication, ignoring signs ie: 3813 x 8

Now think: a negative x positive ➔ negative (different signs)

The answer is -30504

$$\begin{array}{r} {}^{6\ 1\ 2} \\ 3813 \\ \times\ 8 \\ \hline 30504 \end{array}$$

This example, performed on a calculator, is below.

(–) 3 8 1 3 × 8 = -30504

When multiplying more than two numbers, the first step is to multiply the numbers ignoring signs and then work out the sign of the answer.

For example: $^{-}5 \times {}^{+}4 \times {}^{-}2 \times {}^{-}3$

Multiply, ignoring signs $5 \times 4 \times 2 \times 3 = 120$

Sign: working left to right $-\times+\Rightarrow-$ then $-\times-\Rightarrow+$ then $+\times-\Rightarrow-$

The answer is $^{-}120$

A quicker way to work out the signs is:

An odd number of negatives ➔ negative

An even number of negatives ➔ positive

For example: $(-1)\times 6\times(-2)\times(-4)\times 2$

Multiply, ignoring signs $1\times 6\times 2\times 4\times 2=96$
There are an odd number of negative signs, overall negative.
The answer is $^{-}96$

These examples, performed on a calculator, are below.

(–) 5 × 4 × (–) 2 × (–) 3 = -120

(–) 1 × 6 × (–) 2 × (–) 4 × 2 = -96

 Video ‘Multiplying Integers’

Division of whole numbers

Division is best described as a sharing process. Suppose the cost of dinner is $228 and is to be shared between 4 people, each person would pay $228 ÷ 4 which is $57 each. If 12 people share a lotto win of $1 200 000, each person would receive $1 200 000 ÷ 12 which is $100 000 each.
In this module, answers to division questions will be whole numbers. In the decimal module, remainders will be dealt with. There are two types of division. In this module the focus is to do short division only. Traditionally, dividing by a single digit number was answered by short division and dividing by a two digit number was answered by long division. The thought processes in each are identical, the difference being that these are part of the setting out in long division.

The question 371 divided by 7 can be written as $371 \div 7$ or as $371/7$ which should be written as $\frac{371}{7}$.

The setting out for this question is:

3 divided by 7 cannot be done, so think 37 divided by 7 = 5 and 2 left over.

21 divided by 7 = 3

The answer is 53.

$$\begin{array}{r} 5 \\ 7\overline{)3\,7^{2}1} \end{array} \qquad \begin{array}{r} 5\,3 \\ 7\overline{)3\,7^{2}1} \end{array}$$

For example: $6075 \div 5$

6 divided by 5 = 1 and 1 left over

10 divided by 5 = 2

7 divided by 5 = 1 and 2 left over

25 divided by 5 = 5

The answer is 1215.

$$\begin{array}{r} 1 \\ 5\overline{)6^{1}0\,7\,5} \end{array} \qquad \begin{array}{r} 1\,2 \\ 5\overline{)6^{1}0\,7\,5} \end{array} \qquad \begin{array}{r} 1\,2\,1 \\ 5\overline{)6^{1}0\,7^{2}5} \end{array} \qquad \begin{array}{r} 1\,2\,1\,5 \\ 5\overline{)6^{1}0\,7^{2}5} \end{array}$$

For example: $90702 \div 3$

9 divided by 3 = 3

0 divided by 3 = 0

7 divided by 3 = 2 and 1 left over

10 divided by 3 = 3 and 1 left over

$$\begin{array}{r} 3 \\ 3\overline{)9\,0\,7\,0\,2} \end{array} \qquad \begin{array}{r} 3\,0 \\ 3\overline{)9\,0\,7\,0\,2} \end{array} \qquad \begin{array}{r} 3\,0\,2 \\ 3\overline{)9\,0\,7^{1}0\,2} \end{array} \qquad \begin{array}{r} 3\,0\,2\,3 \\ 3\overline{)9\,0\,7^{1}0^{1}2} \end{array}$$

12 divided by 3 = 4

Answer is 30 234

$$\begin{array}{r} 30234 \\ 3\overline{)90^{1}7^{1}0^{1}2} \end{array}$$

These examples, performed on a calculator, are below:

3 7 1 ÷ 7 = 53

6 0 7 5 ÷ 5 = 12035

9 0 7 0 2 ÷ 3 = 30234

Dividing by multiples of 10

For example: $7\,000 \div 10, 100,$ etc.

Question	Mental calculation	Answer
$7000 \div 10$	Remove a 0	700
$7000 \div 100$	Remove 00	70
$7000 \div 1000$	Remove 000	7

Extending this idea:

Question	Mental calculation	Answer
$900 \div 30$	First $\div 10$, which gives 90, then $\div 3$	30
$1000 \div 50$	First $\div 10$, which gives 100, then $\div 5$	20
$80\,000 \div 200$	First $\div 100$, which gives 800, then $\div 2$	400
$24\,000\,000 \div 20\,000$	First $\div 10\,000$, which gives 2400, then $\div 2$	1200

Video 'Dividing by Multiples of 10'

When reading questions involving operations, phrases can give cues for selecting the correct operation. In division, possible phrases are:

40 divided by 5	5 divided into 40	5 goes into 40
quotient of 40 and 5	per	40 shared between 5

For example:

Jayne drove 5 journeys of 34km and 3 journeys of 57km. She used 31 litres of fuel to cover this distance, how many km per litre did she obtain?

In the first sentence, 'of' means multiply and 'and' means add.
The first sentence translates to $5 \times 34 + 3 \times 57$. The two multiplications must be performed first (See Topic 5 – Order of Operations).

Calculator

[3] [4] [×] [5] [=] 170

A distance of 170 km was covered.

Pen and Paper Method

$$\begin{array}{r} {}^{2}3\,4 \\ \times\, 5 \\ \hline 1\,7\,0 \end{array}$$

Calculator

[5] [7] [×] [3] [=] 171

A distance of 171 km was covered.

Pen and Paper Method

$$\begin{array}{r} {}^{2}5\,7 \\ \times\, 3 \\ \hline 1\,7\,1 \end{array}$$

Calculator

[1] [7] [0] [+] [1] [7] [1] [=] 341

The total distance travelled was 341 km.

Pen and Paper Method

$$\begin{array}{r} {}^{1}1\,7\,0 \\ +1\,7\,1 \\ \hline 3\,4\,1 \end{array}$$

The word 'per' in the second sentence is a cue for division.

Calculator

[3] [4] [1] [÷] [3] [1] [=] 11

The answer is 11. The car covered 11km per litre

Pen and Paper Method

$$31\overline{)3\,4^{3}1}\quad 1\,1$$

Video 'Dividing Whole Numbers'

Division of Integers

The same rules apply here as for multiplication.

If the signs are the same, the answer is positive.

If the signs are different, the answer is negative.

	Example	
positive ÷ positive ➔ positive	$24 \div 4 = 6$	Same signs
negative ÷ negative ➔ positive	$^{-}24 \div {}^{-}4 = {}^{+}6$	Same signs
positive ÷ negative ➔ negative	$24 \div {}^{-}4 = {}^{-}6$	Different signs
negative ÷ positive ➔ negative	$^{-}24 \div 4 = {}^{-}6$	Different signs

Remember, these can always be checked by turning the question into a multiplication. $^{-}24 \div 4 = {}^{-}6$ as a multiplication is $4 \times {}^{-}6 = {}^{-}24$, the sign is correct here, so the division answer sign must also be correct.

For example: $^{-}945 \div {}^{-}7$

Do the division, ignoring the signs ie:945÷7

$$7\overline{)9^{2}45}\quad 1$$

Now think: a negative ÷ a negative ➔ positive

$$7\overline{)9^{2}4^{3}5}\quad 13$$

The answer is $^{+}135$ or 135

$$7\overline{)9^{2}4^{3}5}\quad 135$$

For example: $1233 \div {}^{-}9$

Do the division, ignoring the signs ie:1233÷9

$$9\overline{)12^{3}33}\quad 1$$

Now think: a positive ÷ a negative ➔ negative

$$9\overline{)12^{3}3^{6}3}\quad 13$$

The answer is $^{-}137$

$$9\overline{)12^{3}3^{6}3}\quad 137$$

These examples, performed on a calculator, are below.

(–) 9 4 5 ÷ (–) 7 = 135

1 2 3 3 ÷ (–) 9 = -137

Video 'Dividing Integers'

Activity

1. Perform these multiple of ten multiplications mentally.

(a)	9 x 100	(b)	90 x 10	(c)	770 x 1000
(d)	40 x 50	(e)	220 x 20	(f)	410 x 1000
(g)	5 x 900	(h)	6000 x 50	(i)	40 x 25

2. Perform the operation indicated by both pen and paper and calculator methods.

(a) Find 12 lots of $512
(b) Find the quotient of 50 274 and 7
(c) Find the product of 479 and 7
(d) Share 1686 smarties between 6 people

3. Perform the operation indicated by both pen and paper and calculator methods.

(a)	$^{-}12 \times {}^{+}11$	(b)	$^{-}132 \times {}^{-}5$	(c)	$121 \div {}^{-}11$
(d)	$316 \times {}^{-}24$	(e)	$^{-}96 \div 6$	(f)	$^{-}3807 \div {}^{-}9$

4. Evaluate the following.

(a)	$^{-}3 \times 4 \times {}^{-}5 \times 2$	(b)	$^{+}40 \div {}^{-}5 \div {}^{-}2$	(c)	$^{-}36 \div 6 \times {}^{-}2 \div {}^{-}3$

5. One codeine tablet contains 30mg of active ingredient. How much active ingredient will 12 tablets contain?

6. From a roll of material of length 100m, 16 lengths of 6 metres have been cut off and sold. How much should be left on the roll?

7. An athlete runs a marathon (42km) in 3hrs 30 mins. What was his average time per km? (Hint: convert the time to minutes)

8. John can travel 630 km on a tank of fuel. How many journeys of 35 km can he do?

Topic 5: Order of operations

You might recall a question in the previous section:
Jayne did 5 journeys of 34km and 3 journeys of 57km. She used 31 litres of fuel to cover this distance, how many km per litre did she obtain. The distance covered was $5\times34+3\times57$.

There are two possibilities for how this expression is solved. The answer could have been obtained by working left to right.

$$5\times34+3\times57$$
$$=170+3\times57$$
$$=173\times57$$
$$=9861$$

If the two multiplication operations were performed first before the addition took place, the answer is

$$5\times34+3\times57$$
$$=170+171$$
$$=341$$

To avoid confusion there is a convention that states the order in which operations are performed. When solving an expression containing different operations use this order.

1. Evaluate brackets or other grouping symbols first. If there are nested brackets (brackets within brackets) work from the inner set to the outer set.
2. Evaluate any powers or roots. This is covered in the indices module. An example of a simple power is $3^2=3\times3=9$. An example of a root is $\sqrt{16}=4$ because $4\times4=4^2=16$.
3. Multiplication and division. These operations are equal in priority and an expression containing both operations should be solved working left to right.
4. Addition and subtraction. These operations are equal in priority and an expression containing both operations should be solved working left to right.

For example:

$14-7+2$	Addition and subtraction are equal priority, so work from L to R
$=7+2$	Subtraction first
$=9$	then addition

$14-(7+2)\div 3$ Brackets must be simplified first
$=14-9\div 3$ Division next
$=14-3$ then subtraction
$=11$

$1\times 7+3^2-3\times 7$ The power must be calculated first
$=1\times 7+9-3\times 7$ The two multiplications next
$=7+9-21$ then addition followed by subtraction
$=16-21=\bar{}5$

$2[3\times\bar{}4-(4-10)]$ Inner most brackets first
$=2[3\times\bar{}4-\bar{}6]$ Multiplication in the brackets
$=2[\bar{}12-\bar{}6]$ then addition (remember $-\bar{}$ is the same as +)
$=2[\bar{}12+6]=2\times\bar{}6$ The operation between the 2 and the bracket is multiply
$=\bar{}12$

Modern calculators understand order of operations and will do operations in the correct sequence, however, you still need to use brackets. The examples above, performed on a calculator are:

1 4 − 7 + 2 = 9

1 4 − (7 + 2) ÷ 3 = 11

1 × 7 + 3 x^2 − 3 × 7 = -5

2 (3 × (−) 4 − (4 − 1 0)) = -12

Take care when evaluating number expression like this $\frac{25+4^2}{4.2\times 60}$. The calculator sequence below will produce the incorrect answer;

(2 5 + 4 x^2) ÷ 4 . 2 × 6 0 = 585.71…

This is an order of operations problem. To make sure the numerator is divided by the product of 4.2 and 60, the 4.2 x 60 should be bracketed.

(2 5 + 4 x^2) ÷ (4 . 2 × 6 0) = 0.16269…

Video 'Order of Operations'

Activity

1. Evaluate the following expressions by pen and paper and calculator methods.

(a) $20\div(3+1)$ (b) $3\times(4-6)$ (c) $2(9-2\times 3)$

(d) $20 \div 2 + (5 - 7)$ (e) $3\sqrt{16} + (6 + 9 \div 3)$ (f) $14 + (3^2 + \sqrt{16}) \times 3$

(g) $(4 - 11) \times (9 - 7)$ (h) $^{-}3(4 - 2 \times 8)$ (i) $^{-}50 \div [4 + (2 \times 3)]$

Topic 6: Rounding and estimation

Rounding

Rounding of numbers is an important skill to help estimate an answer to a question. Sometimes a rounded value is more appropriate to use than an exact figure:
For example:

- There were approximately 5000 at the football game.
- It is about 900 km from Brisbane to Sydney.
- Roughly 20 million people live in Australia.

When rounding a number, a place value to which the number must be rounded will be stated.

For example:

Round 4 792 to the nearest hundred.

The number 4 792 is, in size, between 4 700 and 4 800 when the hundreds place is being considered. The purpose of rounding is to say that 4 792 is closer to 4 800 than 4700.

There are three steps to rounding to a stated place value. (Using the example above)

Step 1 Locate the digit in the place value being **stated** and underline it. 4 7̲92

Step 2 Locate the digit to the right of the place value **stated**. Now circle this digit.
4 7̲⑨2
If this digit is 'less than 5' then the digit in the stated place value stays the same.
If this digit is '5 or more' then the digit in the stated place value increases by one.
As this digit is ⑨, the stated value 7 will increase to 8.

Step 3 Change each digit in the place values to the right of stated place value to zero.
The answer is 4 800

Round 42 572 to the nearest thousand

Step 1 Locate the digit in the place value being **stated** and underline it. 4̲2 572

Step 2 Locate the digit to the right of the place value stated. Now circle this digit.
4̲2 ⑤72
As this digit is ⑤, the '5 or more' rule applies, so the stated value 2 will increase to 3.

Step 3 Change each digit in the place values to the right of stated place value to zero.
The answer is 43 000

Round 982 340 to the nearest thousand

Step 1 Locate the digit in the place value being **stated** and underline it. 98̲2 340

Step 2 Locate the digit to the right of the place value stated. Now circle this digit.
982 ③40
As this digit is ③, the rule 'less than 5' applies, so the stated value 2 will stay the same.

Step 3 Change each digit in the place values to the right of stated place value to zero.
The answer is 982 000

Video 'Rounding Whole Numbers'

Estimation

Estimation is an important technique to judge whether an answer obtained by pen and paper or calculator methods is approximately right. The method used here involves rounding a number based on its most significant digit. In 725, the 7 is the most significant digit, and because the digit in the next place value is a 2, 725 rounds to 700. In 56190, the 5 is the most significant digit, because the next digit is a 6, 56 190 rounds to 60 000.

An addition example: 5642 + 8134 + 319 + 945

8134 is the largest number, so round to the nearest thousand

$5642 \to 6000$
$8134 \to 8000$
$319 \to 300$
$945 \to 1000$

Estimate is $6000 + 8000 + 300 + 1000 = 15\,300$

A subtraction example: 68 712 – 24 277

68 712 is the largest number, so round to the nearest ten thousand

$68\,712 \to 70\,000$
$24\,277 \to 20\,000$

Estimate is $70\,000 - 20\,000 = 50\,000$

A multiplication example: 312 x 67

Round to the leading digit

$312 \to 300$
$67 \to 70$

Estimate is $300 \times 70 = 21\,000$

A division example: $22\,575 \div 495$

Round to the leading digit

$22\,575 \to 20\,000$
$495 \to 500$

Estimate is $20\,000 \div 500 = 40$

Video 'Estimating Answers'

Activity

1. Round these numbers to the place value given in brackets.

(a)	632 (tens)	(b)	4192 (hundreds)	(c)	4192 (thousands)
(d)	4192 (tens)	(e)	4192 (ten thousands)	(f)	23 914 (thousands)
(g)	2 375 000 (million)	(h)	2 375 000 (ten thousands)	(i)	495 673 (thousands)

2. Estimate the answers to the following questions.

(a)	27×412	(b)	$1234 + 3099 + 971$	(c)	$2750 \div 48$
(d)	$3678 - 1213$	(e)	$234 + 789 - 297$	(f)	212×350
(g)	$2112 + 37 \times 51$	(h)	$37 \times 171 \div 375$	(i)	$(6193 + 12\,190) \div 39$

3. Jane used pen and paper methods to obtain the answer to 157×23 . The answer she obtained was 785. Use estimation to show whether Jane was possibly correct or not.

4. In a shipment of containers containing crates of milk, there are 8 containers; each containing 96 crates; each crate contains 12 milk cartons. Estimate the number of milk cartons in the shipment.

5. A cyclist rode 47 km on Monday, 57 on Wednesday, 34 on Friday, 55 on Saturday and 92 on Sunday. Approximately how many km did he do for the week?

Answers to activity questions

Check your skills

1. (a) Three hundred and forty five million, one hundred and nine thousand, and fifty.
   (b) 2 000 537 250

2. (a) 51 is odd (ends with 1) and composite (factors are 1,3,17,51)
   (b) Factors of 24 are: 1, 2, 3, 4, 6, 8, 12, 24.
   (c) First 5 multiples of 7: 7, 14, 21, 28, 35.

3. (a)
$$\begin{array}{r} {\scriptstyle 1\ 1\ \ \ } \\ 2345 \\ 94 \\ +559 \\ \hline 2998 \end{array}$$

(b)
$$\begin{array}{r} {\scriptstyle 5\ 11\ \ \ } \\ 16\not{1}45 \\ -945 \\ \hline 15\ 200 \end{array}$$

(c)
$$\begin{array}{r} {\scriptstyle 5\ 2\ \ \ } \\ \not{2}83 \\ \times 73 \\ \hline 1149 \\ +26810 \\ \hline 27959 \end{array}$$

(d)
$$\begin{array}{r} 528 \\ 9\overline{)47^{2}5^{7}2} \end{array}$$

4. (a)
$$\begin{aligned} &{}^{-}234 + {}^{-}109 \\ &= {}^{-}234 - 109 \\ &= {}^{-}343 \end{aligned}$$

(b)
$$\begin{aligned} &405 - {}^{-}259 \\ &= 405 + 259 \\ &= 664 \end{aligned}$$

(c)
$$\begin{array}{r} {\scriptstyle 5\ 3\ \ \ } \\ 175 \\ \times 7 \\ \hline 1225 \end{array}$$

${}^{-}\times{}^{-} = {}^{+}$ answer is ${}^{+}1225$ or 1225

(d)
$$\begin{array}{r} 753 \\ 11\overline{)82^{5}8^{3}3} \end{array}$$

${}^{+}\div{}^{-} = {}^{-}$ answer is ${}^{-}753$

5.
$$\begin{aligned} &825 + 32 \times (18 + 42) \\ &= 825 + 32 \times 60 \\ &= 825 + 1920 \\ &= 2745 \end{aligned}$$

Place value of large numbers

1. (a) Seventy five thousand, three hundred and twelve.
   (b) Three hundred and ninety five thousand and eighty five.
   (c) One thousand, two hundred and thirty four.
   (d) Four million, seven hundred and eight thousand and twenty
   (e) Two hundred and three million, seven hundred and forty six thousand and fifty.
   (f) Four billion, three hundred and seventy five million, two hundred and fifty thousand.
   (g) Fifty million, two hundred and thirty thousand, four hundred and fifty.
   (h) Four million, nine hundred thousand.
   (i) Six billion, five hundred million.

2. (a) 22 412 (b) 350 080 (c) 4 616 775
(d) 45 000 000 (e) 6 230 570 402 (f) 1 500 000 000
(g) 375 000 000 (h) 900 005 007

3. (a) The 4 is 4 ten millions.
(b) The 6 is 6 tens.
(c) The 1 is 1 hundred thousand.
(d) The 9 is 9 ten thousands
(e) The 5 is 5 hundred million

Some properties of whole numbers

1. (a) 12 is even (b) 85 is odd (c) 234 is even
(d) 1020 is even (e) 5057 is odd (f) $^{-}351$ is odd
(g) 5453 is odd (h) $^{-}12\,512$ is even (i) 15250 is even

2. The first four multiples of 7 are: 7, 14, 21, 28 [1x7, 2x7, 3x7, 4x7]

3. (a) Factors of 20 are 1, 2, 4, 5, 10, 20
(b) Factors of 72 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
(c) Factor of 104 are 1, 2, 4, 8, 13, 26, 52, 104

4. (a) 21 is composite (b) 22 is composite (c) 23 is prime
(d) 37 is prime (e) 39 is composite (f) 43 is prime
(g) 57 is composite (h) 217 is composite (i) 337 is prime

5. (a) 36 is a multiple of 6 TRUE
(b) 7 is a factor of 42 TRUE
(c) 1 is a prime, odd number FALSE 1 is odd but not prime.
(d) Because 119 is divisible by 7, it is a prime number FALSE
(e) All even numbers are composite numbers FALSE 2 is the exception

Adding and subtracting

1. (a)
$$\begin{array}{r} {\scriptstyle 1\ \ 2}\\ 4709\\ 864\\ +21109\\ \hline 26682 \end{array}$$

(b)
$$\begin{array}{r} {\scriptstyle 6\ \ 10}\\ 27\not{0}9\\ -672\\ \hline 2037 \end{array}$$

(c)
$$\begin{array}{r} {\scriptstyle 5\ 9\ 16}\\ 37\not{6}\not{0}\not{0}\\ -488\\ \hline 37118 \end{array}$$

(d)
$$\begin{array}{r} {\scriptstyle 1\ 1\ \ 1\ 1}\\ 294529\\ +57294\\ \hline 351823 \end{array}$$

2. (a)
$$\begin{aligned} & {}^{-}3+{}^{-}4\\ &= {}^{-}3-4\\ &= {}^{-}7 \end{aligned}$$

(b)
$$\begin{aligned} & 46-56\\ &= {}^{-}10 \end{aligned}$$

(c)
$$\begin{aligned} & {}^{+}27-{}^{-}36\\ &= {}^{+}27+36\\ &= 63 \end{aligned}$$

(d)
$$\begin{aligned} & {}^{-}37+{}^{+}16\\ &= {}^{-}37+16\\ &= {}^{-}21 \end{aligned}$$

(e)
$$\begin{aligned} & {}^{-}411-{}^{+}276\\ &= {}^{-}411-276\\ &= {}^{-}687 \end{aligned}$$

(f)
$$\begin{aligned} & 123+{}^{-}204\\ &= 123-204\\ &= {}^{-}81 \end{aligned}$$

(g) $^{-}11 + {}^{-}8 - {}^{-}15 + 12 - {}^{+}9$

$= {}^{-}11 - 8 + 15 + 12 - 9$

$= {}^{-}19 + 15 + 12 - 9$

$= {}^{-}4 + 12 - 9$

$= 8 - 9$

$= {}^{-}1$

or

$^{-}11 + {}^{-}8 - {}^{-}15 + 12 - {}^{+}9$

$= {}^{-}11 - 8 + 15 + 12 - 9$

$= (15 + 12) - (11 + 8 + 9)$

$= 27 - 28$

$= {}^{-}1$

(h) $^{-}234 + {}^{-}76 - {}^{-}127$

$= {}^{-}234 - 76 + 127$

$= {}^{-}310 + 127$

$= {}^{-}183$

3. Increase 3680m by 550m.

The new altitude of the plane is 4230m.

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ } \\ 3680 \\ +550 \\ \hline 4230 \end{array}$$

4. Balance = deposits – withdrawals
Balance = (500+36)-(137+67+247)
Balance = 536-451
Balance =$85
The balance at the end of the week is $85

5. (a)

$$\begin{array}{r} {\scriptstyle 4\ \ 12\ 12\ \ 8\ 11\ \ 6\ 14} \\ 15\not{3}\not{2}\not{9}\not{1}.\not{7}\not{4} \\ -12\,745.28 \\ \hline 2583 5.46 \end{array}$$

The superannuation increased by $25 835.46 from 2007 to 2008

(b) $151 875.28 – $153 291.74

Method 1.
Signs in front of both the numbers are different →subtract

$$\begin{array}{r} {\scriptstyle \ \ 2\ 12\ \ 8\ 11\ \ 6\ 14} \\ 15\not{3}\not{2}\not{9}\not{1}.\not{7}\not{4} \\ -15\,187 5.28 \\ \hline 1416.46 \end{array}$$

As the sign of the largest number is -, the answer is $^{-}1416.46$,
meaning a loss of $ 1 416.46

Method 2.
Using the logic of the situation, a loss has occurred. The size of the loss is the difference between the values, calculated above. Overall, a loss of $ 1 416.46

Multiplying and dividing

1. (a) 900 (b) 900 (c) 770 000
(d) 2000 (e) 4400 (f) 410 000
(g) 4500 (h) 300 000 (i) 1000

2. (a)

$$\begin{array}{r} 512 \\ \times 12 \\ \hline 1024 \\ +5120 \\ \hline 6144 \end{array}$$

(b) $7\overline{)5^{1}0^{5}2^{1}7^{1}4}$ = 7182

(c)

$$\begin{array}{r} {\scriptstyle 5\ 6\ } \\ 479 \\ \times 7 \\ \hline 3353 \end{array}$$

(d) $6\overline{)16^{4}86}$ $= 281$

281 smarties each

3. (a) $^{-}12 \times {}^{+}11 = {}^{-}132$

(b) $^{-}132 \times {}^{-}5$

$$\begin{array}{r} {}^{1\,1}132 \\ \times\ 5 \\ \hline 660 \end{array}$$

$^{-} \times {}^{-} = {}^{+}$

Answer 660

(c) $121 \div {}^{-}11 = {}^{-}11$

(d) $316 \times {}^{-}24$

$$\begin{array}{r} {}^{1}\not{2}16 \\ \times 24 \\ \hline 1264 \\ +6320 \\ \hline 7584 \end{array}$$

$^{+} \times {}^{-} = {}^{-}$

Answer $^{-}7584$

(e) $^{-}96 \div 6$

$6\overline{)9^{3}6}$ $= 16$

$^{-} \div {}^{+} = {}^{-}$

Answer $^{-}16$

(f) $^{-}3807 \div {}^{-}9$

$9\overline{)38^{2}0^{2}7}$ $= 423$

$^{-} \times {}^{-} = {}^{+}$

Answer 423

4. (a) Even number of -

$^{-}3 \times 4 \times {}^{-}5 \times 2$

$= 120$

(b) $^{+}40 \div {}^{-}5 \div {}^{-}2$

$= {}^{-}8 \div {}^{-}2$

$= 4$

(c) $^{-}36 \div 6 \times {}^{-}2 \div {}^{-}$

$= {}^{-}6 \times {}^{-}2 \div {}^{-}3$

$= 12 \div {}^{-}3$

$= {}^{-}4$

5. 12 codeine tablets will contain $12 \times 30 = 360$ mg

6. 16 lengths of 6m gives a total of 96m.
Amount left = 100 – 96 = 4m.

There will be 4m left on the roll.

$$\begin{array}{r} {}^{3}16 \\ \times 6 \\ \hline 96 \end{array} \qquad \begin{array}{r} {}^{0}\not{1}{}^{9}\not{0}{}^{10}\not{0} \\ -96 \\ \hline 4 \end{array}$$

7. Time per km?
Given 3 hrs 30 mins for (per) 42 km
210mins for 42km
5 mins per km or 5 min/km
The athlete averaged 5 min/km.

$42\overline{)210}$ $= 5$

8. How many 35km journeys in 630km?
John can do 18 journeys on a tank of fuel.

$35\overline{)63^{28}0}$ $= 18$

Order of operations

1. (a) $20 \div (3+1)$
$= 20 \div 4$
$= 5$

(b) $3 \times (4-6)$
$= 3 \times {}^{-}2$
$= {}^{-}6$

(c) $2(9-2\times3)$
$= 2(9-6)$
$= 2\times3$
$= 6$

(d) $20 \div 2 + (5-7)$
$= 20 \div 2 + {}^{-}2$
$= 10 - 2$
$= 8$

(e) $3\sqrt{16} + (6+9\div3)$
$= 3\sqrt{16} + (6+3)$
$= 3\sqrt{16} + 9$
$= 3\times4+9$
$= 12+9$
$= 21$

(f) $14 + (3^2 + \sqrt{16}) \times 3$
$= 14 + (9+4)\times3$
$= 14 + 13\times3$
$= 14 + 39$
$= 53$

(g) $(4-11)\times(9-7)$
$= {}^{-}7\times2$
$= {}^{-}14$

(h) ${}^{-}3(4-2\times8)$
$= {}^{-}3(4-16)$
$= {}^{-}3\times{}^{-}12$
$= {}^{+}36$ or 36

(i) ${}^{-}50 \div [4+(2\times3)]$
$= {}^{-}50 \div [4+6]$
$= {}^{-}50 \div 10$
$= {}^{-}5$

Rounding and estimation

1.

(a)	630	(b)	4200	(c)	4000
(d)	4190	(e)	0	(f)	24 000
(g)	2 000 000	(h)	2 380 000	(i)	496 000

2. (a) 27×412
$\begin{pmatrix}27\to30\\412\to400\end{pmatrix}$
$= 30\times400$
$= 12000$

(b) $1234+3099+971$
$\begin{pmatrix}1234\to1000\\3099\to3000\\971\to1000\end{pmatrix}$
$= 1000+3000+1000$
$= 5000$

(c) $2750 \div 48$
$\begin{pmatrix}2750\to3000\\48\to50\end{pmatrix}$
$= 3000 \div 50$
$= 60$

(d) $3678-1213$
$\begin{pmatrix}3678\to4000\\1213\to1000\end{pmatrix}$
$= 4000-1000$
$= 3000$

(e) $234+789-297$
$\begin{pmatrix}234\to200\\789\to800\\297\to300\end{pmatrix}$
$= 200+800-300$
$= 700$

(f) 212×350
$\begin{pmatrix}212\to200\\350\to400\end{pmatrix}$
$= 200\times400$
$= 80000$

(g) $2112+37\times51$
$\begin{pmatrix}2112\to2000\\37\to40\\51\to50\end{pmatrix}$
$= 2000+40\times50$
$= 2000+2000$
$= 4000$

(h) $37\times171\div375$
$\begin{pmatrix}37\to40\\171\to200\\375\to400\end{pmatrix}$
$= 40\times200\div400$
$= 8000\div400$
$= 20$

(i) $(6193+12\,190)\div39$
$\begin{pmatrix}6193\to6000\\12190\to10000\\39\to40\end{pmatrix}$
$= (6000+10000)\div40$
$= 16000\div40$
$= 400$

3. 157×23
$\begin{pmatrix}157\to200\\23\to20\end{pmatrix}$
$= 200\times20$
$= 4000$

Jane's answer is incorrect.

4. $8 \times 96 \times 12$

Number of milk cartons = $\begin{pmatrix} 8 \to 10 \\ 96 \to 100 \\ 12 \to 10 \end{pmatrix}$ Estimate is 10 000 milk cartons.

$= 10 \times 100 \times 10$

$= 10\,000$

5. Kilometres travelled = 47 + 57+ 34 + 55 + 92

$\text{Estimate} = 50 + 60 + 30 + 60 + 90$

$= 290$

The cyclist travelled about 290km during the week.